Hong Kong Physics Olympiad 2009
2009 年香港物理奧林匹克競賽

Written Examination
筆試

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院
The Education Bureau of HKSAR
香港特區政府教育局
The Hong Kong Physical Society
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

May 24, 2009
2009 年 5 月 24 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for further announcement.

   選擇題的答題紙將于比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID number in the field of "I.D. No., and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of "Course number & Section No."

   請依照選擇題答題紙的指示，用HB鉛筆在選擇題答題紙的I.D. No.欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈**完全塗黑**，然後在適當的空格上填上你的英文姓名，最後於"Course & Section No. "欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.
   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用HB鉛筆**完全塗黑**。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of "Course Title", and write your English name in the field of "Student Name" and your 8-dight Participant ID number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.
   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.
   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitational constant, 6.67×10^{-11} (N m^2/kg^2)
e – charge of an electron, -1.6×10^{-19} (A s)
ε_0 – electrostatic constant, 8.85×10^{-12} (A s)/(V m)
m_e – electron mass = 9.11×10^{-31} kg
c – speed of light in vacuum, 3.0×10^8 m/s
Radius of Earth = 6378 km
Sun-Earth distance (= 1 Astronomical Unit (AU)) = 1.5×10^{11} m
Earth-Moon distance = 3.84×10^8 m
Mass of the sun = 1.99×10^{30} kg
Density of water = 1.0×10^3 kg/m^3
Standard atmosphere pressure $p_0 = 1.0 \times 10^5$ N/m^2

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 萬有引力常數, 6.67×10^{-11} (N m^2/kg^2)
e – 電子電荷, -1.6×10^{-19} (A s)
ε_0 – 靜電常數, 8.85×10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11×10^{-31} kg
c – 真空光速, 3.0×10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 (= 1 天文單位) = 1.5×10^{11} m
地球-月球距離 = 3.84×10^8 m
太陽質量 = 1.99×10^{30} kg
水的密度 = 1.0×10^3 kg/m^3
標準大氣壓 $p_0 = 1.0 \times 10^5$ N/m^2

The following conditions will be applied to all questions unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

除非特別注明，否則下列條件將適用于本卷所有問題：

1) 所有物體都處于地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題（每道題 2 分，每道題選擇一個答案）

The MC questions with the '*' sign may require information on page-3.

帶 * 的選擇題可能需要用到第三頁上的資料。

MC1

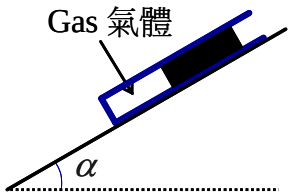
Spaceship-A moves at velocity 40($\vec{x}_0 + \vec{y}_0$) km/s relative to a fixed coordinate system. Spaceship-B moves at 30 ($\vec{x}_0 - \vec{y}_0$) km/s as seen by Spaceship-A. Find the velocity of Spaceship-B in the fixed coordinate system in the unit of km/s.

飛船-A 以 40($\vec{x}_0 + \vec{y}_0$)公里/秒的速度相對于一固定坐標行駛，飛船-A 看到飛船-B 以 30 ($\vec{x}_0 - \vec{y}_0$)公里/秒的速度行駛。求飛船-B 相對于固定坐標的速度（以公里/秒爲單位）。

(a) $10\vec{x}_0 + 10\vec{y}_0$ (b) $10\vec{x}_0 + 70\vec{y}_0$ (c) $10\vec{x}_0 + 10\vec{z}_0$ (d) $70\vec{x}_0 + 10\vec{y}_0$
(e) $10\vec{x}_0 - 10\vec{y}_0$

MC2*

A light glass tube with a sealed lower end and cross section area S = 2.5 cm^2 contains a column of mercury of mass m = 2 kg. Between the mercury and the lower end is some trapped gas. The glass tube is sliding down a slope with an inclining angle α = 30°. The dynamic friction coefficient between the tube and the slope is $\mu = \sqrt{3}/6$. Find the pressure of the trapped gas in terms of the standard atmosphere pressure p_0.



下端封閉、橫截面積 S = 2.5 cm^2 的輕玻璃管內裝有質量 m = 2 kg 的水銀柱。一些氣體被封閉在水銀柱與下端之間。玻璃管沿傾角α = 30°的斜面下滑，玻璃管與斜面間的滑動摩擦系數爲 $\mu = \sqrt{3}/6$。求被封閉氣體的壓強（以標準大氣壓 p_0 爲單位）。

(a) 1.7 (b) 1.2 (c) 1.9 (d) 2.3 (e) 16

MC3*

Saturn revolves around the sun at 29.5 years per revolution. Find the distance between Saturn and Sun **in terms of astronomical unit (A. U.).**

土星每 29.5 年繞太陽運行一圈。求土星與太陽間的距離（**以天文單位表達**）。

(a) 5.2 (b) 9.5 (c) 11.9 (d) 15 (e) 3.0

MC4

The pixel size of a digital camera is 5.0 × 10^{-3} mm. The focal length of the lens is 10 mm. The image plate from the lens is 15 mm. Find the minimum size of an object the camera can resolve.

一數碼相機的像素尺寸爲 5.0 × 10^{-3} mm，相機鏡頭的焦距爲 10 mm，像面離鏡頭 15 mm。求相機能分辨的最小物體。

(a) 5.0 × 10^{-3} mm (b) 1.5 × 10^{-3} mm (c) 1.5 × 10^{-2} mm
(d) 2.5 × 10^{-3} mm (e) 1.0 × 10^{-2} mm

MC5

Following MC4, if the diameter of the camera lens is doubled, what will be the increase of light intensity on the image plate?

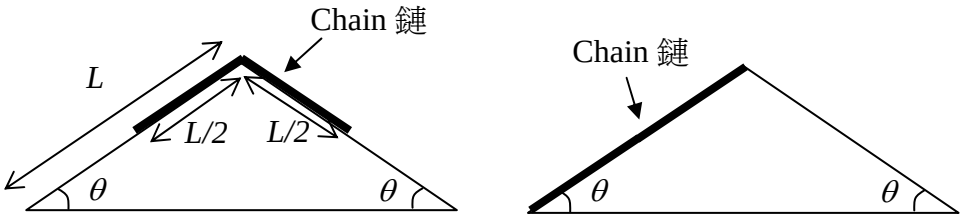
(a) 4 times (b) 2 times (c) 1.5 times (d) ½ times (e) ¼ times

接 MC4。若相機鏡頭的直徑加倍，則像面上的光强爲原來的_____。

(a) 4 倍 (b) 2 倍 (c) 1.5 倍 (d) ½ (e) ¼

MC6

A uniform chain of mass m and length L is originally placed mid-way on the top of a fixed smooth double-sided wedge (Figure-A). The length of each side of the wedge is L. It is then given a slight push. Find the kinetic energy of the chain when the whole chain has just slid to the left side of the wedge (Figure-B).



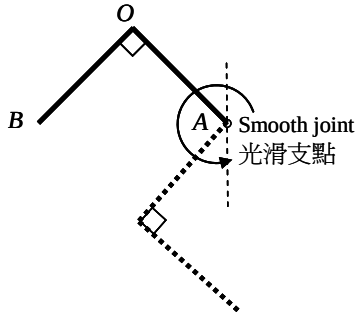
一質量爲 m 長度爲 L 的均勻鏈條， 原來放在一邊長爲 L 的固定的光滑等邊雙斜面上（圖-A）。現將鏈條輕輕推一下，求當鏈條剛好全部滑到左斜面時（圖-B）的動能。

(a) $mgL\sin\theta$ (b) $\frac{mgL\sin\theta}{2}$ (c) $\frac{mgL\sin\theta}{4}$ (d) $\frac{mgL\sin\theta}{8}$

(e) $2mgL\sin\theta$

MC7

A uniform "L"-shaped rigid body AOB of mass m, where $AO = OB = l$ and $\angle AOB$ is a right angle, is hinged to a smooth joint at Point A of the body and can swing freely in a vertical plane. Initially, the body is released from rest with AB horizontal. Find the ***maximum kinetic energy*** of the body.



一質量爲 m 的均勻"L"-形剛體 AOB，$AO = OB = l$，角 $\angle AOB$ 爲直角。剛體的一端（點-A）挂在光滑支點上，剛體可在垂直面內自由擺動。剛體處于初始位置時 AB 連綫是水平的。求將剛體從該位置放開後剛體可得的***最大動能***。

(a) $\left(\frac{\sqrt{2}+\sqrt{10}}{4}\right)mgl$ (b) $\left(\frac{\sqrt{2}+\sqrt{5}}{2}\right)mgl$ (c) $\left(\frac{\sqrt{3}+\sqrt{10}}{4}\right)mgl$

(d) $\left(\frac{\sqrt{3}+\sqrt{5}}{2}\right)mgl$ (e) $\left(\frac{\sqrt{2}}{2}\right)mgl$

MC8

A 20 μF capacitor charged to 2.0 kV and a 40 μF capacitor charged to 4.0 kV are connected to each other, with the positive plate connected to the negative plate, and the negative plate to the positive plate. What is the final charge on the 20 μF capacitor?

一個 20 μF 的電容充電至 2.0 kV 的電壓，一個 40 μF 的電容充電至 4.0 kV 的電壓。現將兩電容的正極與負極相連，負極與正極相連，問在 20 μF 的電容上有多少電荷?

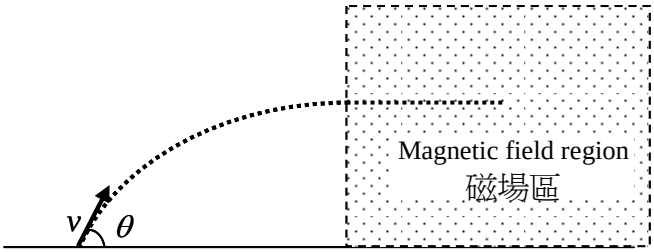
(a) 50 mC (b) 200 mC (c) 40 mC (d) 67 mC (e) 120 mC

MC9

A particle of mass M and carrying charge Q is launched with initial speed v and at an angle of θ relative to the horizontal direction. When it reaches the maximum height it enters a region of uniform magnetic field. In the region it moves at constant velocity in the horizontal direction. Determine the direction and strength of the magnetic field.

(a) outward, $\frac{Mg}{Qv\cos\theta}$ (b) inward, $\frac{Mg}{Qv\cos\theta}$ (c) outward, $\frac{Mg}{Qv\sin\theta}$

(d) inward, $\frac{Mg}{Qv\sin\theta}$ (e) inward, $\frac{Mg}{Qv\tan\theta}$



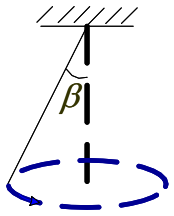
一質量爲 M 帶電量爲 Q 的粒子以速度 v 相對于水平綫角度θ射出。粒子到達最高點後進入一均勻磁場區，幷以恒定速度沿水平綫運動。求磁場的方向和大小。

(a) 向外, $\frac{Mg}{Qv\cos\theta}$ (b) 向內, $\frac{Mg}{Qv\cos\theta}$ (c) 向外, $\frac{Mg}{Qv\sin\theta}$

(d) 向內, $\frac{Mg}{Qv\sin\theta}$ (e) 向內, $\frac{Mg}{Qv\tan\theta}$

MC10

As shown, one end of a light thread is fixed on the ceiling and the other end tied to a small sphere. The angle between the thread and the vertical direction is β. When β = α and α is a small angle, the sphere is in simple harmonic motion like a pendulum with period T. When β = α_1 or α_2 ($\alpha < \alpha_1 < \alpha_2$), the sphere is in a uniform circular motion in a horizontal plane with period T_1 or T_2, respectively. Then the correct relation is_____.



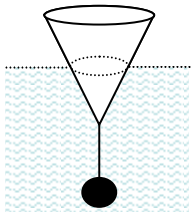
一根輕細綫，上端固定在天花板上，下端連接一小球，小球在水平面上做勻速圓周運動。設偏斜角可取值α_1和α_2，與之對應的小球圓周運動的周期爲 T_1和 T_2。若$\alpha < \alpha_1 < \alpha_2$，其中$\alpha$是小球做單擺簡諧振動的擺角，$T$是簡諧振動的周期，則正確的關係式是_____。

(a) $T < T_1 < T_2$ (b) $T = T_1 = T_2$ (c) $T > T_1 > T_2$ (d) $T_1 < T < T_2$
(e) $T_1 > T > T_2$

MC11

A cone of height H with a mass attached is floating upside down in water, as shown. The water reaches $H/2$ when in equilibrium. Ignore friction. Find the vibration frequency after the cone is slightly pushed downwards.

一圓錐體高度爲 H，頂端系一重物，倒浮在水裏。平衡時水浸到 $H/2$ 處。不計摩擦，求將錐體輕輕往下一按後的振動頻率。



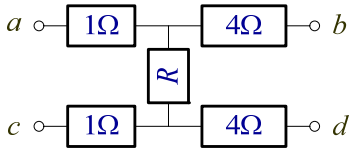
(a) $\frac{1}{2\pi}\sqrt{\frac{6g}{H}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{g}{H}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{3g}{H}}$

(d) $\frac{1}{2\pi}\sqrt{\frac{8g}{H}}$ (e) $\frac{1}{2\pi}\sqrt{\frac{2g}{H}}$

MC12

In the circuit, when the voltage between a and c is 20 V, the voltage between b and d is 10 V. When the voltage between b and d is 20 V, what is the voltage between a and c?

在如圖所示的電路中，當 a、c 兩端加上 20V 的電壓時，測得 b、d 兩端的電壓爲 10V；若改在 b、d 兩端加上 20V 的電壓，測得 a、c 兩端的電壓爲多少?

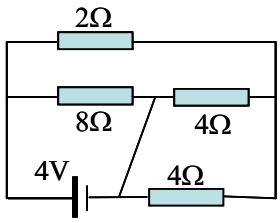


(a) 4V (b) 5V (c) 8V (d) 10V (e) 20V

MC13

In the circuit, the electric current through the battery is _____ Ampere(s).

在電路中，流經電池的電流爲_____ 安培。



(a) 1 (b) 2 (c) 2.5 (d) 4 (e) 1.5

MC14

Object-A is dropped from a height h. At the same instant object-B is thrown vertically upward from the ground. Right before they collide in mid-air, the speed of A is twice the speed of B. Determine the height where the collision occurs.

物體-A 從高度爲 h 的高處釋放，物體-B 同時從地上垂直往上拋。兩物體在空中相撞前瞬間，物體-A 的速率是物體-B 速率的二倍。求它們相撞的高度。

(a) $\frac{2h}{3}$ (b) $\frac{h}{\sqrt{3}}$ (c) $\frac{3h}{4}$ (d) $\frac{h}{2}$ (e) $\frac{h}{4}$

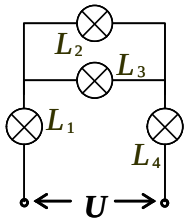
MC15

An object of mass m is placed on a horizontal floor. The static friction coefficient between the object and the floor is $\mu = 1$. Find the minimum force that can move the object.

質量爲 m 的物體置于水平地面上，物體和地面間的靜摩擦係數$\mu = 1$，求能使該物體移動的最小外力。

(a) $\frac{mg}{2}$ (b) $\frac{mg}{\sqrt{2}}$ (c) mg (d) $\sqrt{2}\,mg$ (e) $2mg$

MC16



In the circuit L_1, L_2, L_3, and L_4 are four light bulbs. L_1 and L_2 are labeled "220V, 25W". L_3 and L_4 are labeled "220V, 60W". When voltage U is applied, the sequence of the brightness of the four bulbs is, from the brightest to the dimmest, _____.

四盞電燈接入如圖所示的電路中，其中 L_1 和 L_2 標稱爲"220V, 25W", L_3 和 L_4 的標稱爲"220V, 60W"。加上電壓 U 後時電燈由亮至暗的排列次序是_____。

(a) 1,2,3,4 (b) 2,3,4,1 (c) 4,2,1,3 (d) 3,4,1,2
(e) 1,4,3,2

MC17

A charged object is launched inside a time varying electric field. Its motion is recorded by a video camera on a video tape. When it is at a certain moment *A*, its position vector ***r***, velocity ***v***, and acceleration ***a*** are measured. A student watches the video at a later time but mistakenly plays the tape in the reverse direction. What are the position, velocity, and acceleration of the object at moment *A* observed by the student?

一帶電粒子在一隨時間變化的電場中的運動被錄影機記錄在錄影帶上。在某一時刻 A 它的位置是 ***r***、速度是 ***v***、加速度是 ***a***。一學生後來在播放錄影帶時把播放方向弄反了。問到了 A 時刻他看到粒子的位置、速度、加速度是什麼？

(a) ***r***, ***v***, ***a*** (b) ***r***, -***v***, -***a*** (c) ***r***, -***v***, ***a*** (d) ***r***, ***v***, -***a*** (e) -***r***, -***v***, -***a***

MC18

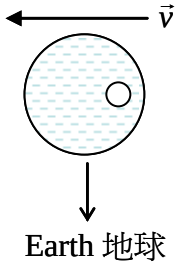
An object of mass *m* is attached to a spring. The restoring force of the spring is $F = -\lambda x^3$, where *x* is the displacement. The oscillation period now depends on the oscillation amplitude. Suppose the object is initially at rest. If the initial displacement is *D* then its period is τ. If the initial displacement is 2*D*, find the period. (Hint: use dimension analysis.)

一質量爲 *m* 的物體系在一彈簧上，彈簧的回復力與位移 *x* 的三次方成正比，即 $F = -\lambda x^3$，因此物體的振動頻率與它的振幅有關。若物體都由靜止釋放，振幅爲 *D* 時的振動周期爲 τ，問振幅爲 2*D* 時的振動周期爲多少?（提示：利用量綱分析。）

(a) 8τ (b) 2τ (c) τ (d) $\tau/2$ (e) $\tau/8$

MC19

A small air bubble is inside a drop of water residing in a space station on an orbit around Earth. The direction to Earth is downwards and the space station is moving to the left relative to Earth, as shown. The air bubble will _____ relative to the water drop.



(a) move to the left (b) move to the right (c) move up
(d) move down (e) not move

在一個繞地球運轉的空間站裏有一相對于空間站靜止的水珠，水珠裏有一小氣泡。空間站相對地球往左運動，空間站下方是地球。則氣泡相對與水珠的運動是_____。

(a) 向左 (b) 向右 (c) 向上 (d) 向下 (e) 不動

MC20

A satellite of mass *m* is at a distance *a* from a star of mass *M*. The speed of the satellite is *u*. Suppose the law of universal gravity is $F = -G\frac{Mm}{r^{2.1}}$ instead of $F = -G\frac{Mm}{r^2}$, find the speed of the satellite when it is at a distance *b* from the star.

一質量爲 *m* 速率爲 *u* 的衛星位于離一質量爲 *M* 的星球距離爲 *a* 的位置。若萬有引力爲 $F = -G\frac{Mm}{r^{2.1}}$ 而不是 $F = -G\frac{Mm}{r^2}$ ，求衛星離星球距離爲 *b* 時的速率。

(a) $\sqrt{u^2 + 2GM\left(\frac{1}{b^{1.1}} - \frac{1}{a^{1.1}}\right)}$

(b) $\sqrt{u^2 + GM\left(\frac{1}{a^{1.1}} - \frac{1}{b^{1.1}}\right)}$

(c) $\sqrt{u^2 + \frac{2}{1.1}GM\left(\frac{1}{b^{1.1}} - \frac{1}{a^{1.1}}\right)}$

(d) $\sqrt{u^2 + \frac{2}{2.1}GM\left(\frac{1}{b^{1.1}} - \frac{1}{a^{1.1}}\right)}$

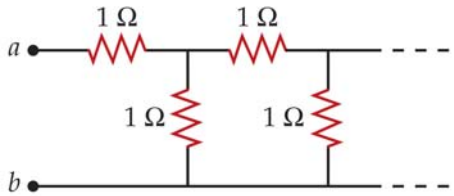
(e) $\sqrt{u^2 + \frac{2}{1.1}GM\left(\frac{1}{b} - \frac{1}{a}\right)}$

《End of MC's 選擇題完》

Open Problems 開放題
Total 5 problems 共 5 題

The Open Problem(s) with the '*' sign may require information on page-3.
帶 * 的開放題可能需要用到第三頁上的資料。

Q1 (8 points)
Consider the figure of an infinite ladder of resistors as shown in the figure. Calculate the equivalent resistance between point *a* and point *b*.



題 1（8 分）
右圖爲一無限長電阻綫路。求 *a*、*b* 點之間的等效電阻。

Q2 (12 points)
A firework exploded into *N* fragments and converted chemical energy *E* into the kinetic energies of the fragments. The masses of the fragments are $\{m_i\}$, $i = 1, \ldots N$. The spinning motions of the fragments are ignored.

(a) Consider the 2nd to the Nth fragments as a whole system, find the velocity of the center-of-mass of the system $\vec{v}_c$ in terms of the fragment masses and the velocity of the 1st fragment $\vec{v}_1$. (4 points)

(b) Let the velocities of the 2nd to the Nth fragments relative to the center-of-mass of the system be $\{\vec{v}_i\}$, $i = 2, \ldots N$. Find their total kinetic energy in terms of their masses, $\vec{v}_c$, and $\{\vec{v}_i\}$, $i = 2, \ldots N$. (4 points)

(c) Find the maximum kinetic energy the 1st fragment can have in terms of *E* and the masses of the fragments. (4 points)

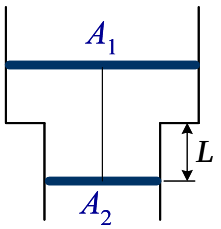
題 2（12 分）
一烟花爆炸後形成 *N* 塊碎片，并將化學能 *E* 轉化成碎片的動能。碎片的質量爲$\{m_i\}$, $i = 1, \ldots N$，它們的轉動可以忽略。

(a) 將第二到第 N 塊碎片看做是一個系統，用第一塊碎片的速度$\vec{v}_1$和各碎片的質量來表達該系統的質心的速度$\vec{v}_c$。 (4 分)

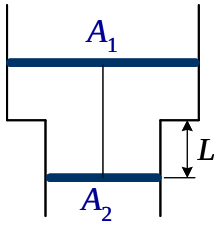
(b) 若第二到第 N 塊碎片相對于系統質心的速度爲 $\{\vec{v}_i\}$, $i = 2, \ldots N$，求它們的總動能，并以$\vec{v}_c$, $\{\vec{v}_i\}$, $i = 2, \ldots N$，和碎片的質量來表達。 (4 分)

(c) 求第一塊碎片可得的最大動能，并以 *E* 和碎片的質量來表達。 (4 分)

Q3 (15 points)
Two vertical pistons of total mass *M* and different diameters are connected by a rigid light rod of length *H*. Trapped in between is *N* mol of ideal gas. The area of the upper piston is A_1 and that of the lower piston is A_2. The air pressure outside is p_0. The wall of the cylinder is smooth. The gas constant is *R*. (a) Find the distance between the bottom of the large cylinder and the lower piston *L* when the system is in equilibrium. (b) Find the vibration frequency of the pistons near the equilibrium position.
(Hint: for $x << 1$, $(1+x)^n \approx 1+nx$, where *n* can be a fraction number or an integer.)



題 3（15 分）



直立的氣缸由半徑不同的兩個薄圓筒連接而成，由剛性的長爲 H 的輕杆連接的兩個活塞把 N mol 的理想氣體封閉在氣缸內。兩個活塞的總質量爲 M，上活塞截面積爲 A_1、下活塞爲 A_2，氣缸外部大氣的壓強爲 p_0。活塞與氣缸壁的接觸是光滑的，普適氣體恆量爲 R。(a)求平衡時下活塞到大圓筒底部的距離 L。(b)求在平衡點附近的振蕩頻率。

(提示: 當 $x << 1$, $(1+x)^n \approx 1+nx$, 其中 n 可以是分數或整數。)

Q4* (15 points)

(a) Use suitable parameters on Page-3, calculate the mass of Earth. (3 points)

(b) Geostationary satellites seem stationary in the sky all the time. One such satellite is right above Singapore on the Equator. Find the orbit radius of the satellite. (5 points)

(c) Hong Kong is 22° N in latitude. To maintain a geostationary satellite of mass m right above Hong Kong, the orbit of the satellite is no longer around the Earth center and a constant force f from the small rocket engines on the satellite must be provided. Find the height of the satellite when the force f is minimum. (For A, B, $x > 0$, the function $y = \dfrac{\sqrt{1+A\left(1-Bx^3\right)^2}}{x^2}$ is minimum when $x = \left(\dfrac{\sqrt{1+8\left(1+1/A\right)}-1}{2B}\right)^{1/3}$) (7 points)

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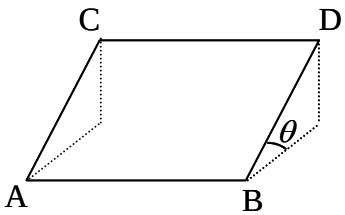
題 4*（15 分）

(a) 利用第三頁上的參數，求地球的質量。 (3 分)

(b) 同步衛星看上去就像固定在天上一樣。有一顆同步衛星正好在位于赤道的新加坡上空。求該衛星的軌道半徑。 (5 分)

(c) 香港位于北緯 22° 。要在香港上空維持一顆質量爲 m 的同步衛星，則該衛星的軌道不再以地球中心爲圓心，幷要由衛星上的小火箭提供一作用力 f 。求使 f 爲最小值時衛星的高度。 (若 A, B, $x > 0$, 函數 $y = \dfrac{\sqrt{1+A\left(1-Bx^3\right)^2}}{x^2}$ 在 $x = \left(\dfrac{\sqrt{1+8\left(1+1/A\right)}-1}{2B}\right)^{1/3}$ 時爲最小值。) (7 分)

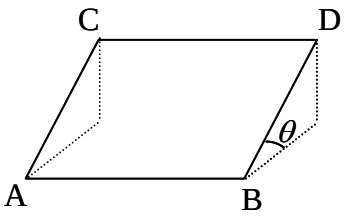
Q5 (10 points)



According to quantum mechanics, a particle of momentum p can be regarded as a plane matter wave with wavelength $\lambda = h/p$, where h is the Planck Constant. As shown in the figure, ABDC is a flat square of side length L at an inclined angle θ to the horizontal plane. A neutron beam of initial kinetic energy E_0 is divided into two beams at point-A. One beam moves along the path ACD and the other beam along the path ABD. When the two beams meet at point-D they interfere. The mass of a neutron is m. How many times can one get maximum neutron number readings at point-D when θ changes from 0° to 90°?

題 5（10 分）

根據量子力學，一動量爲 p 的粒子可當作是一平面物質波，其波長爲 $\lambda = h / p$ ，h 爲普朗克常數。如圖所示，ABCD 爲一邊長爲 L 的平面正方形，與水平面的夾角爲 θ。一束初始動能爲 E_0 的中子在 A 點被分成兩束，一束沿路徑 ACD 傳播，一束沿路徑 ABD 傳播。到達 D 點的兩束波相互干涉。一個中子的質量爲 m。把 θ 從 0° 轉到 90° 在 D 點可測到幾次中子數的最大值？



《END 完》