Hong Kong Physics Olympiad 2014
2014 香港物理奧林匹克

Organisers 合辦機構

Education Bureau
教育局
The Hong Kong Academy for Gifted Education
香港資優教育學苑
The Hong Kong University of Science and Technology
香港科技大學

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Advisory Organisations 顧問機構

The Physical Society of Hong Kong
香港物理學會
Hong Kong Physics Olympiad Committee
香港物理奧林匹克委員會

11 May, 2014
2014 年 **5** 月 **11** 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English, but only ONE language should be used throughout the whole paper.

   所有題目均爲中英對照。你可選擇以中文或英文作答，惟全卷必須以單一語言作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將於比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你即可開始作答開放式題目，而無須等候任何宣佈。

3. On the cover of the answer book and the multiple-choice answer sheet, please write your HKID Number in the "Seat Number" box, your 8-digit Contestant number in the "Student Number" box and your English Name in the "Name" box.

   在答題簿封面及選擇題答題紙上，請於 Seat Number 欄中填上你的身份證號碼，於 Student Number 欄中填上你的 8 位數字參賽者號碼及於 Name 欄上填上你的英文姓名。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. The open problems are long. Please read the whole problem first before attempting to solve them. If there are parts that you cannot solve, you are allowed to treat the answer as a known answer to solve the following parts.

   開放式問答題較長，請將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來解答其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別註明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.8 m/s^2
Gravitational constant 萬有引力常數	G	6.67×10^{-11} N m^2/kg^2
Radius of Earth 地球半徑	R_E	6378 km
Sun-Earth distance 太陽-地球距離 (= 1 Astronomical Unit (AU)) (= 1 天文單位(AU))	r_E	1.5×10^{11} m
Earth-Moon distance 地球-月球距離	r_m	384400 km
Mass of Sun 太陽質量	M_{Sun}	1.99×10^{30} kg
Mass of Earth 地球質量	M_E	5.98×10^{24} kg
Density of Air 空氣密度	ρ_0	1.2 kg/m^3
Density of Water 水密度	ρ_w	1000 kg/m^3
Sea Water Density 海水密度	ρ_{sea}	1022 kg/m^3

Trigonometric Identities:
三角學恆等式：

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

Multiple Choice Questions

Select one answer in each question. For each question, 2 marks for correct answer, 0 mark for no answer, minus 0.25 mark for wrong answer, but the lowest mark of the multiple choice section is 0 mark.

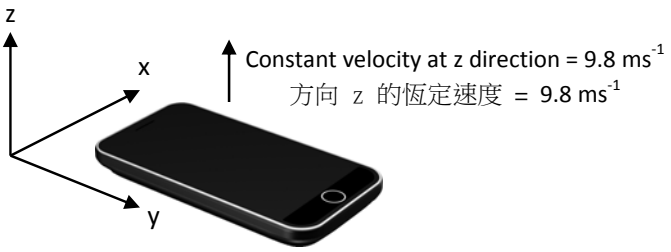
選擇題

每題選擇一個答案，每題答對 2 分，不答 0 分，答錯扣 0.25 分，但全部選擇題最低為 0 分。

1. An accelerometer is a device to measure acceleration. At the earth sea level, the acceleration due to gravity is g = 9.8 ms^{-2}. Most of the smartphones such as Android and Apple iPhone consist of a built-in accelerometer. If a smartphone is moving vertically upward at sea level with constant velocity 9.8 ms^{-1}, the instantaneous scalar readings of the x, y, and z components from the accelerometer are:

   加速度計是一種用來測量加速度的裝置。在地球海平面的重力加速度是 g = 9.8 ms^{-2}。 大部分的智能手機如 Android 和蘋果 iPhone 都有一個內置的加速度計。 如果智能手機正在 以均速 9.8 ms^{-1} 在地球海平面垂直向上移動，加速度計 x, y, and z 的瞬時讀數是：

   A. x = 0 g, y = 0 g, z = 0 g.
   B. x = 1 g, y = 0 g, z = 0 g.
   C. x = 0 g, y = 1 g, z = 0 g.
   D. x = 0 g, y = 0 g, z = 1 g.
   E. x = 1 g, y = 1 g, z = 1 g.



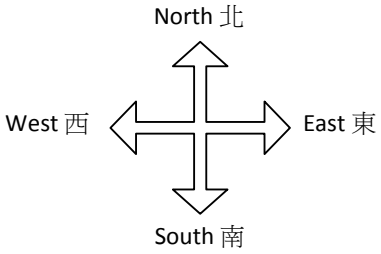
2. A toy rocket is launched upward from rest with a net acceleration of 2.5 g, where g is the acceleration due to gravity. The engine is turned off after 5 seconds while the rocket continues to move upward with negligible air resistance. Assume that the mass of the rocket is kept constant, what maximum elevation does the rocket reach in terms of g?

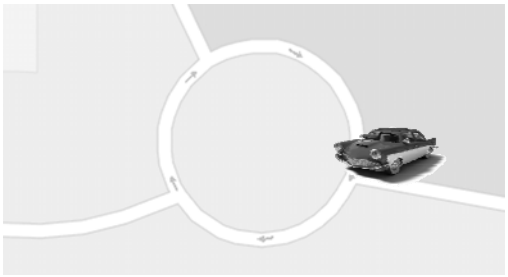
   一個玩具火箭從靜止被向上發射，其淨加速度為 2.5 g，而 g 是重力加速度。沒有空氣阻力下，發動機 5 秒後被關閉，而火箭繼續向上爬升。假設火箭質量保持不變，以 g 來推導火箭可達的最大高度。

   A. (31 g) m
   B. (50 g) m
   C. (78 g) m
   D. (109 g) m
   E. (159 g) m

3. A car (mass 200 kg) is travelling clockwisely around a flat roundabout (diameter 10 m) at a constant speed 5 m/s, as shown in the figure. What is the acceleration of the car?

   如圖所示，一輛汽車 (質量 200 kg) 正在以 5 m/s 的恆定速度圍繞著一個平坦的迴旋處順時針方向行駛 (直徑 10 m)。這輛汽車的加速度是什麼？





A. Zero 零
B. 5 m/s^2 East 東
C. 5 m/s^2 West 西
D. 10 m/s^2 East 東
E. 10 m/s^2 West 西

4. A machine gun fires bullets (10 g each) at a rate of 200 bullets per minute to hit a target. The speed just before hitting the target is 600 m/s, and the bullets are stopped in the target. The cross sectional area of the target is 2 m^2. Find the average force exerted on the target.

一挺機關槍每分鐘發 200 顆子彈（每顆子彈為 10 g），並以 600 m/s 的速度擊中目標，目標的橫切面積為 2 m^2。假設子彈停留在目標中，求子彈打在目標上的平均力。

A. 10 N.
B. 20 N.
C. 40 N.
D. 80 N.
E. 100 N.

5. Tides are mainly resulted from the difference in the Moon's gravitational field between the Earth's near side and its far side. What is the percentage gravitational force difference?

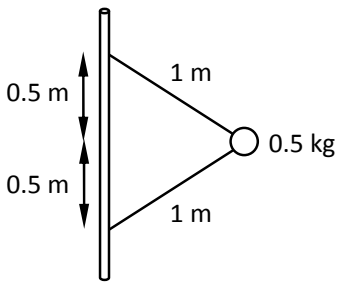
潮汐的產生主要是由於月球對地球的近端和遠端的引力場差異所成。這個引力場的差異百分比是多少?

A. 1%.
B. 3%.
C. 5%.
D. 7%.
E. 9%.

6. As shown in the following figure, a mass (0.5 kg) is tied to two wires (1 m) and revolves in a horizontal circle at a constant speed of 10 m/s. Calculate the tension of the upper wire.

如圖所示，一個質量物 (0.5 kg) 被連接到兩條線 (1 m) 並以 10 m/s 的恆定速度圍繞在水平轉圈。計算上方線的拉力。

A. 28 N
B. 38 N
C. 66 N
D. 105 N
E. 115 N



7. A person is pulling a heavy box with a 50 N force on a horizontal floor at constant velocity for 5 m. He then feels exhausted and the force exerted by him decreases linearly from 50 N to near 0 N for the next 5 m. What is the total work done by the person on the box?

一個人以 50 N 的力拉著一個放在水平地板的沉重箱子並以等速拉了 5 m。然後他感到筋疲力盡，而他的拉力以線性函數的方式從 50 N 到 0 N 慢慢地減少。計算此人在盒子上所做的總功。

A. 250 J.
B. 300 J.
C. 325 J.
D. 350 J.
E. 375 J.

8. A mass m_1 at one end of a spring establishes simple harmonic motion with a period T_1. If the mass is replaced with a different mass m_2, what is the period of this oscillation?

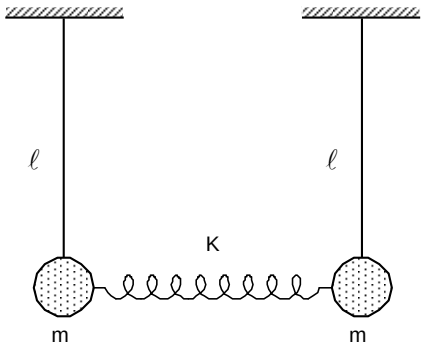
一個週期 T_1 的簡諧運動由一個質量物 m_1 和一個彈簧組立。 假設那個質量物被替換為另一 質量物 m_2，問該振盪的週期是什麼？

A. $2\pi\sqrt{m_2/m_1}$
B. $2\pi\sqrt{m_1/m_2}$
C. $2\pi\sqrt{T_1}$
D. $T_1\sqrt{m_1/m_2}$
E. $T_1\sqrt{m_2/m_1}$

9. As shown in the following figure, two pendulums are constructed with identical massless springs and mass objects (mass *m*). The initial distance between the masses is the equilibrium length of the spring (spring constant K). What is the frequency of the system if the two balls are oscillating out of phase?

如圖所示，兩個鐘擺由相同且無質量的彈簧和質量物 (質量 m) 所構成。質量物之間的初始 距離是彈簧 (彈簧常數 K) 的平衡長度。如果這兩個球以異相的方式振盪，這個系統的頻率 是什麼？

A. $\sqrt{\frac{g}{l}}$
B. $\sqrt{\frac{K}{2m}}$
C. $\sqrt{\frac{2K}{m}}$
D. $\sqrt{\frac{g}{l}+\frac{2K}{m}}$
E. $\sqrt{\frac{2g}{l}+\frac{K}{2m}}$



10. What is the minimum rotation period of a spherical neutron star with uniform mass density ρ such that material will not fly off from the equator? (G = Universal gravitational constant)

一個均勻質量密度 ρ 的球形中子星，它的自轉週期最小值應是什麼才能使得其物質不會從它的赤道飛出去? (G = 萬有引力常數)

A. $\left(\frac{3\pi}{\rho G}\right)^{1/2}$

B. $\left(\frac{\pi}{\rho G}\right)^{1/2}$

C. $\frac{3}{4\pi G}$

D. $\frac{4}{3\pi G}$

E. $\left(\frac{3}{8\pi\rho G}\right)^{1/2}$

11. A solid ball weighs 10 N in air, but 6 N when it is submerged in water. If the ball weighs 2 N when it is submerged in an unknown liquid, find the specific gravity of the unknown liquid.

一個實心球在空氣中的重量為 10 N，但當它被淹沒在水中時為 6 N。如果那個球被淹沒在一種不明液體時為 2N，求不明液體的比重。

A. 1.0.
B. 1.5.
C. 2.0.
D. 2.5.
E. 3.0.

12. A bullet (mass 10 g) is shot vertically and upward into a brick (2 kg). The bullet is embedded in the brick, and the brick is lifted upward 5 mm in a time interval of 1.2 ms. Assume that the force on the bullet is constant during penetration and air resistance is negligible. Find the initial kinetic energy of the bullet.

一顆子彈 (質量 10 g) 垂直向上射向一塊磚 (2 kg)。子彈藏在磚塊中，並用了 1.2 ms 把磚頭 向上抬起 5 mm。 假設在穿過過程中的力是恆定的而且空氣阻力的因素可忽略， 計算子彈的 初始動能。

A. 5 J.
B. 10 J.
C. 15 J.
D. 20 J.
E. 25 J.

13. A ball is hit from an inclined plane making an angle θ with the horizontal direction towards the downward direction. The ball has an initial velocity v_o and flies at an initial angle θ above the horizontal direction. Assume that the air resistance is negligible, at what horizontal distance from the initial point of hitting does the ball touch the inclined plane again?

一個球從一斜坡向下被擊出，斜坡與水平方向的角度為 θ。 球的初始速度為 v_o， 初始方向 高於水平方向的角度為 θ。 假設空氣阻力可忽略，當球擊中斜坡水平時， 從起初點計的距離 是多少？

A. $\dfrac{4v_o^2 \tan\theta}{g}$

B. $\dfrac{4v_o^2 \sin\theta\cos\theta}{g}$

C. $\dfrac{4v_o^2 \sin\theta}{g}$

D. $\dfrac{4v_o^2 \cos\theta}{g}$

E. $\dfrac{4v_o^2 \cot\theta}{g}$

14. Three identical blocks of mass m are stacked vertically. Block 1 is on top and rests on Block 2, which rests on Block 3, which rests on a table. What is the net force acting on Block 1? The weight of the 3 Blocks is measured to be 3 N.

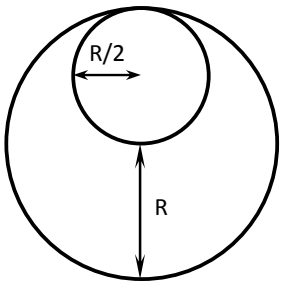
三個相同的方塊 (質量 m) 垂直堆疊在一起。 方塊 1 位於且靜止在方塊 2 的上方， 而它們同時靜止且位於方塊 3 的上方， 這三個方塊都靜止在桌子上。 問方塊 1 的淨作用力 是什麼？ 假設這三個方塊的重量被測量為 3 N。

A. Zero.
B. 1/3 N.
C. 1 N.
D. 2 N.
E. 3 N.

15. The following figure is a spherical styrofoam of radius R. A cavity of radius R/2 is made in the sphere. If the cavity is filled with a solid material of density 5 times of styrofoam and with the same thickness, where is the new center of mass?

圖中是一個半徑 R 的球形發泡膠。一個半徑 R/2 的球形空穴在裡面形成。如果這個空穴被填滿了一種固體材料，其密度為發泡膠的 5 倍而厚度與發泡膠相同，問新的質心在哪？

A. R / 2 upward from the center of the styrofoam sphere.
B. R / 3 upward from the center of the styrofoam sphere.
C. R / 4 upward from the center of the styrofoam sphere.
D. R / 5 upward from the center of the styrofoam sphere.
E. R / 6 upward from the center of the styrofoam sphere.



A. 從發泡膠球的中心向上起計 R / 2 的位置。
B. 從發泡膠球的中心向上起計 R / 3 的位置。
C. 從發泡膠球的中心向上起計 R / 4 的位置。
D. 從發泡膠球的中心向上起計 R / 5 的位置。
E. 從發泡膠球的中心向上起計 R / 6 的位置。

16. What is the potential energy for a tide cycle per square meter of ocean surface, if the sea water level difference between a high tide and a low tide is Δh (in meter)?

假設漲潮和退潮時的海水水位差是 Δh（以米作單位），以每平方米海面計算，一個潮週期的位能是多少？

A. 1.0 Δh^2 Watt-hour.
B. 1.2 Δh^2 Watt-hour.
C. 1.4 Δh^2 Watt-hour.
D. 1.6 Δh^2 Watt-hour.
E. 1.8 Δh^2 Watt-hour.

17. The weight of an object on the Moon surface is 1/6 of that on the Earth surface. A pendulum clock on the Earth environment ticks once per second. On the Moon environment, the clock would tick once every

一個物體在月球表面上的重量是其在地球表面的 1/6 倍。一個擺鐘在地球上每秒擺動一次。在月球上，這個時鐘擺動一次之時間為

A. $1/6$ s.
B. $1/\sqrt{6}$ s.
C. 1 s.
D. $\sqrt{6}$ s.
E. 6 s.

18. Geostationary satellite is its orbital period the same as the Earth's rotation period. Calculate the height of a geostationary satellite above the Earth's surface in terms of G (Gravitational constant), M_{Earth} (mass of the Earth), and R_{Earth} (radius of the Earth), where G, M_{Earth} and R_{Earth} are in SI units.

地球同步衛星是指它的軌道週期和地球自轉週期時間一樣。 以 G (萬有引力常數)， M_{Earth} (地球質量)，和 R_{Earth} (地球半徑) 來推導地球同步衛星在地球表面上的高度。其中，G， M_{Earth}，和 R_{Earth} 是以國際單位制為基礎。

A. $\dfrac{1231\times\left(G\cdot M_{\text{Earth}}\right)}{\pi}$

B. $\dfrac{1231\times\left(G\cdot M_{\text{Earth}}\right)^{1/3}}{\pi^{2/3}}-R_{\text{Earth}}$

C. $\dfrac{1231\times\left(G\cdot M_{\text{Earth}}\right)^{2/3}}{\pi^{1/3}}-R_{\text{Earth}}$

D. $\dfrac{1231\times\left(G\cdot M_{\text{Earth}}\right)^{1/3}}{\pi^{2/3}}+R_{\text{Earth}}$

E. $\dfrac{1231\times\left(G\cdot M_{\text{Earth}}\right)^{2/3}}{\pi^{1/3}}+R_{\text{Earth}}$

19. In comparison of a pendulum with large-angle oscillations (period T_L and magnitude A_L, maximum tension in the string F_L) to a pendulum with small-angle oscillations (period T_S and amplitude A_S, maximum tension in the string F_S), which of the following is true?

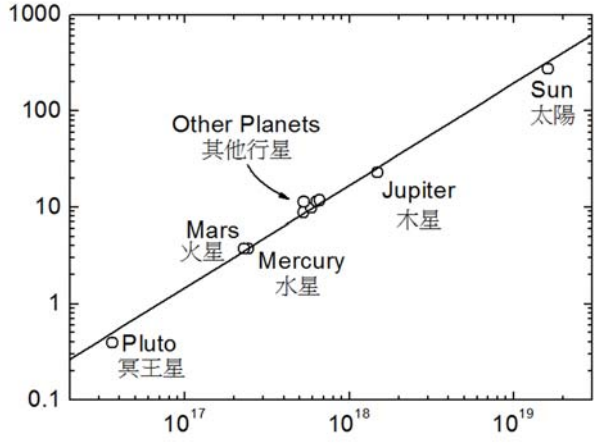
大角度 (週期 T_L， 幅度 A_L ， 繩子最大張力 F_L) 與小角度鐘擺 (週期 T_S， 幅度 A_S ， 繩子最大張力 F_S) 作比較，下列哪項是正確的？

A. $T_L > T_S$ and $F_L < F_S$.
B. $T_L < T_S$ and $F_L > F_S$.
C. $T_L > T_S$ and $F_L > F_S$.
D. $T_L < T_S$ and $F_L < F_S$.
E. $T_L = T_S$ and $F_L = F_S$.

20. The following figure is most probably a plot of
A. square of the orbital period of a planet against the cube of the semi-major axis of its orbit.
B. cube of the orbital period of a planet against the square of the semi-major axis of its orbit.
C. surface gravity of a planet against (M/R^2), where M is the mass and R is the equatorial radius of a planet.
D. surface gravity of a planet against (M^2/R), where M is the mass and R is the equatorial radius of a planet.
E. distance of a planet from the Galactic Center in Astronomical Unit against the mass of a planet.

以下的圖表最有可能是表示

A. 行星軌道週期之平方與其軌道半長軸之立方的關係。
B. 行星軌道週期之立方與其軌道半長軸之平方的關係。
C. 行星表面引力與 (M/R^2) 之關係，M 和 R 分別是行星之質量和赤道半徑。
D. 行星表面引力與 (M^2/R) 之關係，M 和 R 分別是行星之質量和赤道半徑。
E. 行星從銀河系中心的距離 (以天文單位計) 與其質量之關係。



--- END OF MULTIPLE CHOICE SESSION 選擇題完 ---

Open Problems 開放題
Total 5 Problems 共 5 題

1. Venus Transit (15 Marks)

Venus transit is an astronomical phenomenon when the planet Venus passes directly between the Sun and Earth. As illustrated in Fig. 1, for two different observation points A and B on the Earth, Venus appears as two separate black dots (A' and B') on the Sun surface.

(a) Suppose Venus takes 225 days to orbit the sun, calculate the ratio of $a_{\text{Earth}}/a_{\text{Venus}}$, where a_{Earth} and a_{Venus} are the mean distances of the planets from the Sun, respectively.

(b) On a Venus transit day, two observations were made in Hong Kong and Bangkok. They are geographically separated by 1800 km. Bangkok is 37° South of West of Hong Kong. Calculate the distance A'B'.

(c) Another observation finds that the Sun diameter is 290 times the distance of the two separate black dots. Calculate the diameter of the Sun.

(d) Calculate the time difference between the exit of Venus transit as observed in Hong Kong and Bangkok. Give your answer in minutes. (Hint: calculate the velocity of Earth relative to Sun, then the velocity of Sun and Venus relative to Earth, then the velocity of the shadow of Venus projected on Sun's surface, then the velocity of the shadow of Venus sweeping on Sun's surface.)

金星凌日 (15 分)

金星凌日是一種天文現象，當地球、金星、太陽成一直線時，便會出現金星凌於太陽表面的天象。 如圖 1 所示， 對於兩個不同在地球表面的觀測點 A 和 B， 金星會顯示在太陽表面兩個 獨立的黑點 (A' 和 B') 。

(a) 假設金星需要 225 天圍繞太陽，計算 $a_{\text{Earth}}/a_{\text{Venus}}$ 之值。 其中 a_{Earth} 和 a_{Venus} 分別是行星與太陽之平均距離。

(b) 在金星凌日當天，兩個觀察分別在香港和曼谷同時進行了。 在地理上， 這 2 個 城市的距離為 1800 km，而曼谷位於香港西方向南 37°。 計算 A' 與 B' 之距離。

(c) 假設太陽的直徑是兩個單獨黑點距離的 290 倍。計算太陽的直徑。

(d) 試計算香港和曼谷兩地觀測到金星凌日結束的時差，答案請以分鐘表達。
(提示：計算地球相對於太陽的速度，然後計算金星和太陽相對於地球的速度、 金星在太陽表面影子的速度、金星影子掃過太陽表面的速度。)

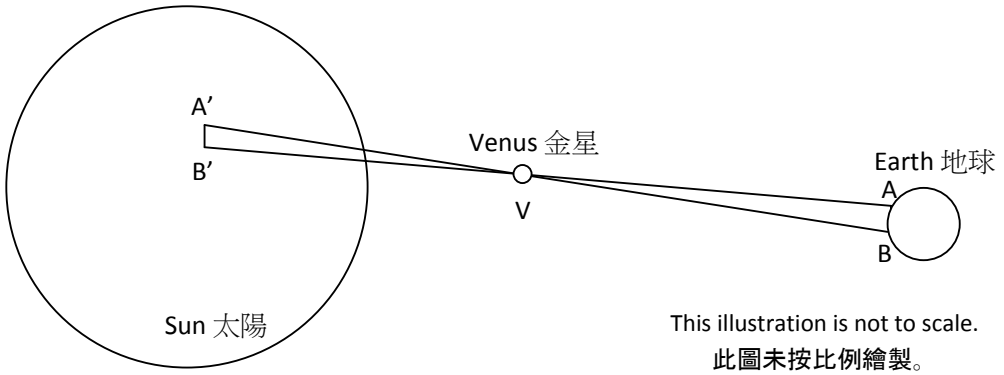


Fig. 1 – Venus Transit
圖 1 – 金星凌日

2. Terminal Velocity of Free Falling Object (10 Marks)

A spherical styrofoam of mass 2 g and radius 2 cm is released for free falling. The only forces acting on the styrofoam is the gravitational force and a drag force. The drag force is due to air resistance and is velocity (v) dependent. At low velocities, air resistance is negligible; but at high velocities, constant falling velocity is expected.

(a) Suppose the drag force is linear proportional to its falling velocity, write down the equation of motion. State the velocities in the limits of both low and high velocities. Sketch the graph of falling velocity against time with the indications of both the velocity limits.

(b) Suppose that the drag force is given by $F_d = -\frac{1}{2} C_d \cdot \rho \cdot A \cdot v^2$; where C_d is the drag coefficient, which is ~ 0.5 for a spherical object; ρ is the air density; A is the total cross-sectional area. Estimate the terminal velocity.

(c) Estimate the time taken by the falling styrofoam sphere to reach a velocity comparable to the terminal velocity.

自由下落物的終極速度（10 分）

一個質量 2 g 半徑 2 cm 的球形發泡膠正在自由下落。 作用於發泡膠的力只有引力和 後曳力。後曳力是由於空氣阻力並且是速度 (v) 的函數。 在低速度下，空氣阻力可忽略; 但在高的速度下，以匀速下降是可預期的。

(a) 假設後曳力是以線性正比於它的降落速度，寫下運動方程式。闡明在極低和極高速度 狀況下的速度值。草繪下降速度與時間的關係圖，在圖中標明極低和極高速度的狀況。

(b) 假設該後曳力的計算公式是 $F_d = -\frac{1}{2} C_d \cdot \rho \cdot A \cdot v^2$; 而 C_d 是阻力係數，對於一個球形物體大約等於 0.5 ; ρ 為空氣密度; A 是總橫截面面積。估計終端速度。

(c) 估算發泡膠球從靜止至達到相當於終極速度所需的時間。

3. Total Energy in a Surface Wave (15 Marks)

Consider a surface wave traveling on the sea (Fig. 2). At an instant, its surface profile can be approximated by $z = A\sin\left(\frac{2\pi x}{\lambda}\right)$, where A is the amplitude and λ is the wavelength.

(a) Find the potential energy per unit wavelength per unit width of the surface wave in terms of ρ (sea water density), g (acceleration due to gravitation), and A.

Hint: $\int \sin^2\left(\frac{2\pi x}{\lambda}\right) dx = \frac{x}{2} - \frac{\lambda \sin(4\pi x/\lambda)}{8\pi}$.

(b) Assume equipartition of energy, that is, the average potential energy equals the average kinetic energy of surface wave. Find the total energy over a whole wavelength.

(c) Given that the wave velocity $v_g = \frac{\lambda}{2T}$ and the relationship $\lambda = \frac{gT^2}{2\pi}$, derive the power of a wave period per unit width in terms of ρ, g, A, T (period of a wave), and H (wave height).

表面波的總能量（15 分）

在海面上有一表面波（圖 2），其表面輪廓在某時刻可表示為 $z = A\sin\left(\frac{2\pi x}{\lambda}\right)$。其中，$A$ 是幅度而 λ 是波長。

(a) 計算表面波每波長及每單位寬度的勢能。答案請以 ρ (海水密度)， g (重力加速度)，和 A 表達。

提示：$\int \sin^2\left(\frac{2\pi x}{\lambda}\right) dx = \frac{x}{2} - \frac{\lambda \sin(4\pi x/\lambda)}{8\pi}$.

(b) 假設能量是均分的，即表面波的平均勢能等於其平均動能。 計算一整體波長每單位寬度的總能量。

(c) 已知波速為 $v_g = \frac{\lambda}{2T}$，及關係 $\lambda = \frac{gT^2}{2\pi}$， 推導出一個週期表面波每單位寬度的功率。 答案請以 ρ， g， A， T （波週期），與及 H（波浪高度）表達。

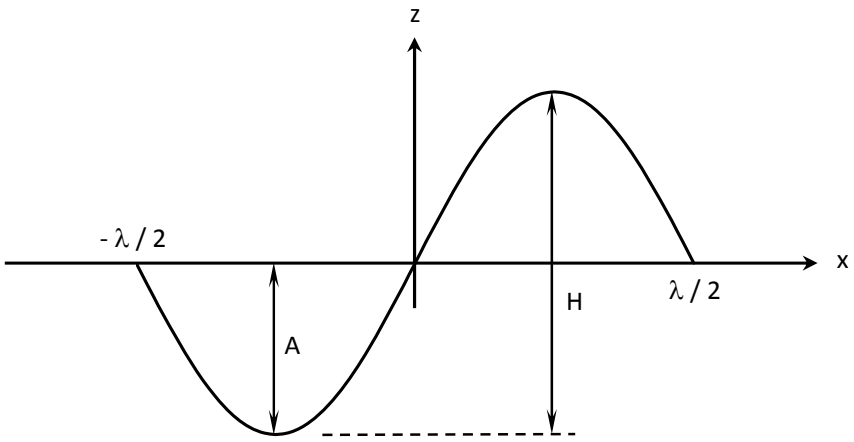


Fig. 2 – Surface Wave
圖 2 – 表面波

4. A Sliding Block up a Slope Platform (10 Marks)

A solid cube (length L each) of mass m starts to slide up a stationary slope platform from the bottom. The slope platform has a mass M, an inclination angle θ and the slope is smooth (Fig. 3). The slope platform is free to slide along the horizontal surface without friction.

(a) Calculate the initial velocity v_0 of the sliding block in terms of h, θ, m, M, L, and g, such that the cube just reaches the upper end of the slope.

(b) The cube then slides down the slope platform. What is the final kinetic energy of the cube and the slope platform when the cube reaches the lower end of the slope platform?

一個依住斜坡平台向上滑動的方塊 (10分)

如圖 3 所示， 一個質量為 m 的立方體 (每邊長度 L)，正從靜止的斜坡台的底部開始向上滑動。斜坡台的質量 為 M，傾斜角為 θ ，表面是光滑的。斜坡台可以自由地沿水平表面滑動。

(a) 已知立方體剛剛能到達斜坡的上端，試計算滑動立方體的初始速度 v_0， 答案請以 h, θ, m, M, L 和 g 表達。

(b) 其後立方體在斜坡台上向下滑動，當立方體到達斜坡台下端時，立方體和斜坡台的總動能是什麼?

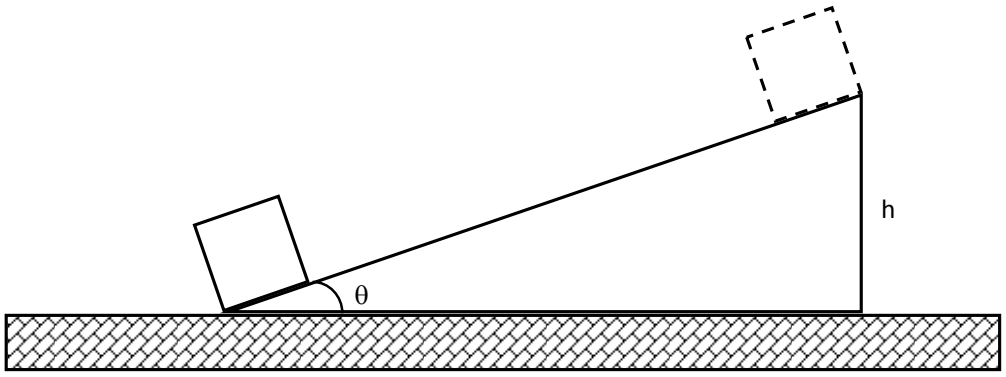


Fig. 3 – Sliding Block up a Slope Platform
圖 3 – 斜坡平台滑動的方塊

5. A Physical Pendulum Motion (10 Marks)

Two identical uniform thin rods (mass m, and length L) are connected at right angle to form a rigid upside-down "T" shape. The center of one rod is connected to one end of another rod. The upside-down "T" shape is suspended and allowed to form a pendulum motion, as shown in Fig. 4.

(a) The kinetic energy of the system is $\frac{17}{24}mv^2$, where v is the velocity of the meeting point of the two rods. Derive the equation of the pendulum motion in terms of m, L, and g.

(b) Calculate the period of the system if θ is small. (Hint: $\sqrt{1-\frac{x^2}{L^2}} \approx 1-\frac{x^2}{2L^2}$ when x << L.)

(c) At t = 0, the meeting point of the two rods is displaced by x_0. What is the time when the displacement becomes $x_0/2$ the first time?

一個物理鐘擺的運動 (10 分)

兩個相同均勻的薄桿 （ 質量 m， 長度 L ）被連接成直角，以形成一個硬性倒掛的 "T" 形。 一個桿的中心被連接到另一個桿的末端。 如圖 4 所示， 這個倒掛的 "T" 形被懸掛並容許成一 個鐘擺運動。

(a) 已知該系統的動能是 $\frac{17}{24}mv^2$， 其中 v 為兩桿交點的速度。以 m，L，和 g 來推導鐘擺運動方程式。

(b) 如果 θ 的數值是很小，求系統的週期。(提示：當 x << L 時，$\sqrt{1-\frac{x^2}{L^2}} \approx 1-\frac{x^2}{2L^2}$)

(c) 當 t = 0 時，兩桿交點的位移為 x_0。問位移第一次為 $x_0/2$ 時，時間是什麼？

Axis of oscillation: perpendicular to paper
振盪軸： 垂直於紙

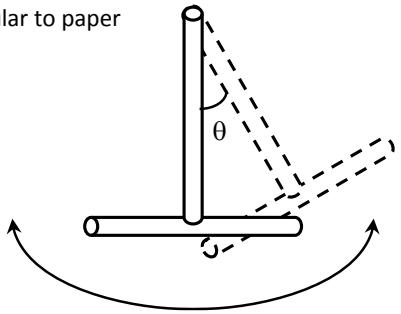


Fig. 4 – Physical Pendulum Motion
圖 4 - 物理鐘擺運動

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