Hong Kong Physics Olympiad 2011
2011 香港物理奧林匹克

(Junior Level 初級組)

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院
The Education Bureau of HKSAR
香港特區政府教育局
The Physical Society of Hong Kong
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

March 19, 2011
2011 年 3 月 19 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將于比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID Number in the field of "I. D. No.", and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of "Course & Section No."

   請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 "I. D. No." 欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈**完全塗黑**，然後在適當的空格填上你的英文姓名，最後於 "Course & Section No. " 欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆**完全塗黑**。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of "Course Title", and write your English name in the field of "Student Name" and your 8-dight Participant I. D. Number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。 答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.

   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.

   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 m/s^2
G – gravitational constant, 6.67×10^{-11} N m^2/kg^2
e – charge of an electron, -1.6×10^{-19} C
ε_0 – electrostatic constant, 8.85×10^{-12} C/(V m)
m_e – electron mass = 9.11×10^{-31} kg
c – speed of light in vacuum, 3.0×10^8 m/s
Radius of Earth = 6378 km
Sun-Earth distance (= 1 Astronomical Unit (AU)) = 1.5×10^{11} m
Mass of Jupiter = 1.9×10^{27} kg
Mass of the sun = 1.99×10^{30} kg
Air Density = 1.2 kg/m^3
Water Density = 1.0×10^3 kg/m^3
Standard atmosphere pressure $p_0 = 1.013 \times 10^5$ N/m^2

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 m/s^2
G – 萬有引力常數, 6.67×10^{-11} N m^2/kg^2
e – 電子電荷, -1.6×10^{-19} C
ε_0 – 靜電常數, 8.85×10^{-12} C/(V m)
m_e – 電子質量, 9.11×10^{-31} kg
c – 真空光速, 3.0×10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 (= 1 天文單位(AU)) = 1.5×10^{11} m
木星質量 = 1.9×10^{27} kg
太陽質量 = 1.99×10^{30} kg
空氣密度 = 1.2 kg/m^3
水密度 = 1.0×10^3 kg/m^3
標準大氣壓 $p_0 = 1.013 \times 10^5$ N/m^2

The following conditions will be applied to all questions unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

除特別註明外，下列條件將適用于本卷所有問題：

1) 所有物體都處于地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

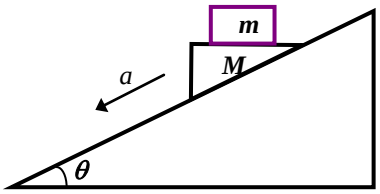
選擇題（每題 2 分，每題選擇一個答案。）

The MC questions with the '*' sign may require information on page-3.

帶 * 的選擇題可能需要用到第三頁上的資料。

MC1

As shown in the figure, a wedge of mass M is placed on a smooth inclined ramp that makes an angle θ to the horizontal. An object of mass m rests on top of the wedge. The system is sliding down the ramp at acceleration a. Determine the apparent weight of the object as it slides down. Note that there is friction between the object and the wedge so that the object remains relatively at rest on the wedge.



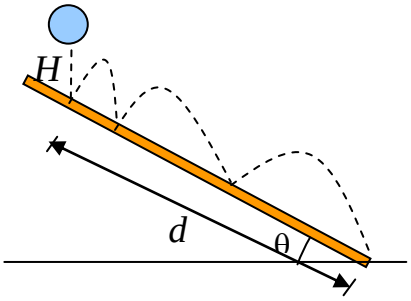
如圖所示，質量為 M 的楔子被置於平滑的斜坡上，斜坡相對於水平的傾角為θ。一件質量為 m 的物體置於楔子上。這系統以加速度 a 滑落斜坡。求該物體的表面重量。注意：物體和楔子之間的摩擦力令物體與楔子相對靜止。

(a) $mg\cos\theta$ (b) $mg\cos^2\theta$ (c) $mg\sin\theta\cos\theta$ (d) $mg\tan\theta$ (e) mg

MC2

A ball is released vertically from a height H above an inclined plane and makes several bounces. The angle of the inclined plane is θ. Assume the ball bounces elastically in each hit. Calculate the distance from the first hit to the fourth hit on the inclined plane.

一個球體從高於斜坡 H 的位置垂直掉落到斜坡上，並反彈數次。斜坡的傾角為θ。假設球體與斜坡的碰撞為彈性碰撞。求第一次碰撞到第四次碰撞的落點之間的距離。



(a) $3H\sin\theta$
(b) $24H\sin\theta$
(c) $30H\cos\theta$
(d) $36H\cos\theta$
(e) $48H\sin\theta$

MC3*

The first two extra-solar planets were discovered in 1992 to be revolving around a pulsar of 1.5 solar mass. The period of the circular orbit of one of the two planets is 98 days. Ignore the gravity interaction between the planets. Find the distance between the pulsar and the planet in terms of astronomical units (AU).

首兩個太陽系外行星於 1992 年被發現。其中一顆行星環繞 1.5 太陽質量的脈衝星作圓形軌跡運動，週期為 98 日。忽略行星之間的引力。求脈衝星與該行星的距離，以天文單位(AU)表達。

(a) 0.11 (b) 0.17 (c) 0.36 (d) 0.40 (e) 0.48

MC4

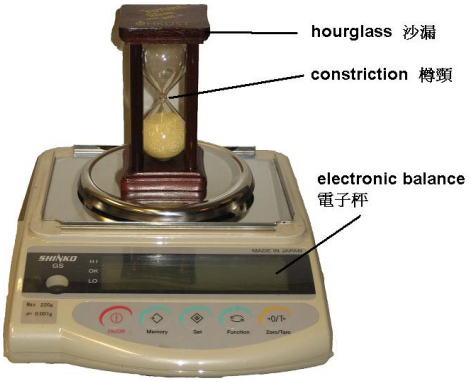
A ball hits a horizontal plane in a direction making an angle α with the horizontal, where $\sin\alpha = 3/5$. The coefficient of friction between the ball and the plane surface is 1/2. If 5/16 of the kinetic energy of the ball remains after the impact, then the ball bounces off the plane in a direction making at an angle θ with the horizontal. Find θ.

一球體與水平平面碰撞，方向與水平的夾角為α ， $\sin\alpha = 3/5$。球體與平面的摩擦系數為 1/2。如果碰撞過程後餘下的動能只有原來的 5/16，求球體反彈的方向與水平的夾角θ 。

(a) 23^o (b) 37^o (c) 53^o (d) 63^o (e) 83^o

MC5

As shown in the figure, an hourglass is put on an electronic balance. At time $t < 0$, a clot at the constriction prevents the sand from passing through. At time $t = 0$, the clot clears and the sand begins to drip down from the upper part to the lower part at a constant rate, until all sand is collected in the lower part after one hour. Assuming that the sand powder is extremely fine, how does the force acting on the electronic balance depend on the dripping process?

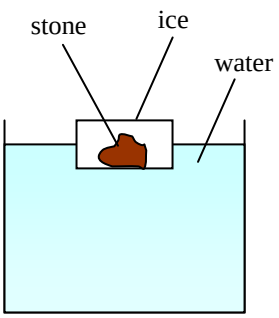


(a) The force increases to a higher value during the dripping process and then returns to the equilibrium value after the process.
(b) The force decreases to a lower value during the dripping process and then returns to the equilibrium value after the process.
(c) The force increases momentarily at the beginning and restores to the equilibrium value, and remains the same thereafter.
(d) The force increases momentarily at the beginning and restores to the equilibrium value, and decreases momentarily at the end of the dripping process, and remains at the equilibrium value thereafter.
(e) The force decreases momentarily at the beginning and restores to the equilibrium value, and increases momentarily at the end of the dripping process, and remains at the equilibrium value thereafter.

如圖所示，一沙漏放在電子秤上。在時間 $t < 0$ 時，沙漏的樽頸位置被堵塞，沙粉無從通過。在時間 $t = 0$ 時，堵塞被移走，沙粉開始已均勻速率從上方漏向下方，直至一小時後，所有沙粉都漏到下方。假設沙粉極度幼滑，問作用在電子秤上的作用力在漏沙過程中如何改變？
(a) 作用力在漏沙過程中增至一個高值，然後在過程後回復至平衡值。
(b) 作用力在漏沙過程中降至一個低值，然後在過程後回復至平衡值。
(c) 作用力在過程開始時瞬間增加，然後回復至平衡值，其後再沒有變動。
(d) 作用力在過程開始時瞬間增加，然後回復至平衡值，再在過程完結時瞬間降低，然後在過程後回復至平衡值。
(e) 作用力在過程開始時瞬間降低，然後回復至平衡值，再在過程完結時瞬間增加，然後在過程後回復至平衡值。

MC6

A piece of ice with an embedded stone floats on the surface of water in a glass. After the ice has melted, the stone sinks to the bottom of the glass. Compared with the initial water level, what is the change of the water level in the glass, first during the period the ice is melting, and second after the stone sinks to the bottom?



一片冰塊(ice)浮於杯中的水(water)面，冰塊內有一塊石子(stone)。當冰塊融解後，石子沉到杯底。與初始的水面高度比較，水面高度在冰塊融解時，和石子沉到杯底後有何改變？
(a) Remains the same then rises. 先不變，然後升高。
(b) Remains the same then falls. 先不變，然後降低。
(c) Remains the same all the way. 從始至終都不變。
(d) Rises then falls. 先上升，然後降低。
(e) Falls then rises. 先降低，然後上升。

MC7

A student observed the presence of strong winds generated by moving trains in the tunnels of subway. She suggested that wind turbines could be installed in the tunnels to generate electricity. Which of the following statement(s) is(are) correct?

I. The electricity generated by the turbines can be supplied to the air conditioners of the underground train stations, thereby saving energy.

II. The electricity generated by the turbines can be supplied to the electric motors of the underground trains, thereby saving energy.

III. The air resistance experienced by the trains will not be affected.

有學生觀察到地鐵隧道內，地鐵火車的運動產生很強的氣流。她提議在隧道內裝設渦輪機，利用氣流發電。下列哪些句子是正確的？

I. 渦輪機產生的電力可以供應給地鐵站的空調系統，以節省能源。

II. 渦輪機產生的電力可以供應給地鐵火車的電動機，以節省能源。

III. 地鐵火車遇到的空氣阻力不會受影響。

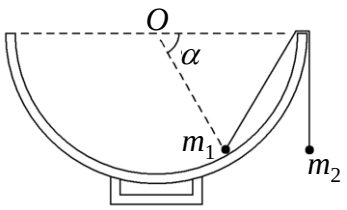
(a) All are not true 所有都不正確　(b) Only I is true 只有 I 正確

(c) Only II is true 只有 II 正確　(d) Only III is true 只有 III 正確

(e) All are true 所有都正確

MC8

As shown in the figure, a hemispherical bowl is placed horizontally on a table. Point-*O* is the center of the hemisphere. The edge and the surface of the bowl are smooth. A particle of mass m_1 is placed in a bowl and is tied to a string with negligible mass. The other end of the string is tied to another particle of mass m_2 hanging outside the bowl. When the system is in equilibrium, the line joining the particle m_1 and Point-*O* makes an angle $\alpha = 60^o$ with the horizontal. Find the ratio m_1/m_2.

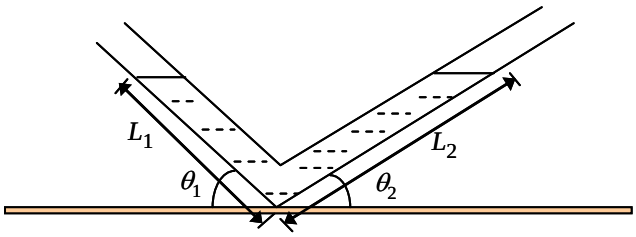


如圖所示，一個半球形的碗放在桌面上，碗口水平，*O* 點為其球心，碗的內表面及碗口是光滑的。一根細線跨在碗口上，線的兩端分別繫有質量為 m_1 和 m_2 的小球，當它們處於平衡狀態時，小球 m_1 與 *O* 點的連線與水平線的夾角為 $\alpha = 60^0$。求兩小球的質量比 m_1/m_2。

(a) 0.71　(b) 0.87　(c) 1.15　(d) 1.41　(e) 1.73

MC9

A non-viscous liquid of density ρ, as shown in figure, is filled in a '***V***' shape tube with A, L_1 and L_2 being the area of cross section and arm lengths respectively. If the liquid is slightly depressed in one of the arms, find the oscillation frequency of the liquid column.



如圖所示，'***V***'形管內有非粘滯的液體，密度為 ρ。管的截面積和兩臂長度分別為 A, L_1 和 L_2。如令液體水平在其中一臂輕微降低，求液體柱產生振動的頻率。

(a) $\frac{1}{2\pi}\sqrt{\frac{g(\sin\theta_1+\sin\theta_2)}{L_1+L_2}}$　(b) $\frac{1}{2\pi}\sqrt{\frac{\rho gA(\sin\theta_1-\sin\theta_2)}{L_1+L_2}}$　(c) $\frac{1}{2\pi}\sqrt{\frac{L_1\sin\theta_1+L_2\sin\theta_2}{\rho gA}}$

(d) $\frac{1}{2\pi}\sqrt{(\frac{L_1\sin\theta_1}{L_2\sin\theta_2})\cdot\rho gA}$　(e) $\frac{1}{2\pi}\sqrt{\frac{g}{\frac{L_1}{\sin\theta_1}+\frac{L_2}{\sin\theta_2}}}$

MC10

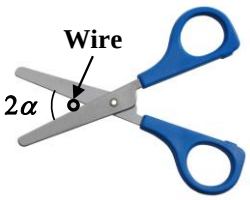
The kinetic energy of a particle in a simple harmonic motion is $\frac{1}{2}av^2$, its potential energy is $\frac{1}{2}bx^2$, where x is the coordinate for the position of the particle and v is its speed. Find the frequency of the motion.

一質點在簡諧運動中的動能為 $\frac{1}{2}av^2$，位能為 $\frac{1}{2}bx^2$，其中 x 為質點的位置，v 為其速度。求運動的頻率。

(a) $\frac{1}{2\pi}\sqrt{\frac{a}{b}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{b}{a}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{a}{b}+\frac{b}{a}}$ (d) $\frac{1}{2\pi}\sqrt{ab}$ (e) $\frac{1}{2\pi}\sqrt{\frac{1}{ab}}$

MC11

Someone is using a scissors to cut a wire of circular cross section and negligible weight. The wire slides in the direction away from the hinge until the angle between the scissors blades becomes 2α. Find the coefficient of friction between the blades and the wire.

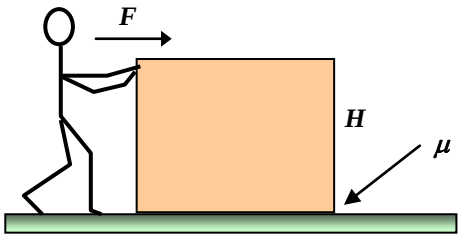


用一把剪刀，去剪一條圓截面的導綫。導綫先會向外滑動，直到剪刀之間的角度爲 2α，求剪刀和導綫之間的摩擦係數。

(a) $\sqrt{1-\tan\alpha}$ (b) $2\cos\alpha$ (c) $2\tan\alpha$ (d) $\tan\alpha$ (e) $\sqrt{2\cos^2\alpha-1}$

MC12

A person exerts a horizontal force F at the upper edge of a box to push the box of uniform mass density, length L, and height H across the floor. The friction coefficient between the box and the floor is μ. If $\mu > \mu_o$, the box will overturn before it slides. Determine the value of μ_o.

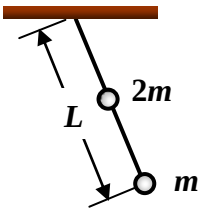


有人嘗試以作用力 F 沿水平方向作用於箱子的頂邊去推動箱子。箱子質量密度均勻，長度為 L，高度為 H。箱子與地面的摩擦系數為μ。若$\mu > \mu_o$，箱子不會滑動，反而會翻倒。求μ_o的值。

(a) $\frac{L}{2H}$ (b) $\frac{L}{H}$ (c) $\frac{H}{L\mu}$ (d) $\frac{\mu L}{H}$ (e) $\frac{2H}{\mu L}$

MC13

A compound pendulum is made of a light and rigid rod of length L with one end attached to a hinge on the ceiling. A small ball of mass m is attached to the other end of the rod, and another small ball of $2m$ is attached to the middle of the rod. Find the frequency of the simple harmonic oscillation of the pendulum.



有一複合擺錘，一端繫於天花板上，錘身長度為 L，重量可忽略。錘另一端繫有質量為 m 的小球，錘身中央另繫有質量為 $2m$ 的小球。求擺錘簡諧振動的頻率。

(a) $\frac{1}{2\pi}\sqrt{\frac{g}{2L}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{4g}{3L}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{3g}{2L}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{9g}{4L}}$

(e) $\frac{1}{2\pi}\sqrt{\frac{9g}{2L}}$

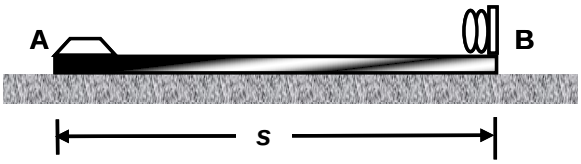
MC14

A small object is initially at the bottom of a plane inclined at an angle α with the horizontal. It is projected upward along the inclined plane with an initial velocity, and reaches the maximum height after time t_1. It then slides downward and returns to the initial position after time t_2. If the coefficient of sliding friction between the object and the surface is μ, find the ratio t_2/t_1.

一個傾角為 α 的斜面底部有一小物塊。現給物塊一個初速率，使它沿斜面向上滑動，經過時間 t_1 到達最高點，之後它又自動滑回到底部，所用時間為 t_2。若物體與斜面間的滑動摩擦係數為 μ，求比例 t_2/t_1。

(a) $\sqrt{\dfrac{\sin\alpha-\mu\cos\alpha}{\sin\alpha+\mu\cos\alpha}}$ (b) $\sqrt{\dfrac{\sin\alpha+\mu\cos\alpha}{\sin\alpha-\mu\cos\alpha}}$ (c) $\dfrac{\sin\alpha-\mu\cos\alpha}{\sin\alpha+\mu\cos\alpha}$ (d) $\dfrac{\sin\alpha+\mu\cos\alpha}{\sin\alpha-\mu\cos\alpha}$ (e) $\sqrt{\dfrac{\sin\alpha}{\mu\cos\alpha}}$

C15

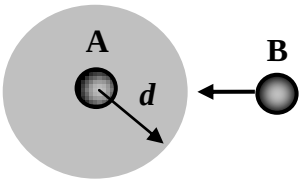


As shown in the figure, AB is a board of mass M = 4 kg and length s = 2 m, placed on a smooth horizontal surface. A bumper of negligible mass is fixed at end-B. A peg of mass m = 1 kg is placed at end-A. The coefficient of kinetic friction between the peg and the board is μ = 0.2. With both the board initially at rest, the peg is ejected with an initial velocity of v_0 = 10 m/s in contact with the board surface until it hits the bumper at end-B. After the collision, it just returns to end-A without falling off the board. Find the mechanical energy loss in the process.

如圖所示，AB 為位於光滑水平面上的長木板，質量為 M = 4 kg，長度為 s = 2 m。B 端有一固定擋板，A 端放有一小滑塊，其質量 m = 1 kg。小滑塊與木板間的動摩擦系數為 μ = 0.2。當木板處於靜止的初始狀態時，小滑塊以初速度 v_0 = 10 m/s 緊貼木板表面射出，直到和 B 端的擋板相撞。碰撞後，小滑塊恰好回到 A 端而不脫離木板。求此過程中損失的機械能。

(a) 20 J (b) 24 J (c) 28 J (d) 32 J (e) 40 J

MC16

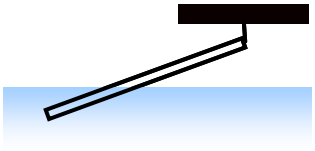


The elastic collision between two bodies, A and B, can be considered using the following model. A and B are free to move along a common line without friction. When their distance is greater than d = 1 m, the interacting force is zero; when their distance is less than d, a constant repulsive force F = 6 N is present. The mass of body A is m_A = 1 kg and it is initially at rest; the mass of body B is m_B = 3 kg and it is approaching body A head-on with a speed v_0 = 2 m/s. Find the minimum distance between A and B.

兩物體 A 和 B 的彈性碰撞過程可以如下的模型考慮。A 和 B 可沿同一直線自由運動，不用考慮摩擦力。當它們之間的距離大於 d = 1 m 時， 相互作用力為零; 當它們之間的距離小於 d 時， 存在大小恒為 F = 6N 的斥力。設 A 物體質量為 m_A = 1 kg， 開始時靜止在直線上某點; B 物體質量 m_B = 3 kg, 以速率 v_0 = 2 m/s 從遠處沿該直線向 A 運動。求 A、B 間的最小距離。

(a) 0.25 m (b) 0.50 m (c) 0.75 m (d) 1 m (e) 1.25 m

MC17



A uniform rod is hung at one end and is partially submerged in water. If the density of the rod is 5/9 that of water, find the fraction of the length of the rod above water.

一端懸掛的均勻竿，部分浸在水中。若竿密度是水密度的 5/9，求在平衡狀態時，竿在水面上部分的長度比例。

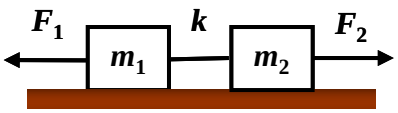
(a) 0.25 (b) 0.33 (c) 0.50 (d) 0.67 (e) 0.75

MC18

An observer stands next to the front end of the first carriage of a train. When the train starts to accelerate uniformly, it takes 5 seconds for the first carriage to pass the observer. Assuming that all carriages are of the same length, what is the time taken by the tenth carriage to pass the observer?
一位觀察者站在靜止的列車第一節車廂的前端。當列車以等加速度開動時，第一節車廂經過其旁需 5 s。假設所有車廂長度相同，求第十節車廂經過其旁的時間。
(a) 1.18 s (b) 1.07 s (c) 0.98 s (d) 0.91 s (e) 0.81 s

MC19

As shown in the figure, two blocks of masses m_1 and m_2 are connected by a light string, and are placed on a horizontal smooth surface. Forces of magnitude F_1 and F_2 act on them respectively, causing them to move linearly. The force constant of the light string is k, and $F_1 > F_2$. What is the extension x of the light string?

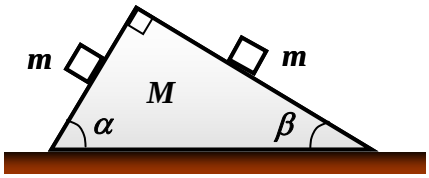


如圖所示，兩個用輕線相連的位於光滑水平面上的物塊，質量分別為 m_1和 m_2，拉力 F_1和 F_2 方向相反，令系統沿同一水平直線運動。輕線的剛度係數為 k，且 $F_1 > F_2$。求在兩個物塊運動過程中輕線伸長的長度 x。

(a) $\frac{F_1 m_1 + F_2 m_2}{k(m_1 + m_2)}$ (b) $\frac{F_1 m_2 + F_2 m_1}{k(m_1 + m_2)}$ (c) $\frac{|F_1 m_2 - F_2 m_1|}{k(m_1 + m_2)}$ (d) $\frac{|F_1 m_1 - F_2 m_2|}{k(m_1 + m_2)}$ (e) $\frac{F_1 m_1 + F_2 m_2}{k|m_1 - m_2|}$

MC20

As shown in the figure, a triangular wooden block of mass M is fixed on a horizontal table. Its top angle is 90^o, and the base angles are α and β. Two small pieces of wood, each of mass m, are located on the inclined smooth surfaces. When the wood pieces slide down the inclined surfaces, what is the normal force acting on the table by the triangular block?



如圖所示，一質量為 M 的三角形木塊固定在水平桌面上，它的頂角爲 90^o，兩底角為 α、β，兩個質量均為 m 的小木塊位于両側光滑的斜面上。當兩小木塊沿斜面下滑時，求三角形木塊對水平桌面的垂直作用力。
(a) Mg (b) $2mg$ (c) $Mg+mg$ (d) $Mg+2mg$ (e) $Mg+mg(\sin\alpha + \sin\beta)$

《END OF MC's 選擇題完》

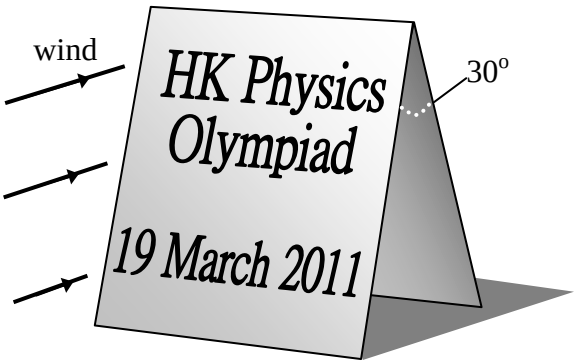
Open Problems 開放題

Total 5 problems 共 5 題

The Open Problem(s) with the '*' sign may require information on page-3.
帶 * 的開放題可能需要用到第三頁上的資料。

Q1 (10 points) **題 1 (10 分)**

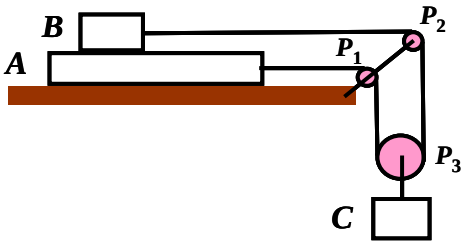
A "HK Physics Olympiad sandwich board" sign consists of two uniform pieces of metal后 50.0 cm wide and 1.0 m high, each of mass M = 3.0 kg, joined at the top forming an angle $\theta = 30^o$. The sign looks like an inverse letter 'V' when viewed sideways. Assume that the friction between the sign and the ground is sufficiently strong to prevent the sign from sliding on the ground, and that the board surface can stop any horizontal movement of air hitting it. The density of air is ρ = 1.2 kg/m^3. Find the minimum wind speed v_{min} above which the sign will turn over.



"香港物理奧林匹克"廣告牌由兩塊均勻金屬板組成，每塊金屬板的質量為 M = 3.0 kg，寬 50.0 cm，高 1.0 m，在頂端以$\theta = 30^o$的角度連接。 從側面看，廣告牌仿似倒置的'V'字。假設廣告牌與地面的摩擦足夠強，令廣告牌不致在地面滑行，而當空氣吹向它時，牌面可以終止空氣的任何水平運動。空氣的密度為ρ = 1.2 kg/m^3。計算可以把廣告牌吹翻的最低風速 v_{min}。

Q2 (10 points) **題 2 (10 分)**

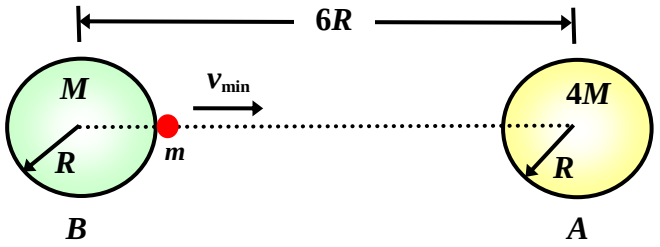
Rectangular block A of mass $3m$ is placed on the rough surface of a table. Another block B of mass m with rough surfaces is placed on top of block A. A light inextensible string connects blocks A and B, and winds through the massless smooth pulleys P_1 and P_2 fixed at the edge of the table. A massless smooth pulley P_3 hangs from the string segment between pulleys P_1 and P_2, and block C of mass m hangs on pulley P_3. Let μ be the coefficient of friction between all contact surfaces.



(a) Find the range of μ within which the system stays in equilibrium.
(b) Find the range of μ within which block C moves downwards, while block A remains stationary.

將一塊質量為 $3m$ 的粗糙長方體物塊 A 置於粗糙水平桌上，並將一塊質量為 m 的粗糙物塊 B 置於 A 之上。一根不可伸長的輕繩連接 A 和 B。繩子繞過兩個固定於桌邊的無重光滑小滑輪 P_1 和 P_2，兩滑輪之間的繩子懸掛著一個光滑的滑輪 P_3。一質量為 m 的物塊 C 懸於 P_3 之下。設μ為所有接觸面之間的摩擦係數。
(a) 若該系統保持平衡，求摩擦係數μ可取值的範圍。
(b) 若 A 不動而 C 向下運動，求摩擦係數μ可取值的範圍。

Q3 (15 points) **題3 (15分)**



As shown in the figure, spheres *A* and *B* have equal radii *R* and mass 4*M* and *M* respectively. They are separated center to center by a distance of 6*R*. A projectile of mass *m* is ejected from the surface of sphere *B* in the direction towards the center of sphere *A*.

(a) Find the minimum speed v_{min} of the projectile so it can reach the surface of sphere *A*.

(b) If the projectile is ejected with the minimum speed v_{min}, calculate the speed of the projectile when it reaches the surface of sphere *A*.

(c) If the projectile can be launched from the surface of sphere *B* in any direction so as to escape to infinity, find the speed of the projectile when it leaves sphere *B*.

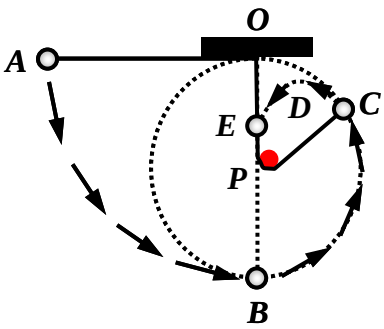
如圖所示，兩均勻球體 *A* 和 *B* 半徑均為 *R*，質量分別為 *4M* 和 *M*，兩球中心距離為 6*R*。有拋射體質量為 *m*，在 *B* 球表面朝 *A* 球中心發射。

(a) 計算拋射體可以達到 *A* 球表面的最小速率 v_{min}。

(b) 若拋射體以最小速度 v_{min} 發射，計算拋射體到達 *A* 球表面時的速率。

(c) 若拋射體可以從 *B* 球朝任何方向發射以逃逸至無限遠，計算拋射體離開 *B* 球表面時的速率。

Q4 (15 points) **題 4 (15 分)**



As shown in the figure, a pendulum is made up of a bob *A* suspended from a fixed point *O* by a light inextensible string of length *L*. A nail *P* is located at a distance *L*/2 vertically below *O*. The pendulum is lifted with the string taut until line *OA* is horizontal and then released. When the pendulum swings to the vertical position at point *B*, only the portion below point *P* can swing further.

(a) When the interrupted pendulum swings further, the string becomes loose at point *C*. Find the angle between the line *PC* and the vertical direction.

(b) After the string becomes loose, the bob continues to move and reaches its maximum height at point *D*. Find the maximum height of the bob above point *P*.

(c) The bob then passes through point *E* which is right below *O*. Find the distance between *E* and *O*.

如圖所示的擺錘，擺線一端固定在點 *O*，另一端吊掛著擺球 *A*，擺線不能伸展，長度為 *L*，質量可忽略。在 *O* 點正下方距離 *L*/2 處有一固定釘子 *P*。現將擺球拉到 *OA* 線成水平的位置，擺線保持拉直狀態，然後放手。當擺球運動到垂直位置 *B* 時，釘子擋住擺線，結果只有釘子以下的部分可以繼續運動。

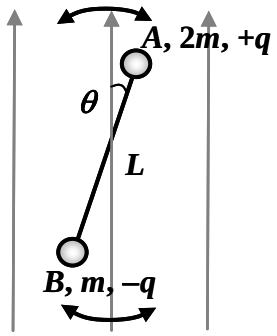
(a) 當受釘子阻擋的擺錘到達 *C* 點時，擺線開始鬆弛。求 *PC* 線與垂直方向的夾角。

(b) 擺線鬆弛後，擺球繼續運動，到達最高位置 *D* 點。求擺球相對於 *P* 點的最高高度。

(c) 擺球通過位於 *O* 點正下方的 *E* 點，求 *O* 點與 *E* 點之間的距離。

Q5 (10 points) **題 5 (10 分)**

As shown in the figure, a particle A of mass $2m$ and carrying charge q is connected by a light rigid rod of length L to another particle B of mass m and carrying charge $-q$. The system is placed in an electric field $\vec{E}$. The electric force on a charge q in an electric field $\vec{E}$ is $\vec{F} = q\vec{E}$. After the system settles into equilibrium, one particle is given a small push in the transverse direction so that the rod makes a small angle θ_0 with the electric field.



(a) Find the period of the angular oscillation.

(b) Find the maximum tension in the rod.

如圖所示，質點 A 的質量為 $2m$，帶電荷 q。另一質點 B 的質量為 m， 帶電荷 $-q$。兩質點由一輕而剛硬的棒連接，棒的長度為 L。現把系統放在電場 $\vec{E}$ 中，電場 $\vec{E}$ 作用於電荷 q 上的作用力為 $\vec{F} = q\vec{E}$。系統達到平衡後，把一質點向橫向輕輕推一下，使棒與電場形成夾角 θ_0。

(a) 求夾角的振動頻率。

(b) 求連接棒的最大張力。

《END 完》