Hong Kong Physics Olympiad 2008
2008 年香港物理奧林匹克競賽

Written Examination
筆試

Jointly Organized by

Education Bureau
教育局
The Hong Kong Physical Society
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

May 25, 2008
2008 年 5 月 25 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for further announcement.

   選擇題的答題紙將於比賽開始後一小時三十分收回。完成選擇題後，你即可開始作答開放題。同學若果提早完成了選擇題後，即可爭取時間開始作答開放式問答題，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID number in the field of "I.D. No., and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of "Course number & Section No."

   請依照選擇題答題紙的指示，用HB鉛筆在選擇題答題紙的I.D. No.欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈**完全塗黑**，然後在適當的空格上填上你的英文姓名，最後於"Course & Section No. "欄內填上你的身分證號碼。

4. On the cover of the answer book, please write your Hong Kong ID number in the field of "Course Title", and write your English name in the field of "Student Name" and your 8-dight Participant ID number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 填上你的 8 位數字參賽號碼。 答題簿可雙面使用。

5. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.
   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用HB鉛筆**完全塗黑**。

6. The information provided in the text and in the figure of a question should be put to use together.
   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.
   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitational constant, 6.67×10^{-11} (N m^2/kg^2)
e – charge of an electron, -1.6×10^{-19} (A s)
ε_0 – electrostatic constant, 8.85×10^{-12} (A s)/(V m)
m_e – electron mass = 9.11×10^{-31} kg
c – speed of light in vacuum, 3.0×10^8 m/s
Radius of Earth = 6378 km
Sun-Earth distance (= 1 Astronomical Unit (AU)) = 1.5×10^{11} m
Earth-Moon distance = 3.84×10^8 m
Mass of the sun = 1.99×10^{30} kg
Density of water = 1.0×10^3 kg/m^3
Density of iron = 7.7×10^3 kg/m^3
Density of mercury = 13.6×10^3 kg/m^3
Speed of sound in air = 340 m/s

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 萬有引力常數, 6.67×10^{-11} (N m^2/kg^2)
e – 電子電荷, -1.6×10^{-19} (A s)
ε_0 – 靜電常數, 8.85×10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11×10^{-31} kg
c – 真空光速, 3.0×10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 (= 1 天文單位) = 1.5×10^{11} m
地球-月球距離 = 3.84×10^8 m
太陽質量 = 1.99×10^{30} kg
水的密度 = 1.0×10^3 kg/m^3
鐵的密度 = 7.7×10^3 kg/m^3
水銀的密度= 13.6×10^3 kg/m^3
空氣中聲速= 340 m/s

The following conditions will be applied unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

除非特別注明，否則下列條件將適用於本卷所有問題：

1) 所有物體都處於地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題（每道題 2 分，每道題選擇一個答案）

The MC questions with the '*' sign may require information on page-3.

帶 * 的選擇題可能需要用到第三頁上的資料。

MC1

An object in constant acceleration motion with zero initial velocity covers 1 meter within the 1st second. How many meters will it cover within the 4th second?

一物體作初速度爲零的勻加速運動，在第一秒鐘內走過的距離爲 1 米。問物體在第四秒鐘內走過的距離爲多少米？

(a) 7 (b) 1 (c) 9 (d) 3 (e) 16

MC2

Spaceship A moves at velocity $40(\vec{x}_0 + \vec{y}_0)$ km/s, while spaceship B moves at $30\ (\vec{x}_0 - \vec{y}_0)$ km/s. Find the relative velocity in the unit of km/s between the two ships.

飛船-A 以 $40(\vec{x}_0 + \vec{y}_0)$每秒公里的速度行駛，飛船-B 以 $30\ (\vec{x}_0 - \vec{y}_0)$每秒公里的速度行駛。求兩飛船之間的相對速度（以每秒公里爲單位）。

(a) $10\vec{x}_0 + 10\vec{y}_0$ (b) $10\vec{x}_0 + 70\vec{y}_0$ (c) $70\vec{x}_0 + 10\vec{y}_0$ (d) $10\vec{x}_0 + 10\vec{z}_0$

(e) $10\vec{x}_0 - 10\vec{y}_0$

MC3*

Jupiter revolves around the sun at 11 years and 315 days per revolution. Find the distance between Jupiter and Sun **in terms of astronomical unit (A. U.).**

木星每 11 年 315 天繞太陽運行一圈。求木星與太陽間的距離（**以天文單位表達**）。

(a) 5.2 (b) 1.0 (c) 11.9 (d) 15 (e) 3.0

MC4

An object is in motion with constant acceleration. Which of the following statement is correct?

(a) The object must move in a straight line.
(b) The distance traveled must be proportional to the square of the time.
(c) The speed of the object must increase uniformly with time.
(d) The velocity must be constant.
(e) The total external force must do work to the object.

一物體作勻加速運動。以下哪句論述是對的？

(a) 該物體必須作直線運動。
(b) 該物體運動的距離必須與時間平方成正比。
(c) 該物體的速率必須是隨時間均勻增加的。
(d) 該物體的速度必須是常數。
(e) 合外力必須對物體作功。

MC5

An object is placed on the left side of a positive lens and the real image is formed on the right side of the lens. If the object is moved slightly to the right, the image will _____.

(a) not move (b) move to the left (c) move to the right

(d) move up (e) move down

一物體放在一正透鏡的左邊，它的實像在透鏡的右邊。若將物體向右移動一點，則像會_____。

(a) 不動 (b) 向左移動 (c) 向右移動

(d) 向上移動 (e) 向下移動

MC6

Same initial condition as MC5 above. If the upper half of the lens is covered with black cloth, the image will _____ .

(a) totally disappear (b) move up (c) move down
(d) become brighter (e) become dimmer

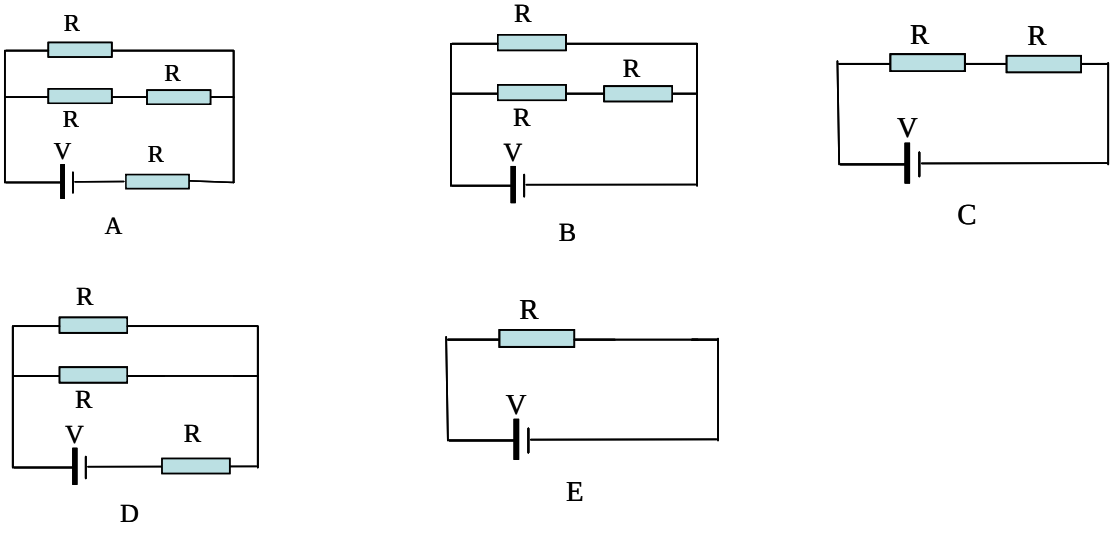
初始情況與 MC5 相同。若將透鏡上半部用黑布遮住，則像會_____。

(a) 完全消失 (b) 向上移動 (c) 向下移動 (d) 亮一點
(e) 暗一點

MC7

Five circuits are shown below. The batteries all have the same voltage *V* and all resistors have the same resistance *R*. In which circuit does the battery output the most power?

在下列電路圖中，所有電池有相同的電壓 *V*，所有電阻器有相同的電阻值 *R*。問哪個電路中的電池輸出功率最大？



(a) A (b) B (c) C (d) D (e) E

MC8

A 20 μF capacitor charged to 2.0 kV and a 40 μF capacitor charged to 4.0 kV are connected to each other, with the positive plate connected to the positive plate, and the negative plate to the negative plate. What is the final charge on the 20 μF capacitor?

一個 20 μF 的電容充電至 2.0 kV 的電壓，一個 40 μF 的電容充電至 4.0 kV 的電壓。現將兩電容的正極與正極相連，負極與負極相連，問在 20 μF 的電容上有多少電荷?

(a) 50 mC (b) 200 mC (c) 40 mC (d) 67 mC (e) 120 mC

MC9

A cylinder is filled with ideal gas with pressure *P*. The gas is then heated up by an external source and has doubled its volume and temperature, while half the gas has leaked out. Find the pressure of the remaining gas.

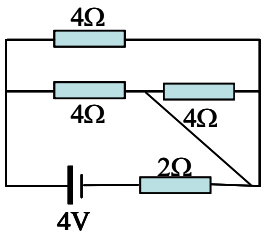
一汽缸充滿理想氣體，初始壓強爲 *P*。一外部熱源對氣體加熱，氣體的體積和溫度都增加到原來的兩倍，但有一半的氣體漏掉了。求剩下氣體的壓強。

(a) 2*P* (b) 4*P* (c) *P* (d) 0.5*P* (e) 0.25*P*

MC10

In the circuit, the electric current through the battery is _____ Amperes.

在電路中，流經電池的電流爲_____ 安培。



(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

MC11

Which of the process below DOES NOT involve energy transfer?

(a) Lifting a cup (b) Turning on a lamp (c) Burning some wood
d) Motion of a simple pendulum (e) Moon revolving around Earth in circular motion

以下哪個過程<u>沒有</u>能量轉換？

(a) 拿起個杯子 (b) 開亮盞燈 (c) 燒木柴
d) 單擺運動 (e) 月亮繞地球作圓周運動

MC12*

Ignore friction between water and pipe wall. Water is pumped from a reservoir up to a height of 10 m at the rate of 30 kg per second through a pipe of 2.0×10^{-3} m^2 in cross section area. Find the pumping power.

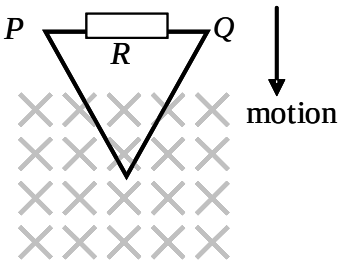
不考慮水與管壁的摩擦力。抽水機經一橫截面面積爲 2.0×10^{-3}平方米的水管將水庫的水以每秒 30 公斤的速率抽上 10 米的高處。求抽水機的功率。

(a) 6315 (b) 2940 (c) 3573 (d) 1305 (e) 4960

MC13

As shown, a triangular coil is moving into a uniform magnetic field region (field pointing into the paper) at uniform velocity. Which of the following statements is/are correct?

(1) An induced current flows from *P* to *Q* through resistor *R*.
(2) The magnetic flux passing through the coil is increasing at a uniform rate.
(3) The induced emf is increasing at a uniform rate



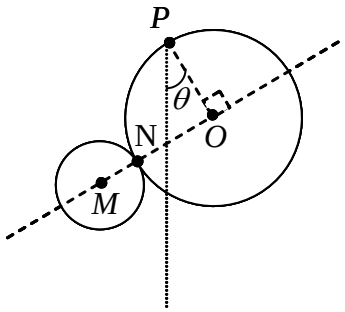
如圖，一個三角型線圈以匀速進入一均匀磁場區域（磁場指向紙面）。問下列哪些論述是正確的？

(1) 感應電流經電阻 *R* 由 *P* 流向 *Q* 。
(2) 穿過線圈的磁通量以匀速增加。
(3) 感應電動勢以匀速增加。

(a) (1) (b) (3) (c) (1) and (2) (d) (2) and (3)
(e) (1) and (3)

MC14

Two uniform circular discs of the same material and thickness are adhered together at point-N. Both discs lie in the same vertical plane. The rigid body is hinged at point-*P* and it can rotate freely about *P* in the vertical plane. $PO \perp OM$ and $ON = 2\,NM$, where point-*O* and point-*M* are the centres of the discs. When static equilibrium is reached, find the angle θ between *PO* and the vertical direction.

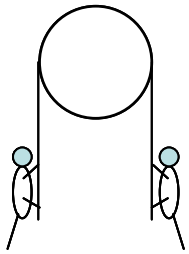


兩個同樣厚度的均匀圓盤，由同樣物質組成，在 N 點粘在一起，組成一剛體。盤面處於垂直面，大圓盤吊在 P 點，整個剛體可繞 P 點自由轉動。*O* 和 *M* 分別是兩圓盤的中心，$PO \perp OM$ ，$ON = 2\,NM$。求當平衡時 *PO* 與垂直方向的夾角θ 。

(a) 16.7^o (b) 26.6^o (c) 30^o (d) 36.9^o (e) 11.7^o

MC15

Two monkeys of the same weight are holding tightly the two ends of a rope with negligible mass. The rope passes through a smooth pulley, as shown in the figure. The two monkeys are initially at rest. Now the left monkey starts to climbs up the rope at an average speed *v* relative to the pulley. Find its average speed relative to the right monkey.

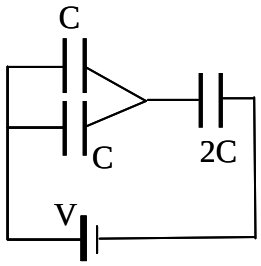


如圖所示，兩隻重量同樣的猴子緊緊抓住無重量細繩的兩端，細繩掛在光滑的滑輪上。現在左邊的猴子開始以相對於滑輪爲 *v* 的平均速度向上爬，求左邊猴子相對於右邊猴子的平均速度。

(a) 0 (b) *v* upwards (c) 2*v* upwards (d) *v* downwards
(e) 2*v* downwards

MC16

As shown, find the charge on one of the capacitors with capacitance *C*.



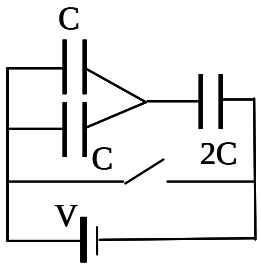
如圖，求其中一個電容值爲 *C* 的電容上的電荷。

(a) $2CV$ (b) CV (c) $\frac{1}{2}CV$ (d) $\frac{1}{4}CV$

(e) $\frac{1}{8}CV$

MC17

Same initial condition as MC16. If the battery is suddenly taken away and the switch shown in the circuit diagram is closed, find the total charge that will pass through the switch.



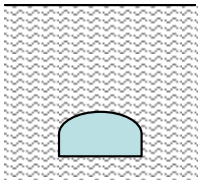
初始情況與 MC16 相同。若突然拿走電池，併且合上電路的開關，求將流過開關的總電荷。

(a) $2CV$ (b) CV (c) $\frac{1}{2}CV$ (d) $\frac{1}{4}CV$

(e) $\frac{1}{8}CV$

MC18*

An object of volume *V* is submerged in a liquid of density ρ. It has a flat bottom of area *A* which is at a depth of *H* in the liquid, and a dome shaped top. Find the total force of liquid acting on the top of the object.



(a) $|\rho HAg - \rho Vg|$ and upward (b) $|\rho HAg - \rho Vg|$ and downward

(c) ρVg and downward (d) ρVg and upward

(e) ρHAg and upward

一體積爲 *V* 的物體完全浸沒在密度爲ρ的液體裏。物體有一圓的拱頂和平的底部，底部處於的深度爲 *H*，底部面積爲 *A*。求液體對拱頂的合力。

(a) $|\rho HAg - \rho Vg|$ 向上 (b) $|\rho HAg - \rho Vg|$ 向下 (c) ρVg 向下

(d) ρVg 向上 (e) ρHAg 向上

MC19

An AC generator consists of 6 turns of rectangular wire coil. Each turn has an area of 0.040 m^2. The coil rotates in a uniform field (B = 0.20 T) at a constant frequency of 50 Hz. Find the maximum induced electro-motive force.

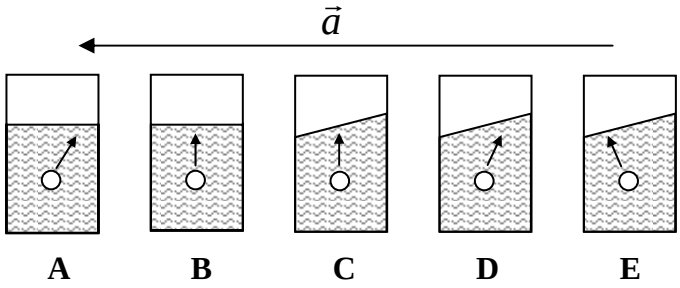
一交流發電機有 6 圈的長方型線圈，每圈面積爲 0.040 平方米。線圈在 0.20 T 的均勻磁場中以每秒 50 周的頻率轉動。 求最大感應電動勢。

(a) 4.8 V (b) 2.5 V (c) 3.0 V (d) 15 V (e) 7.5 V

MC20

A cup of water is placed in a car under constant acceleration to the left, as shown. Inside the water is a small air bubble. The following figures show five situations of the shape of the water surface and the direction of motion of the bubble as indicated by the arrow on the bubble. Which one is correct?

在一輛作匀加速運動的車裏有一杯水，水裏有一個小氣泡。下圖展現了五個水面形狀和以小箭頭表示的氣泡運動方向的情形。哪個是正確的？



(a) A (b) B (c) C (d) D (e) E

《End of MC's 選擇題完》

Open Problems 開放題

Total 5 problems 共 5 題

The Open Problem(s) with the '*' sign may require information on page-3.
帶 * 的開放題可能需要用到第三頁上的資料。

Q1* (10 points)

A particle of mass m carrying charge q is placed mid-way between two fixed point charges, each carrying charge Q. All three particles are on the x-axis. The distance between the fixed charges is $2d$. The particle is confined to move along the x-axis only.

(a) Determine whether the equilibrium position of the particle is stable when q and Q are of the same sign or of opposite sign. (3 points)

(b) In the case of stable equilibrium, find the vibration frequency of small amplitude oscillation of the particle. (7 points)

(Hint: for $x << 1$, $(1+ax+bx^2)^n \approx (1+ax)^n \approx 1+nax$, where n can be a fraction number or an integer, and a and b are constants.)

題 1*（10 分）

一質量爲 m 帶電爲 q 的粒子放在兩個帶電爲 Q 的固定點電荷的正中間。這三個點電荷都在 x-軸上。兩固定點電荷之間的距離爲 $2d$。 粒子只能在 x-軸上作一維運動。

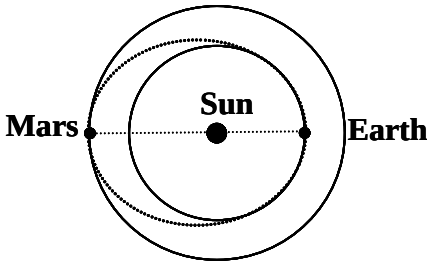
(a) q 和 Q 可爲同號或反號，問哪種情況下粒子的平衡是穩定的？（3 分）

(b) 在粒子的平衡是穩定的情況下，求粒子作小幅振動的頻率。（7 分）

(提示: 當 $x << 1$, $(1+ax+bx^2)^n \approx (1+ax)^n \approx 1+nax$, 其中 n 可以是分數或整數， a 和 b 爲常數。)

Q2* (12 points)

(a) The distance between Mars and Sun is 2.28×10^{11} meters. Find the period of revolution of Mars (in the unit of days) around the sun. (2 points)

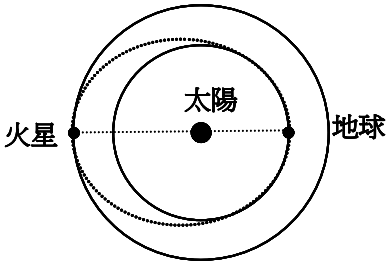


(b) To launch a spaceship from Earth to Mars, the spaceship should follow an elliptical orbit with Earth on one end of the long axis and Mars on the other end. The sun is also on the long axis, as shown. The time it takes to fly half the elliptical orbit is 259 days. Where should Mars be relative to Earth when the spaceship is launched? Express the position of Mars in terms of the angle between the line joining Earth and Sun and the line joining Mars and Sun. (5 points)

(c) To come back from Mars the spaceship will follow the same elliptical orbit and take the same amount of days as the first journey. How many days should the spaceship stay on Mars before it is launched again on its way back to Earth? (5 points)

題 2*（12 分）

(a) 火星離太陽的距離爲 2.28×10^{11}米。求火星繞太陽公轉一周的時間（以天爲單位）。（2 分）

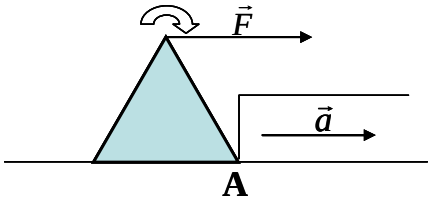


(b) 從地球向火星發射的飛船須沿如圖所示的橢圓軌道運行。地球和火星必須分別處於橢圓長軸的兩端。太陽也在長軸上。飛船運行半個橢圓軌道需時 259 天。若以地球與太陽和火星與太陽連線的夾角來描述地球與火星的相對位置，問應該在什麼相對位置時發射飛船？（5 分）

(c) 回程的飛船軌道也是同樣的橢圓，需時也相同。爲了回到地球，飛船要在火星等多久？（5 分）

Q3* (8 points)

As shown, a uniform block of mass *m* and with equal lateral triangle cross section is placed at the edge of a step on a car floor which is under constant acceleration *a* (< *g*) to the right. A horizontal force on the apex of the triangle $\vec{F}$ is large enough to just tilt the block.



(a) Find the magnitude of $\vec{F}$. (3 points)

(b) Find the force at point-A where the block is in contact with the step edge. (5 points)

題 3*（8 分）

如圖所示，一質量爲 *m* 橫截面爲等邊三角型的均勻物塊放在一汽車地板上的台階邊，汽車以勻加速 *a* (< *g*)向右行駛。一水平力 $\vec{F}$ 作用於三角型物塊的頂上，剛好可以使物塊翻轉。

(a) 求 $\vec{F}$ 的大小。（3 分）

(b) 求台階邊與物塊接觸的 A 點處的作用力。（5 分）

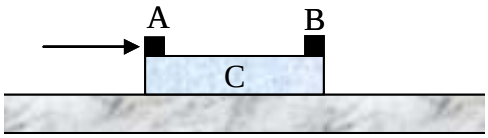
Q4* (14 points)

As shown, a block-C of mass M_C and length L is placed on a smooth horizontal floor. On its two ends are two small blocks A and B with masses of M_A and M_B, respectively. The friction coefficient between block-A and block-C is μ_A, that between block-B and block-C is μ_B. The three objects are originally at rest. Block-A is then given a short impulse I, and starts to move to the right.

(a) Find the condition that block-B will remain rest relative to block-C. (4 points)

(b) Assuming that block-B remains rest relative to block-C, find the minimum impulse I_{min} such that block-A can reach block-B. (5 points)

(c) Assume $I > I_{min}$. Block-A and block-B then collide elastically. Find the condition that block-A's velocity relative to block-C is reversed after the collision. (5 points)



題 4*（14 分）

如圖所示，一質量爲M_C 長度爲L的物塊-C放在光滑的水平面上。在它的兩端有兩個小物塊A和B。物塊-A的質量爲M_A，與物塊-C的磨擦係數爲 μ_A。物塊-B的質量爲M_B，與物塊-C的磨擦係數爲 μ_B。三物體原處於靜止狀態。物塊-A突然受到一短促衝量I，於是開始向右運動。

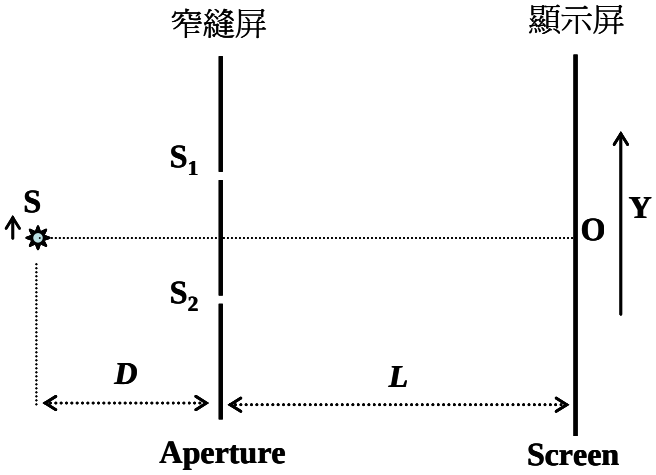
(a) 求物塊-B 與物塊-C 保持相對靜止的條件。（4 分）

(b) 設物塊-B與物塊-C可以保持相對靜止，求能使物塊-A到達物塊-C另一端的最小衝量I_{min}。（5 分）

(c) 設$I > I_{min}$。物塊-A與物塊-B作完全彈性碰撞，求碰撞後能使物塊-A相對於物塊-C的速度反向的條件。（5 分）

Q5 (16 points)

Shown in the drawing is a typical Young's interference experiment setup (not drawn to the true scale, though). A point light source S emitting monochromatic light waves of wavelength λ is placed at equal distance to two narrow slits S_1 and S_2 of width a on an aperture plane. The distance between S and the aperture plane is D. A display screen is parallel to the aperture plane and is at a distance L from it. D and $L >> \lambda$. A Y-coordinate is imprinted upright on the screen, its origin (Y = 0) being at point-O. The line joining S and O is the central axis of the setup. It is perpendicular to the aperture plane, and passes through the aperture plane at the middle point between the two slits, which are separated by a distance d (<< D and L). The light intensity on the screen can be expressed as $I(Y)/I_{max} = f(Y)$, where the maximum value of $f(Y)$ is 1. (Hint: The total intensity due to the interference of two waves of equal intensity I_0 is given by $I = 2I_0(1+\cos\Delta)$, where Δ is the phase difference between the two waves.)



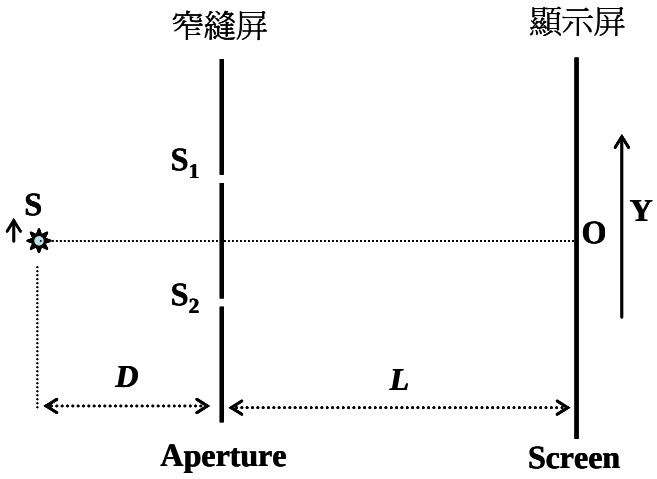
(a) When $a << \lambda$, the amplitude E of the light wave emitted from the slits is constant in all propagation directions. Find $f(Y)$. (4 points)

(b) Now the light source S is moved upwards by a small distance b (<< D) as the small arrow indicates, find $f(Y)$. (6 points)

(c) The light source is moved back to the central axis. When a is comparable to λ, the amplitude E of the light wave emitted from the slits is no longer constant. It is instead given by $E(\theta) = \frac{\sin(\beta)}{\beta}$, where $\beta \equiv \frac{\pi a}{\lambda}\theta$, and θ (<< 1) is the angle between the emitting (propagating) direction and the central axis. Find $f(Y)$ and the position on the screen where the light intensity is maximum. (6 points) (Hint: $\frac{\sin(x)}{x} \to 1$ when $x \to 0$.)

題 5（16 分）

圖中不按比例所畫的是楊氏干涉實驗示意圖。一個點光源 S 發出波長爲λ的單色光，放在離兩個寬度爲a的在窄縫屏上的S_1和S_2 窄縫相等距離的位置，光源離窄縫屏的距離爲D。距離窄縫屏L 處有一個與窄縫屏平行的顯示屏。D和L >> λ。顯示屏上印有豎直的 Y-坐標，其原點(Y = 0) 在O點。S和O的連線是整個實驗裝置的中心軸，與窄縫屏垂直，並經過S_1 和 S_2的中點。S_1 和 S_2之間的距離爲 d (<< D 和 L)。顯示屏上的光強可表達爲 $I(Y)/I_{\max} = f(Y)$，其中 $f(Y)$的最大值爲 1。



（提示：兩個強度均爲I_0的波干涉所產生的總光強爲 $I = 2I_0(1+\cos\Delta)$，其中Δ爲兩波之間的位相差。）

(a) 當 a << λ 時，由窄縫發出的光波的振幅 E 在各個傳播方向都是一樣的，求 $f(Y)$。（4 分）

(b) 現將 S 如箭頭所示向上移一小距離 b (<< D)，求 $f(Y)$。（6 分）

(c) 將 S 移回到中心軸。當 a 和λ 差不多大小時，由窄縫發出的光波的振幅 E 在各個傳播方向是不同的。取而代之的是 $E(\theta) = \frac{\sin(\beta)}{\beta}$, 其中 $\beta \equiv \frac{\pi a}{\lambda}\theta$, θ (<< 1) 是光傳播方向與中心軸的夾角。 求 $f(Y)$，以及顯示屏上的光強最大的位置。（6 分）

(提示: 當 $x \to 0$ ， $\frac{\sin(x)}{x} \to 1$ 。)

《END 完》

Solutions for Multiple Choice Questions

1a 2b 3a 4e 5c 6e 7b 8d 9d 10a 11e 12a 13b 14a 15a 16c 17b 18b 19d 20e

Solutions for Open Questions

Q1

(a) Suppose the charge q's position is r to the midpoint of two Qs.

$$F_1 = \frac{kqQ}{(d+r)^2}, \qquad F_2 = \frac{kqQ}{(d-r)^2}$$

The total force on q is $F_{total} = F_1 - F_2 = -kqQ[\frac{1}{(d-r)^2} - \frac{1}{(d+r)^2}]$. Obviously, the factor $\frac{1}{(d-r)^2} - \frac{1}{(d+r)^2}$ is positive. When q and Q are of the same sign, F is negative, so its effect is to push the charge q back to the midpoint. Therefore q is stable at the midpoint.

On the contrary, when q and Q are of opposite sign, a small deviation from the midpoint will increase the in-balance of the two forces, so q is not stable at the midpoint.

(b) $F = -kqQ[\frac{1}{(d-r)^2} - \frac{1}{(d+r)^2}] = -kqQ\frac{4dr}{(d^2-r^2)^2} = -\frac{4kqQr}{d^3}$

so $\omega^2 = \frac{4kQq}{md^3}$

Q2

(a) The gravity force of the sun provide the acceleration for Mars' circular motion

$$\frac{Gm_M M_S}{r^2} = m_M \omega^2 r = m_M r\left(\frac{2\pi}{T}\right)^2$$

$$\Rightarrow T = \sqrt{\frac{4\pi^2 r^3}{GM_S}} = \sqrt{\frac{4\pi^2(2.28\times10^{11})^3}{6.67\times10^{-11}\times1.99\times10^{30}}} = 5.93 \times 10^7 \text{ s} = 686 \text{ days}$$

(b) Within 259 days, the angular displacement of Mars is $\frac{259}{686}\times360^\circ = 136^\circ$, so Mars should be ahead of Earth by 180º - 136º = 44º.

(c) When the spaceship arrives in Mars, Earth has covered angular distance $\frac{259}{365}\times360^\circ = 255^\circ$. Earth is now AHEAD of Mars by 255º -180º = 75º. When the spaceship is launched again in Mars, Earth will cover the same angular distance. Therefore, Earth should be BEHIND Mars by 255º -180º = 75º, or ahead of Mars by 360º - 75º = 285º. To wait for that to happen, and noted that

the angular speeds of Earth is $\omega_E = \frac{360}{365}$ degree/day and that of Mars is $\omega_M = \frac{360}{686}$ degree/day, the spaceship should stay on Mars for

$$\frac{(360-75)-75}{\frac{360}{365}-\frac{360}{686}} = \frac{365\times 210\times 686}{360\times(686-365)} = 455 \text{ days.}$$

Q3

Using point-A as the pivotal point, the total torque is

$$\tau = mg\cdot\frac{\sqrt{3}}{4}d + ma\cdot\frac{\sqrt{3}}{4}d - F\cdot\frac{\sqrt{3}}{2}d = 0$$

$$\Rightarrow F = \frac{m}{2}(g+a)$$

Use the force equilibrium equation then.

In the vertical direction, $f_v = mg$

In the horizontal plane, $f_h + ma = F = \frac{m}{2}(g+a) \Rightarrow f_h = \frac{m}{2}(g-a)$

The total force at point A includes f_v and f_h.

Q4

(a) The friction force of block-A on Block-C is $\mu_A m_A g$. The acceleration of C and B is then $\frac{\mu_A m_A g}{(m_B + m_C)}$

The condition for B to stay with C is then $\mu_B \geq \frac{\mu_A m_A}{(m_B + m_C)}$.

(b) When block-A reaches the end of Block-C while its velocity becomes the same as C&B, the speed of the system A&B&C is $v_f = \frac{I}{(m_A + m_B + m_C)}$. By energy conservation,

$\frac{I^2}{2m_A} - \mu_A m_A g L \geq \frac{I^2}{2(m_A + m_B + m_C)}$, or $I \geq \sqrt{\frac{2\mu_A m_A^2 g L}{(m_B + m_C)}(m_A + m_B + m_C)}$

(c) Suppose before the collision the speed of A is V_A, and that of B and C is V_B. During the collision process

$$m_A V_A + m_B V_B = m_A V_A{}' + m_B V_B{}'$$

$$m_A V_A{}^2 + m_B V_B{}^2 = m_A V_A{}'^2 + m_B V_B{}'^2$$

From them we have

$$V_A{}' = \frac{V_A(m_A - m_B) + 2m_B V_B}{m_A + m_B}$$

$$V_B{}' = \frac{V_B(m_A - m_B) + 2m_A V_A}{m_A + m_B}$$

The condition for the relative speed of A to C is reversed is $V_A' < V_B$, that is, $V_A' = \frac{V_A(m_A - m_B) + 2m_B V_B}{m_A + m_B} < V_B \Rightarrow V_A(m_A - m_B) < V_B(m_A - m_B)$, since A collides with B means that $V_A > V_B$, the condition is $m_A < m_B$.

Alternatively, in the instantaneous reference where C is at rest, B is initially at rest. So $m_A V_A = m_A V_A' + m_B V_B'$ and $m_A V_A^2 = m_A V_A'^2 + m_B V_B'^2$. Then $V_A' = \frac{(m_A - m_B)}{m_A + m_B} V_A$ and $V_B' = \frac{2m_A}{m_A + m_B} V_A$. So $m_A < m_B$ will make V_A' negative.

Q5

(a) The path difference is $\delta = \sqrt{L^2 + (Y + d/2)^2} - \sqrt{L^2 + (Y - d/2)^2}$. As $d << L$, only the linear terms of d should be kept.

$$\delta = L\sqrt{1 + (\frac{Y}{L} + \frac{d}{2L})^2} - L\sqrt{1 + (\frac{Y}{L} - \frac{d}{2L})^2} = L\sqrt{1 + \left(\frac{Y}{L}\right)^2 + \frac{dY}{L}} - L\sqrt{1 + \left(\frac{Y}{L}\right)^2 - \frac{dY}{L}}$$

$$= L(1 + \left(\frac{Y}{L}\right)^2 + \frac{dY}{2L^2}) - L(1 + \left(\frac{Y}{L}\right)^2 - \frac{dY}{2L^2}) = \frac{dY}{L}.$$

The answer indicates that the pass difference is given by $\delta = d\theta$, where $\theta = \frac{Y}{L}$ is the angle between the emission direction and the central axis.

Using the hint given, we get $f(Y) = \frac{1}{2}\left(1 + \cos\left(\frac{2\pi dY}{\lambda L}\right)\right)$.

(b) Now there is also a path difference for light from the source to the two slits. Using the same calculation procedure as in (a), such path difference is $\delta_1 = \frac{bd}{D}$, and the total path difference is $\delta = \frac{bd}{D} + \frac{dY}{L}$. Finally $f(Y) = \frac{1}{2}\left(1 + \cos\left(\frac{2\pi dY}{\lambda L} + \frac{2\pi db}{\lambda D}\right)\right)$, and the interference fringes shift by a distance $Y_0 = \frac{Lb}{D}$.

(c) Following (a), the emission direction angle is given by $\theta = \frac{Y}{L}$, the amplitude is now direction dependent, so $f(Y) = \frac{1}{2}\left(1 + \cos\left(\frac{2\pi dY}{\lambda L}\right)\right)\frac{\sin(\beta)}{\beta}$, where $\beta \equiv \frac{\pi a}{\lambda}\frac{Y}{L}$. The maximum occurs at $Y = 0$ where $f(Y) = 1$.