Hong Kong Physics Olympiad 2015
2015 香港物理奧林匹克

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Education Bureau
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香港資優教育學苑
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The Physical Society of Hong Kong
香港物理學會
Hong Kong Physics Olympiad Committee
香港物理奧林匹克委員會

10 May, 2015
2015 年 **5** 月 **10** 日

Rules and Regulations 競賽規則

1 All questions are in bilingual versions. You can answer in either Chinese or English, but only ONE language should be used throughout the whole paper.

所有題目均爲中英對照。你可選擇以中文或英文作答，惟全卷必須以單一語言作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

選擇題的答題紙將於比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你即可開始作答開放式題目，而無須等候任何宣佈。

3. On the cover of the answer book and the multiple-choice answer sheet, please write your 8-digit Contestant number and your English Name in the “Name” box.

在答題簿封面及選擇題答題紙上，請填上你的 8 位數字參賽者號碼及於“Name”欄上填上你的英文姓名。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆**完全**塗黑。

5. The open problems are long. Please read the whole problem first before attempting to solve them. If there are parts that you cannot solve, you are allowed to treat the answer as a known answer to solve the following parts.

開放式問答題較長，請將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來解答其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別註明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.8 m/s^2
Gravitational constant 萬有引力常數	G	6.67×10^{-11} N m^2/kg^2
Radius of Earth 地球半徑	R_E	6378 km
Sun-Earth distance 太陽-地球距離 (= 1 Astronomical Unit (AU)) (= 1 天文單位(AU))	r_E	1.5×10^{11} m
Earth-Moon distance 地球-月球距離	r_m	384400 km
Mass of Sun 太陽質量	M_{Sun}	1.99×10^{30} kg
Mass of Earth 地球質量	M_E	5.98×10^{24} kg
Density of Air 空氣密度	ρ_0	1.2 kg/m^3
Density of Water 水密度	ρ_w	1000 kg/m^3

Trigonometric Identities:
三角學恆等式：

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

Tayler Series:
泰勒級數:

$$\sin(x)\approx x-\frac{x^3}{6}+\frac{x^5}{120}-\cdots$$

$$\cos(x)\approx 1-\frac{x^2}{2}+\frac{x^4}{24}-\cdots$$

$$\tan(x)\approx x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots$$

Multiple Choice Questions

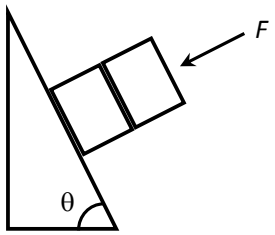
Select one answer in each question. For each question, 2 marks for correct answer, 0 mark for no answer, minus 0.5 mark for wrong answer, but the lowest mark of the multiple choice section is 0 mark.

選擇題

每題選擇一個答案，每題答對 2 分，不答 0 分，答錯扣 0.5 分，但全部選擇題最低為 0 分。

1. A board (mass *m*) is sandwiched between a stationary incline (angle θ) and another identical board. A force acts on the outer board to keep the boards from slipping. If the coefficient of friction between all the surfaces is μ, what is the component of the minimum compression force, *F* normal to the board surface?

   一塊板（質量 *m*）被夾在一固定斜台（角度 θ）與另一相同板子之間。一力作用在外層板子以保持它們不會滑下。假設所有表面之間的摩擦係數為 μ，垂直於外層板子的最小壓縮力的分量 *F* 是多少？

   A. $mg(\sin\theta - 2\mu\cos\theta)$

   B. $\dfrac{mg(\sin\theta - \mu\cos\theta)}{\mu}$

   C. $\dfrac{mg(\sin\theta - 2\mu\cos\theta)}{2\mu}$

   D. $\dfrac{mg(\sin\theta - 3\mu\cos\theta)}{2\mu}$

   E. $(mg\sin\theta + \mu\cos\theta)$

2. A uniform metal ball (mass *m*) is tied to the end of a string (massless, length *L*). The other end of the string is fixed. The ball is moving in a vertical circle centered at the fixed end of the string. Suppose the velocity of the ball at the lowest point is v_o, what is the string tension when it makes an angle θ to the downward vertical position?

   一個均勻質量的金屬球（質量 *m*）被繩子（無質量，長度 *L*）的一端綁著，另一端則固定。金屬球正繞著繩子的固定點垂直圈打轉。假設金屬球在最低點的速度為 v_o，當它成向下 θ 角度時（從垂直線計），繩子的張力是多少？

   A. $\dfrac{mv_o^2}{L} + mg(\cos\theta - 1)$

   B. $\dfrac{mv_o^2}{L} + mg(2\cos\theta - 1)$

   C. $\dfrac{mv_o^2}{L} + mg(3\cos\theta - 2)$

   D. $\dfrac{mv_o^2}{2L} + mg(\cos\theta - 1)$

   E. $\dfrac{mv_o^2}{2L} + mg(2\cos\theta - 1)$

3. The orbital properties of a geostationary satellite include i) its orbit is directly over the Earth equator, and ii) its orbital period is the same as the Earth's rotation period. Suppose a lunar stationary satellite (its orbital period is the same as the Moon's rotation period) is placed over the Moon equator, what is the value of the ratio $(R_{Earth} + H_{Earth}) / (R_{Moon} + H_{Moon})$? R_{Earth} and R_{Moon} are the Earth and the Moon radii, H_{Earth} and H_{Moon} are the satellite heights from the Earth and from the Moon surfaces, respectively. You may assume (Earth mass) / (Moon mass) = 81 and rotation period of the Moon = 27 days.

地球同步衛星的軌道有以下特性，包括 一）它的軌道是直接在地球赤道的上空，與及 二）其軌道週期與地球轉動週期是相同的。假設一月球同步衛星（軌道週期與月球轉動週期相同）被放置在月球的赤道上空，$(R_{Earth} + H_{Earth}) / (R_{Moon} + H_{Moon})$ 的比率值是甚麼？R_{Earth} 和 R_{Moon} 分別是地球和月球之半徑，而 H_{Earth} 和 H_{Moon} 分別是從地球和月球地面上的衛星的高度。你可假設（地球質量）/（月球質量）= 81，而月球的自轉週期為 27 天。

A. (81×27)
B. $(81 \times 27)^2$
C. $(81 / 27)^3$
D. $(1/9)^3$
E. $(1/9)^{1/3}$

4. A dam (height H, width W, one wall inclined at angle θ) is fully filled with water (density ρ). If g is the acceleration due to gravity, the resultant force exerted by the water on the dam wall is given by

一個完全盛載滿了水（密度 ρ）的壩（高度 H，寬度 W，而其中一面之壩壁傾斜角為 θ），假設 g 是重力加速度，水在壩壁所造成的合力是

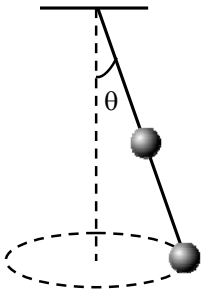
A. $(\rho g H^2 W)/2$
B. $(\rho g H^2 W)/(2\sin\theta)$
C. $(\rho g H W)/(2\cos\theta)$
D. $(\rho g H^2 W)/(2\cos\theta)$
E. $(\rho g H W)/(\sin\theta + \cos\theta)$

Remark: Question 5 is cancelled.

5. A conical pendulum with 2 identical bobs on a massless rod (length L) makes an angle of θ with the vertical. One bob is mounted at $L/2$ of the wire, whereas the other is mounted at the end of the rod. The period of the rotational motion is:

一個以 2 個完全相同的錘子和一根無質量的棒子（長度 L）所組成的錐形鐘擺，旋轉時與垂直方向成角度 θ 。一錘子被安裝在棒子的 $L/2$ 位置，而另一個則被安裝在棒子的末端 。該系統的旋轉週期是：

A. $\pi\sqrt{2L\sin\theta/g}$
B. $\pi\sqrt{3L\cos\theta/g}$
C. $2\pi\sqrt{2L\sin\theta/g}$
D. $2\pi\sqrt{3L\cos\theta/g}$
E. $\pi\sqrt{L\tan\theta/g}$



6. Suppose you are standing in an elevator, the elevator is moving upward at a constant speed of 0.5 m/s, the net force on you is

假設你站在電梯中，電梯正以 0.5 m/s 的速度上升，在你身上的淨力是

A. zero.
B. + 0.5 N.
C. - 9.8 N.
D. + 9.8 N.
E. your weight.

7. A pendulum has a length of 1 m. How many swings does the pendulum perform in an hour?

一個 1 米長的鐘擺，一小時可擺動多少次？

A. 60.
B. 300.
C. 900.
D. 1800.
E. 3600.

8. A ball (mass m_2) is hanging at rest from a wire (negligible mass). A head-on elastic collision occurs when it is struck by a stone (mass m_1, horizontal velocity v_1 before collision), the ball swings to a maximum height (h) above the original level. The maximum height (h) is

起初，一個球（質量 m_2）靜止地掛在繩子上（繩子本身的質量可以忽略不計）。當它被一塊石頭（質量 m_1，碰撞前水平速度為 v_1）擊中並發生對正彈性碰撞時，從原高度計，該球擺動至最大高度（h）。最大高度（h）是

A. $\frac{1}{2g}\left(\frac{m_1}{m_1+m_2}\right)v_1^{\,2}$
B. $\frac{1}{2g}\left(\frac{m_1}{m_1+m_2}\right)^2 v_1^{\,2}$
C. $\frac{2}{g}\left(\frac{m_1}{m_1+m_2}\right)v_1^{\,2}$
D. $\frac{2}{g}\left(\frac{m_1}{m_1+m_2}\right)^2 v_1^{\,2}$
E. $\frac{2}{g}\left(\frac{m_1}{m_2}\right)^2 v_1^{\,2}$

9. An electric water pump draws water from a well of depth 10 m at a rate 0.1 m^3/s. The water is then rejected from the pump with velocity 2 m/s. If the efficiency of the pump is 80%, the power consumption of the pump is

一電動水泵以 0.1 m^3/s 的速率將水從 10 m 深的井泵到地面。水然後以 2 m/s 的速度從泵流出。假設泵的效率是 80%，泵的功率是

A. 8 kW.
B. 10 kW.
C. 12.5 kW.
D. 17.5 kW.
E. 20 kW.

10. A car (mass *m*) undergoes a circular horizontal track (radius *r*) at constant speed *v*. Suppose *h* is the height of the center of mass (CG) above ground, and *a* is the horizontal separations between the left/right wheels and the center of mass. What are the normal forces N_1 and N_2? The direction of friction force *f* is indicated.

一輛汽車（質量 *m*）以恆定速度 *v* 在一圓形且水平的路上行駛（半徑為 r）。假設 *h* 是地面與質心（CG）的高度，*a* 是左/右車輪和質心之間的水平距離。圖中所示是摩擦力 *f* 的方向。問 N_1 和 N_2 的法向力是甚麼？

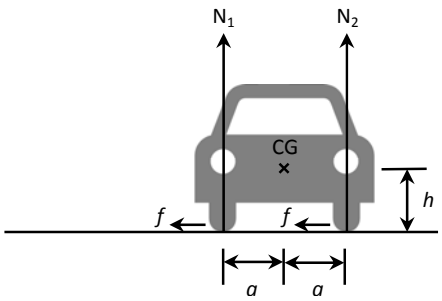
A. $N_1 = \frac{1}{2}\left(mg - \frac{hmv^2}{ar}\right),\ N_2 = \frac{1}{2}\left(mg + \frac{hmv^2}{ar}\right).$

B. $N_1 = \frac{1}{2}\left(mg + \frac{hmv^2}{ar}\right),\ N_2 = \frac{1}{2}\left(mg - \frac{hmv^2}{ar}\right).$

C. $N_1 = \frac{mg}{2} - \frac{hmv^2}{ar},\quad N_2 = \frac{mg}{2} + \frac{hmv^2}{ar}.$

D. $N_1 = \frac{mg}{2} + \frac{hmv^2}{ar},\quad N_2 = \frac{mg}{2} - \frac{hmv^2}{ar}.$

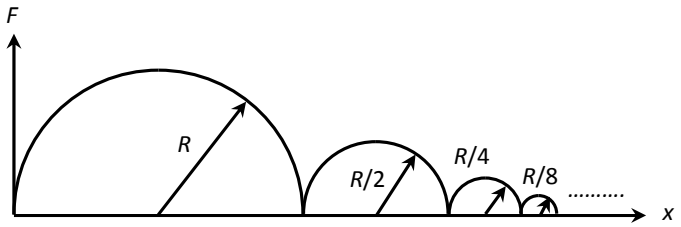
E. $N_1 = \frac{mg}{2} + \frac{hfmv^2}{ar},\quad N_2 = \frac{mg}{2} - \frac{hfmv^2}{ar}.$



11. How much work is done in the following force (*F*) against distance (*x*) graph? The force is an infinite series of semi circles with radii of R, R/2, R/4, R/8, …….

以下的力（*F*）與距離（*x*）的關係圖有多少功？該力的性質是由無窮級數的半圓所組成，半徑為 R， R/2， R/4， R/8， ……。

A. (1 / 3) πR^2.
B. (2 / 3) πR^2.
C. πR^2.
D. (4 / 3) πR^2.
E. (5 / 3) πR^2.



12. A binary star system consists of 2 stars orbiting around their common center of mass. The two stars have identical mass. The orbital speed of each star is 200 km/s and the orbital period of each is 12.2 days. Find the mass of each star.

一雙星體系擁有兩顆星，兩顆星的質量相同，而它們的軌道是圍繞着它們共同的質心。每顆星都以 200 km/s 的軌道速度運行，而它們的軌道週期為 12.2 天。求每顆星的質量。

A. 0.00010 Solar Mass
B. 0.00020 Solar Mass
C. 10.1 Solar Mass
D. 20.2 Solar Mass
E. 40.4 Solar Mass

A. 0.00010 太陽質量
B. 0.00020 太陽質量
C. 10.1 太陽質量
D. 20.2 太陽質量
E. 40.4 太陽質量

13. A block lies on a horizontal and frictionless turntable. The block is connected to a string which is attached to the center of the turntable. The turntable is rotating with slow uniform angular speed ω. An observer sits on the turntable. According to the observer, which of the following is/are true?
i. The block is accelerating.
ii. The block is not accelerating.
iii. The magnitude of the string tension is proportional to ω^2.

一方塊躺在一水平且無摩擦力的轉盤上。該方塊被一條無質量的繩子連接到轉盤的中心位置。轉盤以緩慢均勻角速率 ω 旋轉。一觀察者坐在轉盤上。根據該觀察者，下列那項是正確？

i. 該方塊正在加速。
ii. 該方塊沒有加速。
iii. 繩子的張力與 ω^2 為正比。

A. (i) only.
B. (ii) only.
C. (i) and (iii).
D. (ii) and (iii).
E. (i), (ii), and (iii).

14. A rock is launched vertically on the Earth's surface at 8 km/s. Ignoring air resistance and the Earth rotation, the height of the projectile that can rise is:

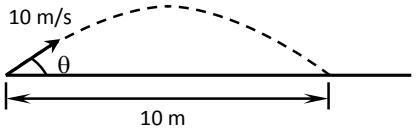
一塊石以 8 km/s 從地球表面垂直發射。在忽略空氣阻力和地球旋轉的情況下，該石塊可達到的高度是：

A. 3,265 km.
B. 6,683 km.
C. 10,597 km.
D. 20,196 km.
E. 40,328 km.

15. A ball is projected at angle(s) θ and at speed 10 m/s, and eventually the range of the projectile is 10 m. The possible value(s) of θ is/are:

一個球以 θ 角度並以 10 m/s 的速度被拋射，最終該球的射程是 10 m。θ 的可能值是：

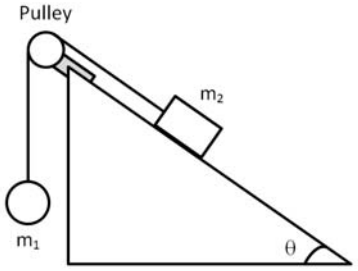
A. 39.25°.
B. 50.75°.
C. 78.5°.
D. 39.25° or 50.75°.
E. 50.75° or 78.5°.



16. As shown in the figure, a ball (mass m_1) and a block (mass m_2) are attached by a cord (massless), which passes over a frictionless pulley. The block lies on a frictionless incline of angle θ. What is the condition that the block will accelerate down the incline?

如圖所示，一個球（質量 m_1）和一方塊（質量 m_2）被連接到一條無質量的線，並繞過一個無摩擦力的滑輪。該方塊躺在一個無摩擦力的斜面上，斜面的角度是 θ。如要方塊向斜面下加速，該斜面角度的條件是甚麼？

A. $\sin\theta > m_1/m_2$.
B. $\cos\theta > m_1/m_2$.
C. $\sin\theta > (m_1/m_2)^2$.
D. $\cos\theta > (m_1/m_2)^2$
E. $(\sin\theta + \cos\theta) > m_1/m_2$.



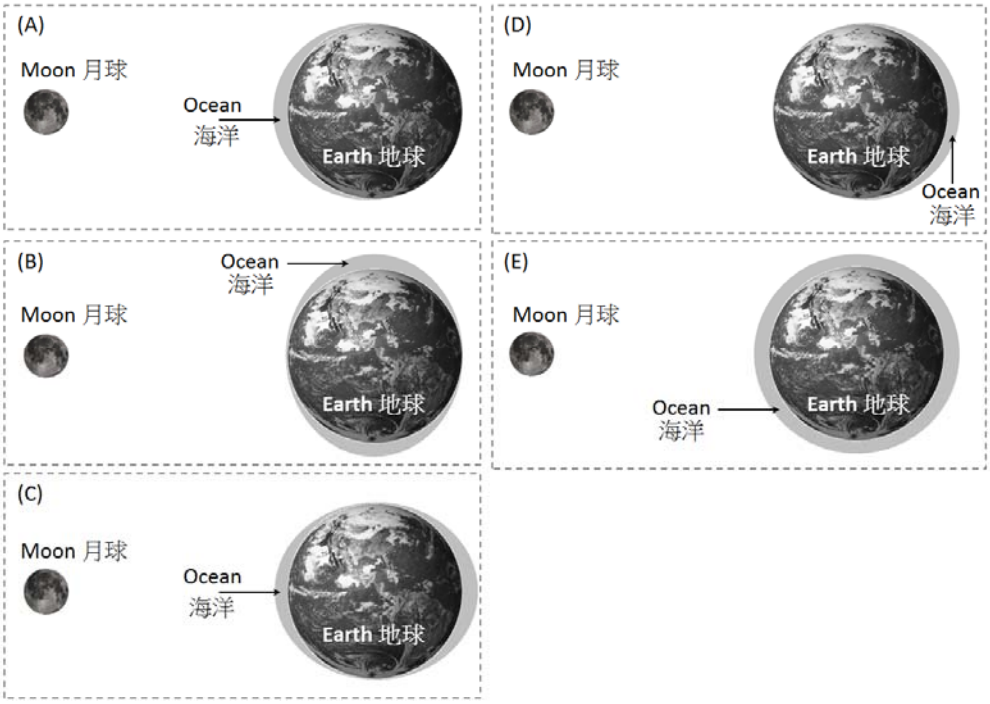
17. A solid sphere is attached at one end of a cord (length *R*) and is set into motion in a vertical circle about the other end which is fixed. Determine the speed of the sphere when it passes over the top of the circle with zero cord tension.

一實心球被連接在一條繩子的末端（長度為 *R*），並以繩子的另一端作固定點垂直圈打轉。當它越過圓圈的頂部，同時繩子的張力為零時，求實心球當時的速度。

A. $\sqrt{gR}$
B. $\sqrt{gR\sin\theta}$
C. $\sqrt{gR\cos\theta}$
D. $\sqrt{gR\sin\theta\cos\theta}$
E. $\sqrt{gR\tan\theta}$

18. By considering the gravitational forces of the Moon and the Earth only, which of the following ocean tide is most possible? The diagram is not drawn to scale.

只考慮月球和地球的引力，下列那一個海潮是最有可能發生的？該圖是不按比例繪製。



19. A small block (mass 0.2 kg) is attached to a massless spring and undergoes a simple harmonic motion on a smooth surface (amplitude A = 0.1 m, angular frequency ω = 0.5 radian/s). The displacement of the particle at time t from its equilibrium position is described by

$$x(t) = A\cos(-\omega t),$$

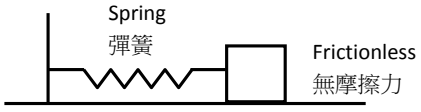
If the potential energy of the spring at the equilibrium position is zero, what is the total energy of the system?

一小方塊（質量 0.2 kg）連接到彈簧並且在光滑水平面上展開簡諧運動（振幅 A = 0.1 m，角頻率 ω = 0.5 radian/s）。其相對於平衡點的位移與時間的關係是

$$x(t) = A\cos(-\omega t)，$$

如果該彈簧的平衡點勢能為零，該系統的總能量是多少？

A. 0.05 mJ.
B. 0.10 mJ.
C. 0.15 mJ.
D. 0.20 mJ.
E. 0.25 mJ.



20. The figure shows a “L” shape object of uniform mass per unit area ρ. Where is the center of mass $\vec{r}_G$?

該圖顯示了一個“L”形的均勻物體，每單位面積的質量是 ρ。質心 $\vec{r}_G$ 在何處？

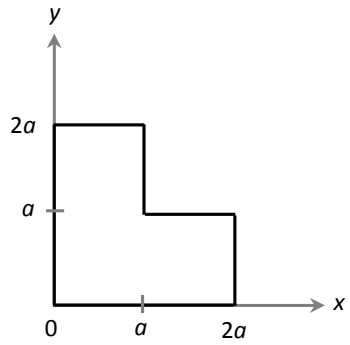
A. $\vec{r}_G = \frac{2a}{3}\left(\hat{i} + \hat{j}\right).$

B. $\vec{r}_G = \frac{3a}{4}\left(\hat{i} + \hat{j}\right).$

C. $\vec{r}_G = \frac{4a}{5}\left(\hat{i} + \hat{j}\right).$

D. $\vec{r}_G = \frac{5a}{6}\left(\hat{i} + \hat{j}\right).$

E. $\vec{r}_G = a\left(\hat{i} + \hat{j}\right).$



--- END OF MULTIPLE CHOICE SESSION 選擇題完 ---

Open Problems 開放題
Total 5 Problems 共 5 題

1. A bouncing ball (10 Marks)

A ball is allowed to drop freely from rest and at a height H_o above the ground. After hitting the ground, the ball bounces to a height H_1. The bouncing process repeats continuously until the bouncing height is too small to be observed.

(a) Calculate the coefficient of restitution, C_R, in terms of H_o and H_1. C_R is defined as:

C_R = Relative speed after collision / Relative speed before collision.

(b) Determine the total distance that the ball travels. Express your answer in terms of H_o and C_R.

(c) Find the total time duration that the ball travels. Express your answer in terms of H_o and C_R.

Hints: You may assume:

i. C_R is constant throughout all the bouncing,
ii. Time of contact between the ball and the ground is negligible, and
iii. Air resistance is negligible.

反彈球（10 分）

一個球從靜止狀態由 H_o 的高度自由下降到地面。撞落地面後，球反彈到高度 H_1。彈跳過程不斷地重複，直至彈跳高度太小而不能被察覺。

(a) 計算恢復係數，C_R。答案請以 H_o 與及 H_1 表達。C_R 的定義為：

C_R = 碰撞後的相對速度 / 碰撞前相對速度。

(b) 求該球移動了的總距離。答案請以 H_o 與及 C_R 表達。

(c) 求該球移動了的總時間。答案請以 H_o 與及 C_R 表達。

提示：你可以假設：

i. 在所有的反彈，C_R 是常數不變的，
ii. 球和地面之間的接觸時間可以忽略不計，和
iii. 空氣阻力可忽略不計。

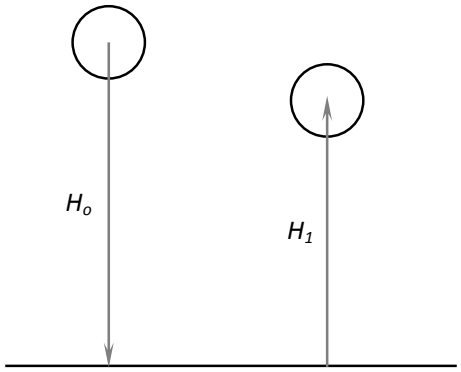


Fig. 1 – Bouncing ball
圖 1 – 反彈球

2. A suspended horizontal platform (15 Marks)

As shown in Fig. 2, a uniform rectangular platform (length L, mass m) is suspended by two ropes making angle θ_1 and θ_2 to the vertical. In order to keep the platform horizontal, a load (mass $2m$) is placed on the platform.

(a) Find the tensions T_1 and T_2 in terms of m, θ_1, θ_2, and/or L.

(b) In terms of m, θ_1, θ_2, and/or L, determine x in order to keep the platform horizontal.

(c) If ($\theta_1 + \theta_2 = 90°$) and $x = L / 8$, find θ_1 and θ_2.

懸吊水平平台（15 分）

如圖 2 所示，一個均勻矩形平台（長度為 L，質量 m）懸掛在兩根繩索上。從垂直計，該角度為 θ_1 和 θ_2。為了保持該平台的水平性，另一負載物（質量 $2m$）被放置在平台上。

(a) 以 m，θ_1，θ_2，和/或 L 作表達，計算 T_1 和 T_2 的張力。

(b) 以 m，θ_1，θ_2，和/或 L 作表達，計算 x 以保持該平台水平性。

(c) 如果 ($\theta_1 + \theta_2 = 90°$) 和 $x = L / 8$，計算 θ_1 和 θ_2。

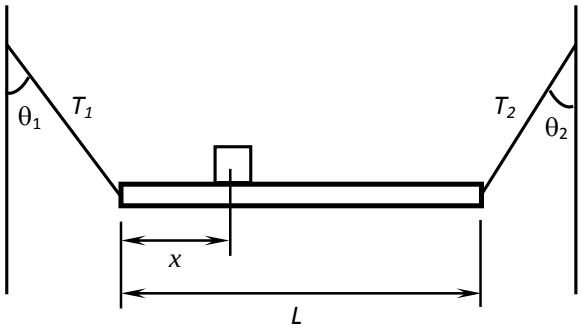


Fig. 2 – Suspended horizontal platform
圖 2 – 懸吊水平平台

3. Oscillating sphere in a cylindrical trough (15 Marks)

A solid sphere (radius *R*, mass *m*, and uniform density) rolls without slipping in a cylindrical trough (radius = 5*R*) with a small angle θ displaced from the equilibrium position. The kinetic energy of the sphere is $KE = \frac{56}{5}mR^2\omega^2$, where ω is the instantaneous angular velocity.

(a) Find the potential energy of the sphere in terms of *m*, *g*, *R*, and θ, where *g* is the acceleration due to gravity.

(b) Find the total energy of the sphere oscillator. Express your answer in the form of energy transformation in simple harmonic mass-spring system, that is, Total Energy $E = \frac{1}{2}mv_x^{\ 2} + \frac{1}{2}kx^2$, where you may consider *x* as an angular displacement.

(c) Determine the effective mass (m_{eff}), effective spring constant (k_{eff}), and period of the oscillation.

在圓筒形槽擺動的球（15 分）

一實心球（半徑 R，質量 m，和均勻密度）在沒有滑動的情況下，以從平衡點計的細小角度 θ 在圓筒形槽（半徑 5R）滾動。已知該球的動能是 $KE = \frac{56}{5}mR^2\omega^2$ ，而 ω 是瞬時角速度。

(a) 以 m，g，R 和/或 θ 作表達，求球體的勢能。

(b) 計算球體擺動系統的總能量。答案請以質量與彈簧的簡諧運動能量轉換形式表達，即是總能量 $E = \frac{1}{2}mv_x^{\ 2} + \frac{1}{2}kx^2$ ，其中你可考慮 *x* 為角位移。

(c) 計算該系統的有效質量（m_{eff}），有效彈簧常數（k_{eff}），與及滾動週期。

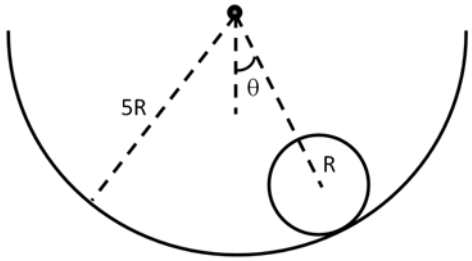


Fig. 3 – Oscillating sphere in a cylindrical trough
圖 3 – 在圓筒形槽擺動的球

4. Maximum gradient that a vehicle can climb up (15 Marks)

An engineer attempts to design a road with an angle of inclination θ to allow a vehicle (mass m) to climb. On one hand the engineer would like to maximize the gradient; on the other hand, the design has to satisfy the specifications that the wheels have to be adhesive to the gradient when the vehicle is a

i. rear wheel drive;

ii. front wheel drive; and

iii. four wheel drive.

The specifications further require a vehicle

iv. to avoid overturning; and

v. to avoid front and rear ends touch the road surface when climbing up a slope from a horizontal road.

A typical vehicle has the following dimensions (Fig. 4). Find the maximum gradients such that the vehicle can climb up at each of the above possibilities (i) to (v). Express your answers in terms of μ_R (the coefficient of static friction between the rear wheels and the road gradient), μ_F (the coefficient of static friction between the front wheels and the gradient), C_R (rear under body clearance), C_F (front under body clearance), H_R (rear over hanging), H_F (front over hanging), h (height between center or mass and the gradient), L_R, L_F, and/or L ($L = L_R + L_F$). You may neglect air resistance and assume that the vehicle is moving at constant speed.

車輛能行駛的最大路面傾斜度（15 分）

一名工程師試圖設計一條傾斜路面（傾斜角 θ）以允許車輛（質量 m）上行。一方面，工程師希望設計一條具有最大傾斜度的道路; 而另一方面，規格要求以下車輪必須緊貼斜路的路面行駛：

i. 後輪驅動 ;

ii. 前輪驅動 ; 與及

iii. 四輪驅動。

規格進一步要求：

iv. 避免車輛翻轉 ;

v. 從水平路面轉上傾斜路面時避免車身前部及後部接觸到路面。

一輛典型的車輛具有如下尺寸（圖 4）。根據上述（i）至（v）所描述的情況，計算每一個情況路面最大的傾斜度以允許車輛行駛。答案請以 μ_R（後輪與傾斜路路面之間的靜摩擦係數），μ_F（前輪與傾斜路路面之間的靜摩擦係數），C_R（車身後方與路面的間隙），C_F（車身前方與路面的間隙），H_R（車身後方懸掛間隙），H_F（車身前方懸掛間隙），h（質心與路面的高度），L_R， L_F，和/或者 L （$L = L_R + L_F$）。空氣阻力可忽略不計，並且假設車輛以恆定速度行駛。

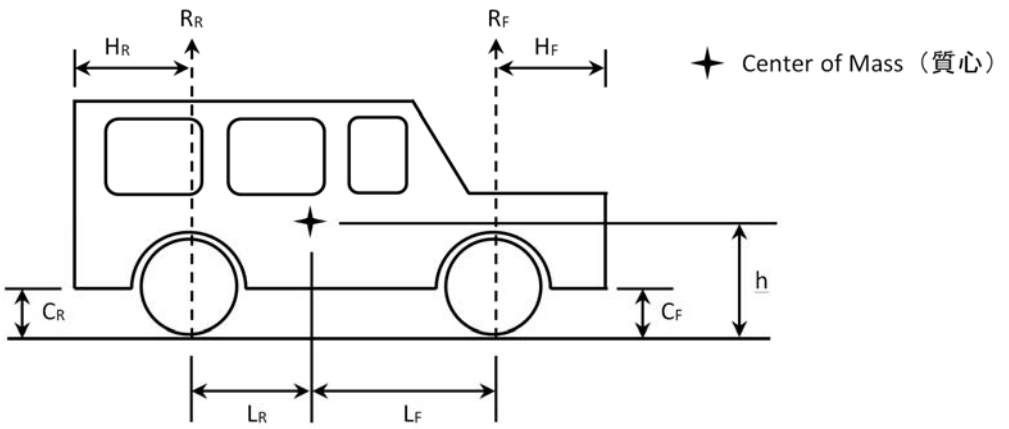


Fig. 4 – Specifications of typical vehicle
圖 4 – 典型車輛的規格

5. Fuel for rocket propulsion (5 Marks)

Consider a rocket in outer space (gravitational forces are negligible). The combustion products are ejected at a constant speed v_e relative to the rocket (total initial mass m_i).

(a) By considering the conservation of momentum only, derive the equation of the rocket velocity change Δv in terms of v_e, total mass change Δm of the rocket, and m_i.
Hint: You may ignore higher order terms.

(b) Given that v_e = 2.5 km/s, and that the rocket generates a constant force 3.5×10^7 N during the fuel burn, calculate the rate of fuel mass change.

火箭推進器之燃料（5 分）

考慮一枚太空中的火箭（引力可忽略不計）。燃料經燃燒後以相對於火箭的恆定速度 v_e 噴出火箭外。火箭的總初始質量為 m_i。

(a) 只考慮動量守恆，試導出火箭速度變化 Δv 的方程。答案請以 v_e，火箭之總質量變化 Δm，與及 m_i 作表達。
提示：你可忽略高階微量項。

(b) 已知 v_e = 2.5 km/s，且火箭在燃料燃燒過程中產生恆力 3.5×10^7 N，試計算燃料質量變化率。

--- END OF EXAM PAPER 全卷完 ---