Hong Kong Physics Olympiad 2017
2017 年香港物理奧林匹克競賽

Organisers 合辦機構

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教育局

The Hong Kong Academy for Gifted Education

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The Physical Society of Hong Kong

香港物理學會

Hong Kong Physics Olympiad Committee

香港物理奧林匹克委員會

14 May, 2017

2017 年 5 月 14 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English, but only ONE language should be used throughout the whole paper.

   所有題目均為中英對照。你可選擇以中文或英文作答，惟全卷必須以單一語言作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將於比賽後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. On the cover of the answer book and the multiple-choice answer sheet, please write your 8-digit Contestant Number, English Name, and Seat Number.

   在答題簿封面及選擇題答題紙上，請填上你的 8 位數字參賽者號碼、英文姓名、及座位號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆**完全塗黑**。

5. The open-ended problems are quite long. Please read the whole problem first before attempting to solve them. If there are parts that you cannot solve, you are allowed to treat the answer as a known answer to solve the other parts.

   開放式問答題較長，請將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來做其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別注明，否則本卷將使用下列符號和常數 ：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.8 m s^{-2}

Trigonometric identities:

三角學恆等式 ：

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = 2\sin\frac{x+y}{2}\sin\frac{y-x}{2}$$

$$\sin x \sin y = \frac{1}{2}\cos\frac{x-y}{2} - \frac{1}{2}\cos\frac{x+y}{2}$$

$$\cos x \cos y = \frac{1}{2}\cos\frac{x-y}{2} + \frac{1}{2}\cos\frac{x+y}{2}$$

$$\sin x \cos y = \frac{1}{2}\sin\frac{x+y}{2} + \frac{1}{2}\sin\frac{x-y}{2}$$

Multiple Choice Questions (2 Marks Each) 選擇題(每題 2 分)

1. The orbital period of the Moon around the Earth is 27.3 days. The orbital period of the Earth around the Sun is 365.3 days. What is the time difference between two full moon?
月球圍繞地球的軌道周期為 27.3 日，地球圍繞太陽的軌道周期為 365.3 日。兩次滿月之間的時間相距是多少?

A. 25.3 days 25.3 日 B. 25.4 days 25.4 日 C. 27.3 days 27.3 日

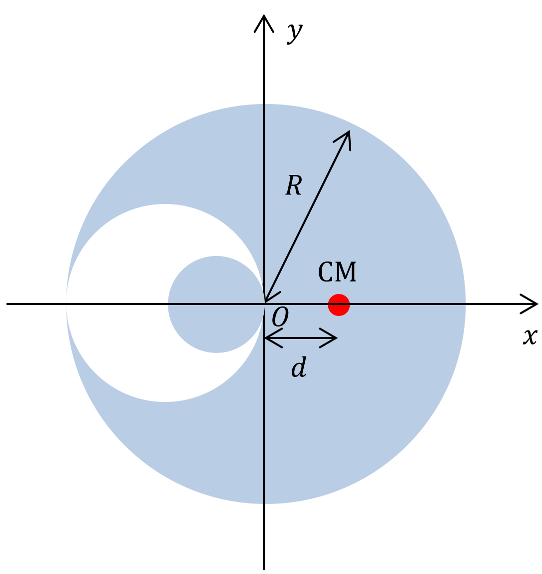
D. 29.3 days 29.3 日 E. 29.5 days 29.5 日

At the full moon, the Moon is collinear with the Earth and the Sun. The angular displacement of the Moon is $2\pi t/27.3$. At the same time, the angular displacement of the Sun relative to the Earth is $2\pi t/365.3$. Hence

$$\frac{2\pi t}{27.3} - \frac{2\pi t}{365.3} = 2\pi$$

$t = \frac{(365.3)(27.3)}{365.3-27.3} = 29.5$ days

2. A circular hole with radius $R/2$ is carved from a uniform circular disk of radius R. A smaller disk with radius $R/4$ with the same uniform density is then put inside the hole, as shown in the figure below. The center of mass of the whole system is located at a distance d from the origin. Find d.
一個半徑為 R 的均勻圓碟，在一邊挖出一半徑為 $R/2$ 的圓孔，然後放置一個半徑為 $R/4$ 的相同均勻密度的小圓碟於孔中，如圖所示。整個系統的質心與原點距離為 d。求 d。



A. $R/13$ B. $5R/52$

C. $3R/26$ D. $7R/52$

E. $2R/13$

$$d = \frac{M \times 0 + (-M/4) \times (-R/2) + (M/16) \times (-R/4)}{M - \frac{M}{4} + \frac{M}{16}} = \frac{7}{52}R$$

3. A seconds pendulum clock loses 20 seconds a day at place A but gains 20 seconds a day at another place B. Find the ratio of the accelerations due to gravity at the two places, $g_B : g_A$. Note: a seconds pendulum has a period of 2 seconds.
一個秒擺鐘於地點 A 每天擺慢 20 秒，但於地點 B 卻每天擺快 20 秒。求這兩地點的重力加速度的比 $g_B : g_A$。註：秒擺的週期為 2 秒。

A. 2159 : 2163　　B. 2159: 4321　　C. 4323 : 4319

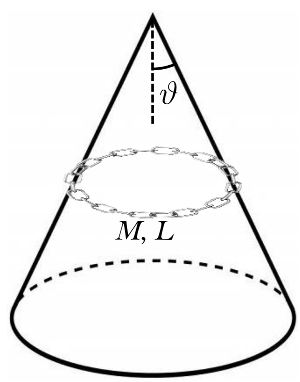
D. 4319 : 4321　　E. 8647 : 8639

$$T_A = 2\pi\sqrt{\frac{l}{g_A}}, \qquad T_B = 2\pi\sqrt{\frac{l}{g_B}}.$$

$$\sqrt{\frac{g_B}{g_A}} = \frac{T_A}{T_B} = \frac{f_B}{f_A} = \frac{(24)(60)(60)+20}{(24)(60)(60)120} = \frac{8642}{8638} = 1 + \frac{4}{8638}$$

$$\frac{g_A}{g_B} = \left(1 + \frac{4}{8638}\right)^2 \approx 1 + \frac{8}{8638} = \frac{4323}{4319}$$

4. A loop of chain of mass M and length (circumference) L rests on the slanted surface of a cone. The chain lies in a horizontal plane, and the half-angle of the cone is ϑ. Assume that the contact surfaces are smooth. What is the tension in the chain?
一條質量為 M、長度（圓周）為 L 的鏈圈被套在一個圓錐體的斜面上。鏈圈平臥在水平面上並處於靜止。圓錐體的半角為 ϑ。假設接觸面是光滑的。鏈圈中的張力為何？



A. $\frac{2\pi Mg}{\vartheta}$

B. $\pi^2 Mg \sec^2\vartheta$

C. $\frac{Mg}{\pi}\sin\vartheta\cos\vartheta$

D. $\frac{Mg}{2\pi}\cot\vartheta$

E. None of the above. 以上選擇都不是。

Let T be the tension in the chain.

Chain length increase $\Delta L \Rightarrow$ eleastic PE increase $T\Delta L$

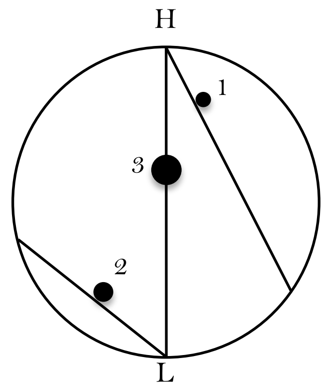
$\Rightarrow$ loop radius increase $\Delta L / 2\pi \Rightarrow$ loop height decrease $\Delta L / (2\pi \tan\vartheta)$

$\Rightarrow$ gravitational PE decrease $Mg\Delta L / (2\pi \tan\vartheta)$

$\Rightarrow$ total PE change $[T - Mg / (2\pi \tan\vartheta)]\Delta L$.

Chain at equilirium $\Rightarrow$ total PE change $= 0 \Rightarrow T = \underline{\underline{Mg / (2\pi \tan\vartheta)}}$.

5. Particle 1 of mass m_1 takes time t_1 to slide down from rest on a chord starting from the highest point H of a vertical circle. Particle 2 of mass m_2 takes time t_2 to slide down from rest on a chord ending at the lowest point L of the circle. Particle 3 of mass m_3 takes time t_3 to drop from point H to point L (along the vertical diameter). Assume that the contact surfaces are smooth. Compare t_1, t_2, and t_3.
質量為 m_1 的粒子 1 從一鉛垂圓的最高點 H 沿任一弦滑下，需時 t_1。質量為 m_2 的粒子 2 沿任一弦滑下至鉛垂圓的最低點 L，需時 t_2。質量為 m_3 的粒子 3（沿着鉛垂直徑）從 H 點掉下至 L 點，需時 t_3。粒子的起始狀態均為靜止。假設所有接觸面都是光滑的。比較 t_1、t_2 及 t_3。



A. $t_1 = t_2 = t_3$.

B. $t_1 = t_2 > t_3$.

C. $t_1 > t_2 > t_3$.

D. $t_2 > t_1 > t_3$.

E. Depends on the relative values of the masses. 取決於粒子的質量的相對值。

$$s = \frac{1}{2}at^2 \quad \text{or} \quad t^2 = \frac{2s}{a}.$$

$$t_1^2 = \frac{2\left(d\cos\theta_1\right)}{g\cos\theta_1} = \frac{2d}{g},\ t_2^2 = \frac{2\left(d\cos\theta_2\right)}{g\cos\theta_2} = \frac{2d}{g},\ t_3^2 = \frac{2d}{g}.$$

Thus $\underline{\underline{t_1 = t_2 = t_3}}$.

6. A satellite is orbiting around the Earth on a circular orbit at velocity v. It is hit by an asteroid. After the impact, the radial velocity of the satellite becomes $v/2$, and the tangential velocity remains at the value v. The new orbit of the satellite is

有衛星沿圓形軌道圍繞地球, 速度為 v。衛星被隕石碰撞，碰撞後衛星的徑向速度為 v/2，切向速度 v 則保持不變。衛星的新軌道是

A. a circular orbit with a larger orbital radius 一個較大圓周的圓形軌道

B. an elliptical orbit 一個橢圓軌道

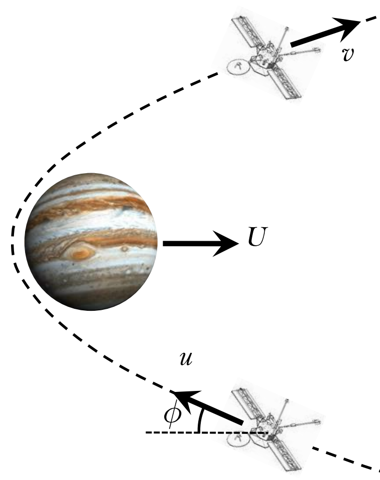
C. a parabolic orbit 一個拋物線軌道

D. a hyperbolic orbit 一個雙曲線軌道

E. an oscillating orbit about a circular trajectory 一個繞著圓形軌跡振盪的軌道

Remark: "asteroid" is more appropriately described as "meteoroid".
註: 題中 "隕石" 應作" 流星" 。

7. A spacecraft moving with initial velocity u approaches Jupiter (which moves with orbital velocity U) at an angle ϕ to the planet's orbital motion. The spacecraft does not enter Jupiter's atmosphere. The gravitational pull from Jupiter causes the spacecraft to swing around the planet and head off with final velocity v in another direction. Note that u and v are the velocities when the spacecraft is sufficiently far away from Jupiter's gravitational influence. Find the final velocity v of the spacecraft.
一艘太空船以初速度 u 飛近木星（木星以軌道速度 U 運行）。太空船的飛行方向與該行星的軌道運行方向成夾角 ϕ。太空船沒有進入木星的大氣層。行星的重力使太空船繞過木星，並以末速度 v 朝另一方向飛走。注意：u 和 v 是指當太空船距離木星足夠遠時（免受其重力所影響）的速度。求太空船的末速度 v。



A. $\sqrt{u^2 + U^2 - 2uU\cos\phi}$

B. $\sqrt{4u^2 + U^2 - 4uU\cos\phi}$

C. $\sqrt{u^2 + 4U^2 - 4uU\cos\phi}$

D. $\sqrt{4u^2 + U^2 + 4uU\cos\phi}$

E. $\sqrt{u^2 + 4U^2 + 4uU\cos\phi}$

In an elastic collision,

Conservation of KE:

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2.$$

Conservation of linear momentum (horizontal):

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

$$\Rightarrow v_1 - v_2 = -(u_1 - u_2) \quad \text{or} \quad v_1 = u_2 - u_1 + v_2.$$

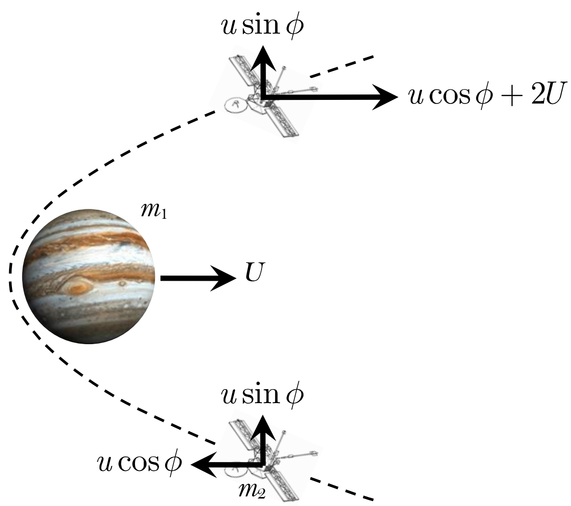
Substitute for v_1 in the momentum equation:

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$
$$= 2u_1 - u_2. \quad (\because m_1 \gg m_2)$$

$$\Rightarrow v_2 = 2(U) - (-u\cos\phi) = 2U + u\cos\phi.$$

Final velocity of the spacecraft:

$$v = \sqrt{(u\sin\phi)^2 + (2U + u\cos\phi)^2}$$
$$= \underline{\underline{\sqrt{u^2 + 4U^2 + 4uU\cos\phi}}}.$$



8. A large piece of granite is put on board a boat, which floats in a swimming pool, and the initial water level of the pool is recorded at equilibrium. Now, if the granite is thrown overboard and sinks into the pool, what will happen to the water level of the pool?
一隻小船浮於泳池上，而船上載有一大塊花崗岩。泳池的初始水位於平衡狀態時被記錄下來。現在這塊花崗岩從船上被拋進池中並沉下。最終的水位於平衡狀態時亦被記錄下來。問泳池的水位有何改變？

A. The water level rises. 水位上升。
B. The water level drops. 水位下降。
C. The water level first rises and then drops to the initial level. 水位先升後降至初始高度。
D. The water level first drops and then rises to the initial level. 水位先降後升至初始高度。
E. Cannot be determined without the exact values of the densities of granite and water.
沒有水和花崗岩的密度的確切值，不能確定。

If the granite was lifted, the pool water level would drop by h_1 corresponding to an amount of water of the granite's weight W_G:

$$W_G = \rho_W A h_1 g \quad \text{or} \quad m_G \not{g} = \rho_W A h_1 \not{g}$$
$$\Rightarrow h_1 = \frac{m_G}{\rho_W A}.$$

If the granite sank, the pool water level would rise by h_2 corresponding to an amount of water of the granite's volume V_G:

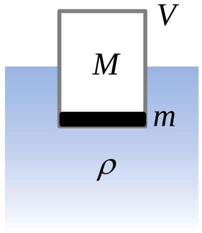
$$V_G = \frac{W_W}{\rho_W g} = \frac{m_W \cancel{g}}{\rho_W \cancel{g}} \quad \text{or} \quad Ah_2 = \frac{m_W}{\rho_W}$$

$$\Rightarrow h_2 = \frac{m_W}{\rho_W A}.$$

Thus, $\frac{h_1}{h_2} = \frac{m_G}{m_W} = \frac{\rho_G \cancel{V_G}}{\rho_W \cancel{V_G}} = \frac{\rho_G}{\rho_W} > 1 \Rightarrow h_1 > h_2 \Rightarrow$ net change in water level: drop.

9. Consider a ship of mass M and volume V floating in water with density ρ. Assume that its density is uniform and its horizontal and vertical cross sections are rectangular. To prevent the ship from overturning, a thin layer of mass m is placed at the bottom floor of the ship. What is the minimum value of m?

一艘質量為 M、體積為 V 的船，浮在密度為ρ的水中。假設船的密度均勻，水平和垂直的切面均為矩形。為防止船翻倒，船底放置了一層質量為 m 的薄物塊。m 的最少值是什麼?



A. $\rho V - M$

B. $\frac{\rho V}{2M} - M$

C. $\frac{\rho V}{M} - M$

D. $\frac{(\rho V)^2}{M} - M$

E. $\sqrt{\rho V M} - M$

Let A be the horizontal cross-section area, H be the height of the ship, and h be the immersion depth of the ship. Using Archimedes' principle,

$$(M + m)g = \rho h A g$$

$$h = \frac{M + m}{\rho A}$$

Height of the center of mass of the ship from the floor $= \frac{MH}{2(M+m)}$

Height of the center of buoyancy of the ship from the floor $= \frac{h}{2}$

Hence minimum mass: $\frac{MH}{2(M+m)} < \frac{h}{2}$

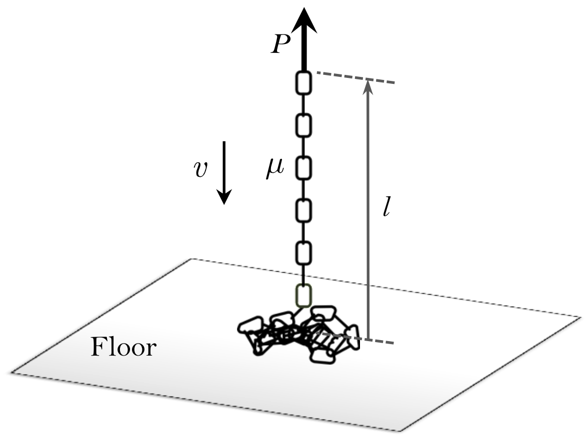
$$\frac{MH}{M + m} < h = \frac{M + m}{\rho A}$$

$$(M + m)^2 > M\rho A H$$

$$m > \sqrt{\rho V M} - M$$

10. A chain, of mass per unit length μ, is hung vertically by an upward pull P acting at the upper end of the chain. Meanwhile, the chain is lowered onto the floor with velocity v. Let l be the length of the free-hanging portion of the chain above the floor. Find the acceleration $\ddot{l}$ of the top end of the chain.

一個向上的拉力 P 作用於一鏈條的上末端，把它垂直吊着。鏈條的每單位長度的質量為 μ。這鏈條同時間以速度 v 被降下到地面。設 l 為鏈條在地面以上懸掛部分的長度。求鏈條頂端的加速度 $\ddot{l}$。



A. $\ddot{l} = -g$

B. $\ddot{l} = \frac{P}{\mu l} - g$

C. $\ddot{l} = \frac{P}{\mu l} + \frac{v^2}{l} - 2g$

D. $\ddot{l} = \frac{P}{\mu l} - \frac{2v^2}{l} - g$

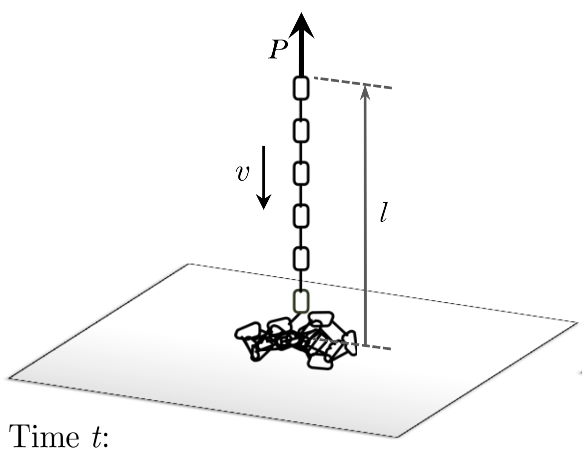
E. $\ddot{l} = \frac{P}{2\mu l} - \frac{2v^2}{l} - g$

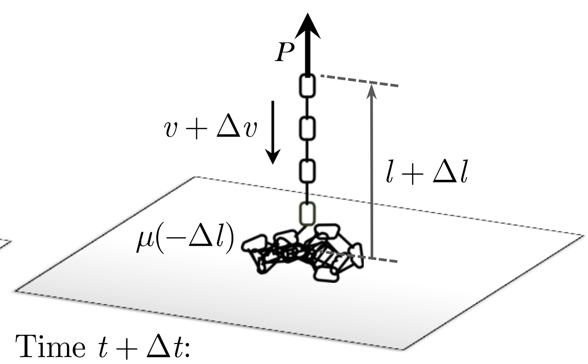
Force = time rate of change of linear momentum:

$$\mu l g - P = \frac{\left[\mu(l+\Delta l)\cdot(v+\Delta v) + \mu(-\Delta l)\cdot v\right] - \mu l \cdot v}{\Delta t} = \frac{\mu l \Delta v}{\Delta t}.$$

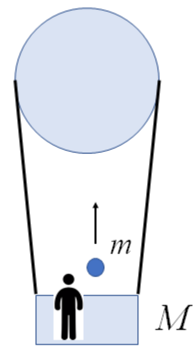
As $\Delta t \to 0$, $\mu l g - P = \mu l \dot{v}$.

But $v = -\dot{l}$, thus $\mu l g - P = -\mu l \ddot{l}$ or $\underline{\underline{\ddot{l} = P/\mu l - g}}$.





11. A man with a ball of mass m on his hand is sitting on the basket hung by a hot air balloon. The total mass of the balloon, the basket and the man is M and the whole system remains at rest in the sky. Suddenly the man throws the ball vertically upward, and the ball falls back to his hand after time t. What is the total work done by the man in throwing the ball?
一人手持質量為 m 的小球乘坐在熱氣球下的吊籃內。氣球、吊籃和人的總質量為 M。整個系統靜止在空中。突然，人將小球上拋，經過時間 t 後小球又返回人手。問人在拋球過程中做了多少功?



A. $\frac{m(M+m)}{8M}g^2t^2$　B. $\frac{m(M+m)}{4M}g^2t^2$　C. $\frac{m}{8}g^2t^2$
D. $\frac{m^2}{8M}g^2t^2$　E. $\frac{M+m}{8}g^2t^2$

Let $Y_M(t)$ and $Y_m(t)$ are the vertical coordinate of the man and the ball respectively. By the conservation of momentum, we have:

$$MV = mv$$

$$Y_M(t) = -Vt + \frac{1}{2}\left(\frac{mg}{M}\right)t^2$$

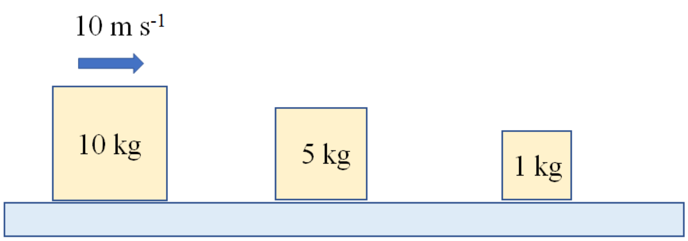
$$Y_m(t) = vt - \frac{1}{2}gt^2$$

$$Y_M(t) = Y_m(t) \Longrightarrow t = \frac{2v}{g} \Rightarrow Y_M(t) = Y_m(t) = 0$$

$$\Rightarrow v = \frac{1}{2}gt \text{ and } V = \frac{1}{2}\frac{m}{M}gt$$

The work done = $\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{m(M+m)}{8M}g^2t^2$

12. Three blocks with masses 10 kg, 5 kg and 1 kg respectively are sitting on the frictionless tabletop as shown in the figure. Initially, the 10 kg block is moving with 10 m s^{-1} to the right. Assume all the collisions are elastic, what is the final velocity of the 1kg mass after collision?
如圖所示，三塊質量分別為10 kg、5 kg 和 1 kg的木板在光滑水平面上。起初，10 kg木板以速度10 m s^{-1}向右移動。假設所有碰撞為彈性碰撞，問1 kg木板在碰撞後的最終速度是多少?



A. 11.1 ms^{-1} B. 22.2 ms^{-1} C. 33.3 ms^{-1}

D. 44.4 ms^{-1} E. 55.5 ms^{-1}

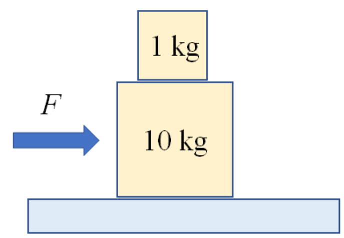
We first consider the collision between the 10kg and 5kg blocks. By conservation of momentum and energy, we have

$$10 \times 10 = 10v_1 + 5v_2$$
$$\frac{1}{2}10 \times 10^2 = \frac{1}{2}10v_1^2 + \frac{1}{2}5v_2^2$$
$$v_2 = 40/3$$

Next, we can consider the collision between the 5kg and 1kg blocks and we get

$$v_4 = \frac{5}{3}v_2 = 22.2 \text{ ms}^{-1}$$

13. Two blocks with masses 10 kg and 1 kg respectively are sitting on the frictionless tabletop as shown. The coefficients of static and kinetic friction on the interface between two blocks are 0.5 and 0.3 respectively. What is the acceleration of the 1 kg block relative to the 10 kg block if the horizontal force F applied 10 kg block is 100 N?



如圖所示，兩塊質量分別為10 kg 和 1 kg的木板在光滑水平面上。兩塊木板之間的靜摩擦系數和動摩擦系數分別為0.5和0.3。現有一強度為100 N的水平推力F作用在10 kg木板上，求1 kg木板相對於10 kg木板的相對加速度。

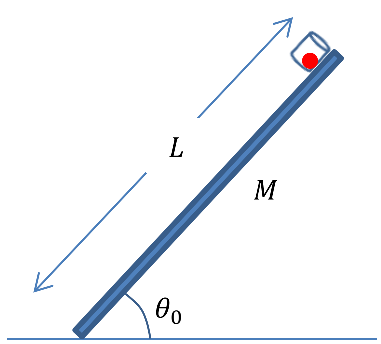
A. 6.77 ms^{-2} to the left B. 6.77 ms^{-2} to the right

C. 4.6177 ms^{-2} to the left D. 4.6177 ms^{-2} to the right

E. 0 ms^{-2}

$$f = 0.3g = a_m$$
$$F - 0.3g = 10a_M$$
$$a_M - a_m = 6.766 \text{ ms}^{-2}$$

14. A uniform thin rod with mass M and length L is initially held at an angle θ_0 with the ground. A small cup is fixed at the higher end of the rod with the mouth facing outward and a small ball is put inside the cup, as shown in the figure. The masses of the cup and the ball are both negligible. The rod is then released from rest to hit the ground. It is assumed that the friction of the ground is large enough so that the contact point of the rod and the ground remains the same when the rod falls. It is given that the rotational kinetic energy of the rod rotating about



its end with angular speed ω is $\frac{1}{6}ML^2\omega^2$. At what angle will the ball starts to move out of the cup, if the friction between the cup and the ball is negligible?

如圖所示，一根質量為 M、長度為 L 的均勻幼棒初始時置於與地面夾角為 θ_0 的位置。一小杯被固定於棒的較高的一端，杯口向外，且杯內放有一小球。小球與杯的質量皆可忽略。現讓幼棒從初始位置下墜，最後擊中地面。假設棒與地面摩擦力足夠大，使得棒與地面的接觸點在下墜過程中保持不變。已知均勻幼棒對其一端以角速度 ω 轉動時其轉動動能為 $\frac{1}{6}ML^2\omega^2$。假設球與杯子之間的摩擦力可忽略，問小球在甚麼角度時會滑出杯子?

A. At all angle

B. At no angle

C. $\sin^{-1}\left(\frac{3}{4}\sin\theta_0\right)$

D. $\sin^{-1}\left(\frac{3}{2}\sin\theta_0\right)$

E. $\cos^{-1}\left(\frac{3}{2}\sin\theta_0\right)$

By energy conservation, $Mg\frac{L}{2}\sin\theta_0 = Mg\frac{L}{2}\sin\theta + \frac{1}{6}ML^2\omega^2$

The ball will leave the cup when $g\sin\theta = \omega^2 L$

Solving the two equations one gets

$$\theta = \sin^{-1}\left(\frac{3}{4}\sin\theta_0\right)$$

15. The Grand Coulee Dam is 1270 m long and 170 m high. The electrical power output from generators at its base is approximately 2×10^9 W. How many cubic meters of water must flow from the top of the dam per second to produce this amount of power if 92% of the work done on the water by gravity is converted to electrical energy? (Each cubic meter of water has a mass of 1000 kg)

大古力水壩長度為 1270 m，高度為 170 m。位於水壩底部的水力發電站功率約為 2×10^9 W。如果引力作用於水的功有 92%能轉化為電能，問每秒從水壩頂部流出的水量為多少立方米才能產生這功率？

A. 1.0×10^3 m^3

B. 1.2×10^3 m^3

C. 1.2×10^6 m^3

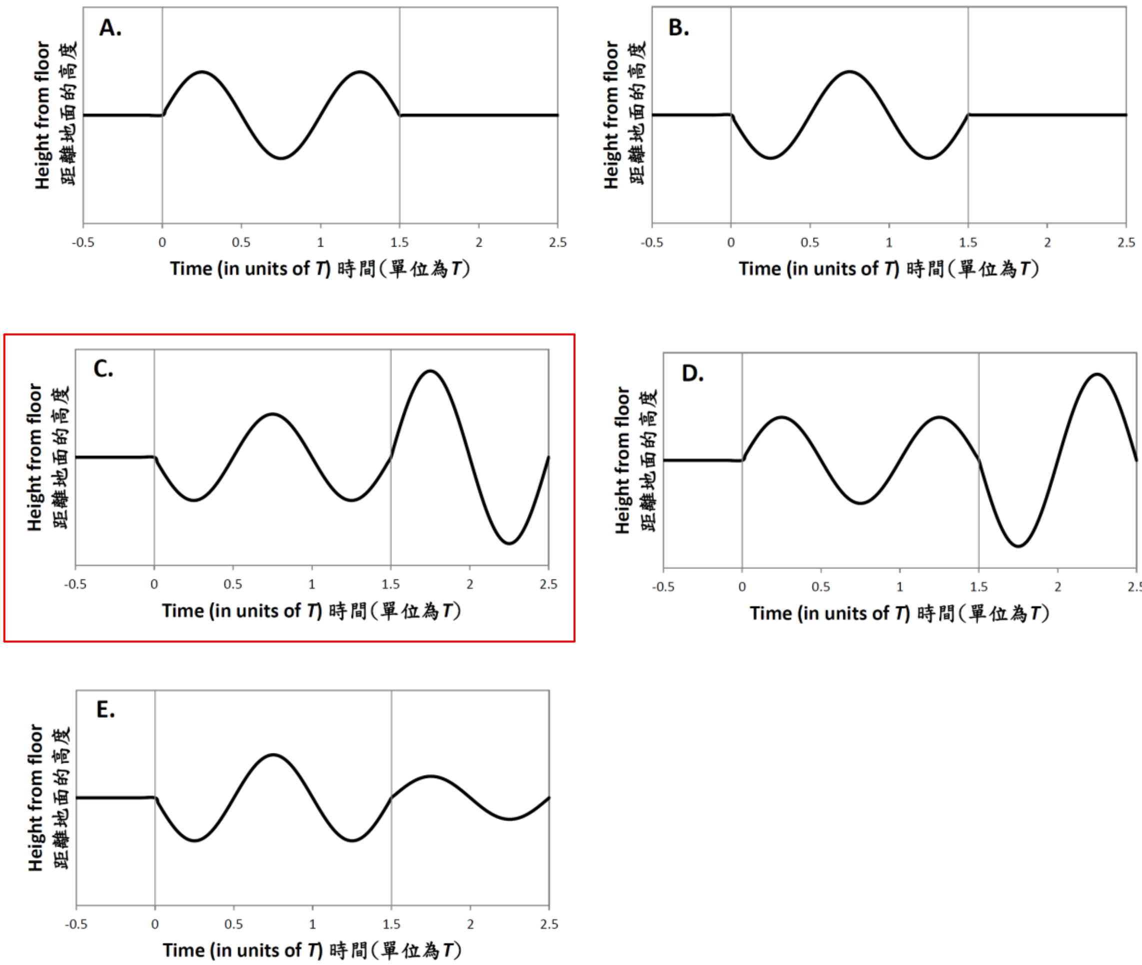
D. 1.3×10^3 m^3

E. 1.3×10^6 m^3

$$mg\times170\times0.92 = 2\times10^9 \rightarrow m = 1.3\times10^6\text{ kg}$$
$$V = 1.3\times\frac{10^6}{1000} = 1.3\times10^3\text{ m}^3$$

16. A mass is hung from the ceiling of a lift by a spring and is initially at rest. The period of oscillation of the mass hung by the spring is T. At time t = 0 the lift starts from rest to move upwards and reaches a constant velocity in a short time. After moving a time interval equal to $3T/2$, the lift stops in a short time. Which one of the following figures represents the time dependence of the height of the mass, measured from the floor of the lift?

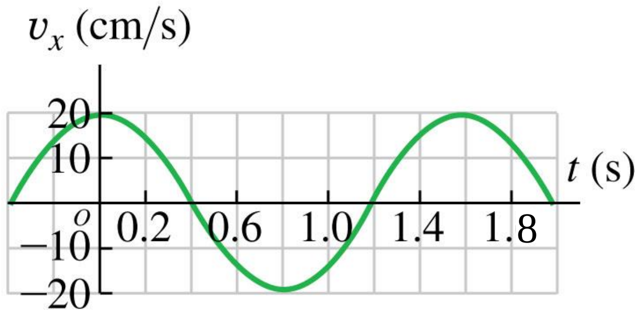
有物體從電梯的頂部以彈簧懸著，處於靜止的初始態。物體被彈簧懸著的振動週期為T。在時間t = 0時，電梯開始上升，在短時間內達到均速。上升了一段為$3T/2$的時段後，電梯在短時間內停止。下面哪張圖代表物體距離電梯地面的高度與時間的關係?



At t = 0, the upward acceleration produces a downward fictitious force. The downward force in a short time produces a downward impulse setting the mass to oscillate downwards. At t = $3T/2$, the upward deceleration produces an upward impulse, further increasing the velocity of the mass.

17. A mass m is attached to a spring and allowed to oscillate. The velocity of the mass is shown in the figure. What is the maximum acceleration magnitude of the mass?

一質量為m的物塊繫於一彈簧上並作振盪。物塊的速度如圖所示。問物塊加速度的最大量值是多少?



A. 39.27 cm s^{-2} B. 78.54 cm s^{-2} C. 157.08 cm s^{-2}

D. 12.5 cm s^{-2} E. 25 cm s^{-2}

$$\text{Period} = 8\times 0.2 = 1.6s = \frac{2\pi}{\omega}$$
$$v(t) = 20\sin(\omega t+\phi)$$
$$x(t) = -\frac{20}{\omega}\cos(\omega t+\phi)$$
$$\text{amplitude} = \frac{20}{\omega} = 5.09\text{cm}$$
$$a(t) = 20\omega\cos(\omega t+\phi) \rightarrow \text{max acceleration} = 20\omega = 78.54\text{ cm s}^{-2}$$

18. An object is released from rest at time $t=0$ at a height of h. Suppose its collision with the ground is inelastic and during every collision the fractional loss in KE is γ, where $1\geq\gamma>0$. After how much time will the object stop and become at rest on the ground?
當時間 $t=0$ 時，一物體從高度 h 開始自由下墜。假設物體與地面的碰撞為非彈性碰撞。每次碰撞後動能的相對損耗為 γ，且有$1\geq\gamma>0$。問經過多少時間後物體會停於地面不動？

A. $\frac{1+\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}}\sqrt{\frac{2h}{g}}$ B. $\frac{1+\sqrt{\gamma}}{1-\sqrt{\gamma}}\sqrt{\frac{2h}{g}}$ C. $2\frac{\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}}\sqrt{\frac{2h}{g}}$

D. $2\frac{\sqrt{\gamma}}{1-\sqrt{\gamma}}\sqrt{\frac{2h}{g}}$ E. ∞

The time between the release and the first collision is $\sqrt{2h/g}$. The kinetic energy after the n-th collision is $(1-\gamma)^n mgh$, where $n=1,2,3,\cdots$.So the speed after the n-th collision is $(1-\gamma)^{n/2}\sqrt{2gh}$. The time taken between the n-th and the $(n+1)$-th collision is $2(1-\gamma)^{n/2}\sqrt{2h/g}$. The total time taken is

$$T=\sqrt{\frac{2h}{g}}+\sum_{n=1}^{\infty}2(1-\gamma)^{n/2}\sqrt{\frac{2h}{g}}=\sqrt{\frac{2h}{g}}+2\sqrt{\frac{2h}{g}}\frac{\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}}=\sqrt{\frac{2h}{g}}\frac{1+\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}}$$

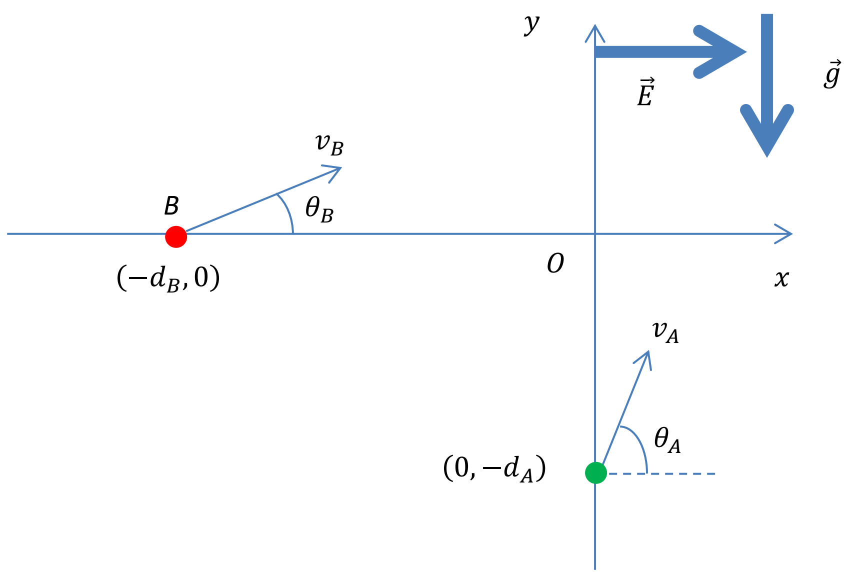
(19.-20.) Two identical particles A and B with charge q and mass m are inside a region of uniform and constant downward gravitational field $\vec{g}=-g\hat{j}$ and uniform time-varying electric field $\vec{E} = E_0 \cos \omega t\, \hat{\imath}$, where $\hat{\imath}$ and $\hat{\jmath}$ are the unit vectors along the x and y axes respectively. At time $t = 0$, particle A is located at $(0, -d_A)$, moving with speed v_A making an angle θ_A with the positive x-axis. Particle B is initially located at $(-d_B, 0)$ and is launched with speed v_B at an angle θ_B measured from the positive x-axis, as shown in the figure below. It is given that

兩全同粒子 A 與 B 各有質量 m 和電荷 q。它們所在的區域有一向下的均勻常引力場 $\vec{g} = -g\hat{\jmath}$ 及一均勻變化電場 $\vec{E} = E_0 \cos \omega t\, \hat{\imath}$，其中 $\hat{\imath}$ 和 $\hat{\jmath}$分別為沿 x 及 y 軸方向的單位矢量。當時間 $t = 0$ 時，如下圖所示，粒子 A 位於$(0, -d_A)$，以速率v_A沿與正 x 軸成夾角θ_A的方向運動;粒子 B 位於$(-d_B, 0)$, 以速率v_B沿與正 x 軸成夾角θ_B的方向運動。已知

$$\theta_A > \frac{\pi}{2} - \tan^{-1}\frac{d_A}{d_B}$$

It is also given that the electric force acting on a charge q under an electric field $\vec{E}$ is $q\vec{E}$. Assume that the electrical and gravitational forces between the particles are negligible.

且電荷 q 於電場 $\vec{E}$ 中所受力為 $q\vec{E}$。假設兩粒子間的電力和萬有引力皆可以忽略。



19. Find the minimum value of v_B so that particle B can hit particle A with certain angle θ_B.
求 v_B 的最小值，使粒子 B 可以某些角度 θ_B 擊中粒子 A。

A. $v_A \sin\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$

B. $-v_A \cos\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$

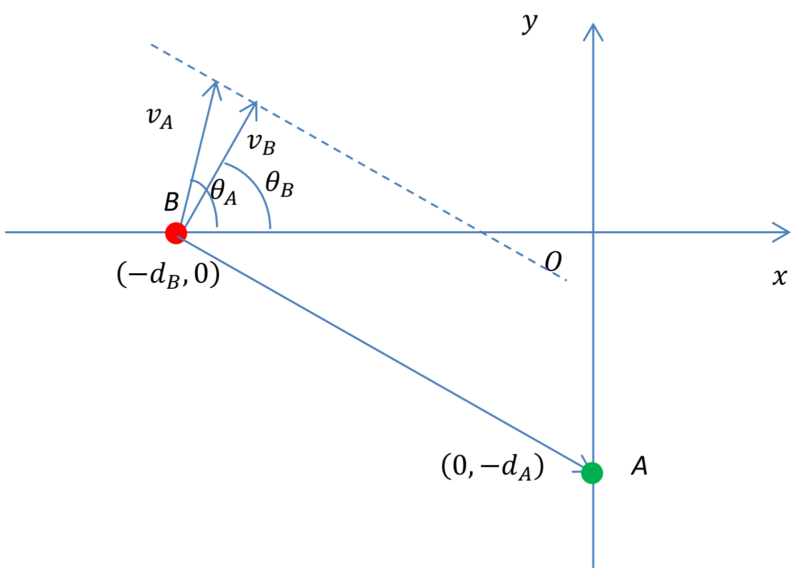
C. $-v_A \sec\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$

D. $v_A \csc\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$

E. $-v_A \frac{mg}{qE_0} \cos\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$

In the frame of B, A has no acceleration. So there will be collision if the initial velocity of A relative to B is antiparallel to the position vector of A measured from B. From the figure below, it can be observed that when v_B is smaller than the length shown, no matter how one chooses θ_B, $\vec{v}_A - \vec{v}_B$ cannot be antiparallel to the position vector of A relative to B. The smallest value of v_B is hence as that shown in the figure. Therefore

$$v_B = v_A \cos\left(\theta_A - \left(\frac{\pi}{2} - \tan^{-1}\frac{d_A}{d_B}\right)\right) = v_A \sin\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$$



20. With the above minimum speed and angle, what is the time at which particle B will hit particle A?
根據上題中的最小速率與對應角度，問粒子 A 在什麼時候擊中粒子 B。

A. $\frac{\sqrt{d_A^2+d_B^2}}{v_A} \sin\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$

B. $-\frac{\sqrt{d_A^2+d_B^2}}{v_A} \cos\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$

C. $\frac{\sqrt{d_A^2+d_B^2}}{v_A} \csc\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$

D. $-\frac{\sqrt{d_A^2+d_B^2}}{v_A} \sec\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$

E. $-\frac{\sqrt{d_A^2+d_B^2}}{\omega d_A} \cos\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$

The speed of A relative to B is

$$v_A \sin\left(\theta_A - \left(\frac{\pi}{2} - \tan^{-1}\frac{d_A}{d_B}\right)\right) = -v_A \cos\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$$

Hence the time taken is

$$-\frac{\sqrt{d_A^2 + d_B^2}}{v_A}\sec\left(\theta_A + \tan^{-1}\frac{d_A}{d_B}\right)$$

Open Problems (12 Marks Each) 開放題(每題 12 分)

1. Spider-Man Saving a Train 蜘蛛俠拯救列車

In the movie *Spider-Man 2*, Spider-Man uses his spider webs to save a runaway train by bringing it to a stop (See the figures). The train is running at 80 mph on a straight railway. It has six carts and the mass of each cart is 58,000 lb. Spider-Man shoots two identical bundles of spider webs, one bundle from each hand, which sticks firmly onto the buildings on both sides of the train. With the help of the webs, the train is eventually stopped in 50 s. The buildings are so close to the train that the webs can be considered parallel to the railway. Assume that each web bundle acts like a spring of spring constant k, and that friction is negligible.

在電影《蜘蛛俠 2》中，蜘蛛俠用他的蜘蛛網拉停並拯救了一台失控的列車（見附圖）。列車以 80 mph 的速度在一條平直的路軌上行駛。列車有六個車卡，而每卡的質量為 58,000 lb。蜘蛛俠發射兩束相同的蜘蛛網（每隻手射出一束）。蜘蛛網牢牢地黏着列車兩旁的建築物。在蜘蛛網的幫助下，列車最終在 50 s 內被停下來。由於建築物和列車很接近，所以蜘蛛網可以被視作與路軌平行。假設每束蜘蛛網的作用有如一根彈簧，其彈簧常數為 k。摩擦力可略去不計。

(a) Find the extension of each web bundle. You may take the extension as the distance travelled by the train during its deceleration by the webs.
求每束蜘蛛網的伸長。你可把列車在被蜘蛛網減速期間所行駛的距離當作網的伸長。

(b) Find the spring constant k of each web bundle.
求每束蜘蛛網的彈簧常數 k。

(c) Find the tension in each web bundle just before the train is stopped.
求列車剛被停下前，每束蜘蛛網中的張力。

Give your answers correct to 3 significant figures, in SI units.
答案準確至 3 位有效數字，並以國際單位制的單位表示。

[Unit conversions: 1 mph (mile per hour) = 1.61 km/h; 1 kg = 2.2 lb (pounds).]
〔單位轉換：1 mph（英里每小時）= 1.61 km/h；1 kg = 2.2 lb（磅）。〕

$m = 6 \times 58000 / 2.2 = 158181.82$ kg.

$$v_i = 80 \text{ mph} = \frac{80 \times 1.61 \times 1000}{60 \times 60} = 35.78 \text{ m/s}.$$

Approach 1: Considering the motion of the train as a simple harmonic motion.

(a) We note that the period of oscillation is (4)(50) = 200 s. In simple harmonic motion, displacement amplitude and velocity amplitude are related by

$$v = \omega x \Rightarrow x = \frac{v}{\omega} = \frac{35.78}{(2\pi / 200)} = 1{,}139 \text{ m} \approx 1{,}140 \text{ m (3 sig. fig.)}$$

(b) Hence the angular frequency is given by

$\omega^2 = \dfrac{2k}{m}$ (the factor of 2 is for two web bundles)

$$\Rightarrow k = \frac{m\omega^2}{2} = \frac{158181.82}{2}\left(\frac{2\pi}{200}\right)^2 = 78.06 \text{ Nm}^{-1} \approx 78.1 \text{ Nm}^{-1} \text{ (3 sig. fig.)}$$

(c) The tension in each bundle $T = kx = (78.06)(1{,}139) \approx 8.89 \times 10^4$ N (3 sig. fig.)

Approach 2: Approximate that the deceleration is constant and using energy conservation.

(a) The extension is

$$x = \bar{v}t = \left(\frac{35.78 + 0}{2}\right)(50) = 894.44 \approx 894 \text{ m (3 sig. fig.)}$$

(b) Energy consideration (Note that the spring force or the deceleration is not constant):

Initial KE of the train is

$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2}(158181.82)(35.78)^2 = 101.25 \times 10^6 \text{ J}.$$

Elastic PE stored in the two web bundles is

$$U = 2 \times \frac{1}{2} k x^2 \Rightarrow k = \frac{U}{x^2}.$$

By conservation of mechanical energy,

$U = K_i$

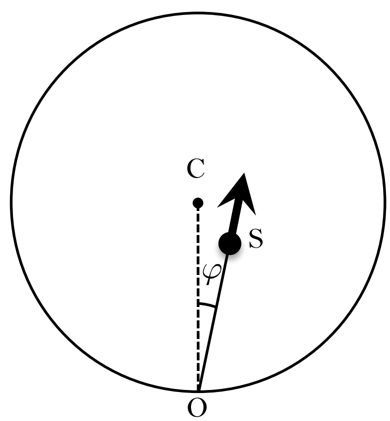
$$k = \frac{U}{x^2} = \frac{K_i}{x^2} = \frac{101.25 \times 10^6}{(894.44)^2} = 126.6 \approx 127 \text{ Nm}^{-1} \text{ (3 sig. fig.)}$$

(c) Tension in each web bundle is
$T = kx = (126.6)(894.44) = 1.13\times10^5\ \text{N}\ (3\,\text{sig. fig.})$

Approach 3: Approximate that the deceleration is *constant* and using Newton's law of motion.
(a) The extension is
$$x = \bar{v}t = \left(\frac{35.78+0}{2}\right)(50) = 894.44 \approx 894\ \text{m}\ (3\,\text{sig. fig.})$$
(b) (b) Deceleration of the train:
$$a = \frac{v}{t} = \frac{35.78}{50} = 0.7156\ \text{m s}^{-2}$$
Noting that the tension in the web bundles is increasing with their extensions, the average force acting on the train during the deceleration is $F = (2k)(x/2) = kx$

Using Newton's law of motion, $kx = ma \Rightarrow$
$$k = \frac{ma}{x} = \frac{(158181.82)(0.7156)}{894.44} = 126.6\ \text{Nm}^{-1} \approx 127\ \text{Nm}^{-1}\ (3\,\text{sig. fig.})$$
(c) The tension in each bundle $T = kx = (126.5)(894) = 113{,}188\ \text{N} \approx 1.13\times10^5\ \text{N}\ (3\,\text{sig. fig.})$

2. A Round Snooker Table 圓型的桌球枱



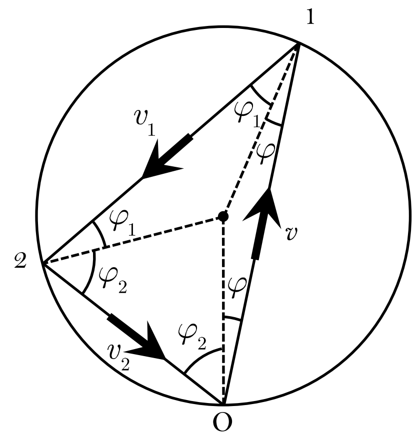
A smooth snooker ball S is struck from point O of a specially-designed circular snooker table. The ball then moves off horizontally in a direction making an angle φ with the radius CO. The ball makes n impacts with the smooth vertical wall of the table before returning to point O. If n = 1, φ = 0. The coefficient of restitution between the ball and the wall is e. (The coefficient of restitution is the ratio of the normal speed after the impact to its value before.)

一張經特別設計的圓型桌球枱上有一粒光滑的桌球從 O 點被擊出。桌球繼而朝着與半徑 CO 成夾角φ的方向水平移動。桌球與球枱的鉛垂牆壁發生 n 次碰撞後返回 O 點。若 n = 1, 則$\varphi = 0$。桌球與球枱之間的恢復係數為 e。（恢復係數是碰撞後與碰撞前的速度垂直分量值的比例。）

(a) If n = 2, find φ in terms of e. 若 $n = 2$，求φ，答案以 e 表示。

(b) If n = 3, find φ in terms of positive exponents of e. 若 $n = 3$，求φ，答案以 e 表示。

(a) n = 2:



From conservation of linear momentum, 1st impact:

$v_1 \sin\varphi_1 = v\sin\varphi.$

From Newton's law of impact, 1st impact:

$v_1 \cos\varphi_1 = ev\cos\varphi.$

Thus $\tan\varphi_1 = e^{-1}\tan\varphi.$

Similarly for 2nd impact: $\tan\varphi_2 = e^{-1}\tan\varphi_1 = e^{-2}\tan\varphi.$

From geometry,

$$2(\varphi + \varphi_1 + \varphi_2) = \pi,$$

$$\tan(\varphi_1 + \varphi_2) = \tan(\frac{\pi}{2} - \varphi)$$

$$\frac{\tan\varphi_1 + \tan\varphi_2}{1 - \tan\varphi_1 \tan\varphi_2} = \frac{1}{\tan\varphi}$$

$$\frac{e^{-1}\tan\varphi + e^{-2}\tan\varphi}{1 - e^{-3}\tan^2\varphi} = \frac{1}{\tan\varphi}$$

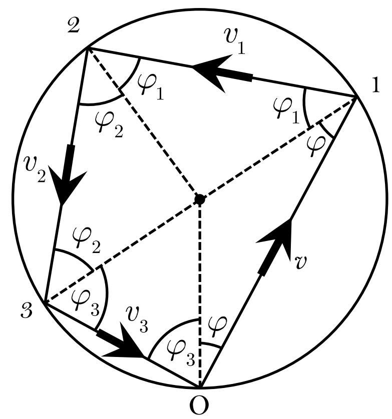
$$\Rightarrow \tan^2\varphi = \frac{1}{e^{-1} + e^{-2} + e^{-3}} = \frac{e^3}{1 + e + e^2}$$

$$\Rightarrow \varphi = \tan^{-1}\frac{1}{\sqrt{e^{-1} + e^{-2} + e^{-3}}} = \underline{\underline{\tan^{-1}\sqrt{\frac{e^3}{1 + e + e^2}}}}.$$

(b) n = 3:

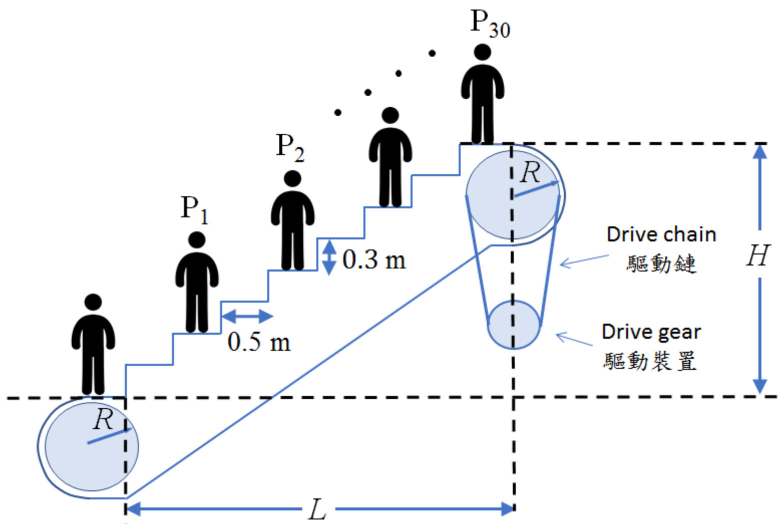
As in (a), from conservation of linear momentum and Newton's law of impact, 3rd impact: $\tan\varphi_3 = e^{-3}\tan\varphi.$

From geometry,



$$2(\varphi+\varphi_1+\varphi_2+\varphi_3)=2\pi,$$
$$\tan(\varphi+\varphi_1)=\tan\left[\pi-(\varphi_2+\varphi_3)\right]=-\tan(\varphi_2+\varphi_3)$$
$$\frac{\tan\varphi+\tan\varphi_1}{1-\tan\varphi\tan\varphi_1}=-\frac{\tan\varphi_2+\tan\varphi_3}{1-\tan\varphi_2\tan\varphi_3}$$
$$\frac{\tan\varphi+e^{-1}\tan\varphi}{1-e^{-1}\tan^2\varphi}=-\frac{e^{-2}\tan\varphi+e^{-3}\tan\varphi}{1-e^{-5}\tan^2\varphi}$$
$$\Rightarrow \tan^2\varphi=\frac{1+e^{-1}+e^{-2}+e^{-3}}{e^{-3}+e^{-4}+e^{-5}+e^{-6}}=e^3$$
$$\Rightarrow \varphi=\underline{\underline{\tan^{-1}e^{3/2}}}.$$

3. Escalator Breakdown 扶手電梯故障



An escalator has a horizontal length L = 30 m and a height H = 18 m. It transports passengers upward through its length in 90 s. Each step on the escalator is 0.5 m deep and 0.3 m high. There are 60 steps on the escalator.

一部自動扶手電梯的長度為 L = 30 m，高度為 H = 18 m。它可以用 90 s 的時間運送乘客上樓。每一級梯級的深度為 0.5 m、高度為 0.3 m。扶手電梯總共有 60 级梯級。

(a) Assume that there is one passenger with mass m = 70 kg standing on alternating steps of the escalator (i.e. there are 30 passengers on the escalator at any time). What is the minimum power of the electric motor to keep the escalator moving in the steady speed?
假設每兩級梯級有一個質量為 m = 70 kg 的乘客站立（任何時間都有 30 個乘客在電梯上）。如果電梯以等速運動，問驅動裝置的最少功率是多少?

The vertical velocity = v_y = 18m/90s
Number of passengers = n = 30
The power required = the power to transport one passenger × (30) = $nmgv_y$ = 4116 W

(b) Suppose the drive chain is suddenly broken and all the braking devices malfunction. The escalator reverses direction, and sends passengers careening downward with an acceleration. Eventually, all passengers will hit on the ground.

假設驅動鏈突然斷裂，且所有的制動裝置都同時失靈。扶手電梯倒轉運動方向，令乘客以加速度向下動。最終，所有乘客著地。

Assume both wheels have mass $M = 7{,}000$ kg, radius $R = 1$ m, and there is one passenger on alternating steps. Initially, the passengers are standing as shown in the figure when the drive chain is broken. What is the velocity v_1 of the passengers when the first passenger (P_1) hits on the ground?

假設兩個大輪的質量為$M = 7{,}000$ kg、半徑為$R = 1$ m, 而每兩級梯級有一個乘客。當驅動鏈最初斷裂時，乘客站立的位置如圖中所示。問第一個乘客(P_1)著地時的速度v_1是多少？

(Hint: The rotational kinetic energy of a wheel is $\frac{1}{4}MR^2\omega^2$ where ω is the angular velocity of the wheel.)

(提示： 一個輪的旋轉動能為$\frac{1}{4}MR^2\omega^2$，其中 ω為輪子的角速度。)

The vertical velocity v_y = 18m/90s

The horizontal velocity v_x = 30m/90s

The velocity $v_0 = \sqrt{v_x^2 + v_y^2} = 0.39\ \text{m s}^{-1}$

Initially, the passengers are moving upward with an initial velocity $v_0 = 0.4\ \text{m s}^{-1}$ and the wheels rotate with an angular velocity $\omega_0 = v_0/R = 0.4$ rad s^{-1}.

The initial energy is $E_0 = 2 \times \frac{1}{4}MR^2\left(\frac{v_0}{R}\right)^2 + 30 \times \frac{1}{2}mv_0^2$ =692 J

When the first passenger hits on the ground, the total energy becomes

$$E_1 = 2 \times \frac{1}{4}MR^2\left(\frac{v_1}{R}\right)^2 + 30 \times \frac{1}{2}mv_1^2 - 30mg(0.6) = E_0$$

By the conservation of energy, we have $v_1 = 1.693\ \text{m s}^{-1}$.

(c) What is the velocity v_2 of the second passenger (P_2) before he/she hits the ground? Assume that the first passenger has left the location.

問第二個乘客(P_2)著地時的速度v_2是多少？假設第一個乘客已經離開所在位置。

The total energy after the first person hits on the ground

$$E_2 = 2 \times \frac{1}{4}MR^2\left(\frac{v_1}{R}\right)^2 + 29 \times \frac{1}{2}mv_1^2 = 12941\ J$$

When the second person hits on the ground, the total energy becomes

$$E_3 = 2 \times \frac{1}{4}MR^2\left(\frac{v_2}{R}\right)^2 + 29 \times \frac{1}{2}mv_2^2 - 29mg(0.6)$$

Hence, we have $v_2 = 2.347\ \text{m s}^{-1}$.

(d) What is the velocity of the last passenger (P_{30}) before he/she hits the ground? Assume that all passengers hitting the ground earlier have left the location.

問最後一個乘客(P_{30})著地時的速度是多少？假設之前所有著地的乘客已經離開所在位置。

(Hint: A useful approximation is $\frac{1}{A+1}+\frac{1}{A+2}+\cdots+\frac{1}{A+N}\approx \ln\left(1+\frac{N}{A+1/2}\right)$.)

(提示：一個有用的近似是$\frac{1}{A+1}+\frac{1}{A+2}+\cdots+\frac{1}{A+N}\approx \ln\left(1+\frac{N}{A+1/2}\right)$。)

In general, we have the recursive formula

$$v_1^2 = v_0^2 + \frac{2(n)mgh}{M+m(n)} = v_0^2 + 2gh\left(1-\frac{M/m}{M/m+30}\right)$$

$$v_2^2 = v_1^2 + \frac{2(n-1)mgh}{M+m(n-1)} = v_1^2 + 2gh\left(1-\frac{M/m}{M/m+29}\right)$$

$$\cdots\cdots\cdots$$

$$v_{30}^2 = v_{29}^2 + \frac{2(n-29)mgh}{M+m(n-29)} = v_{29}^2 + 2gh\left(1-\frac{M/m}{M/m+1}\right)$$

Adding the equations,

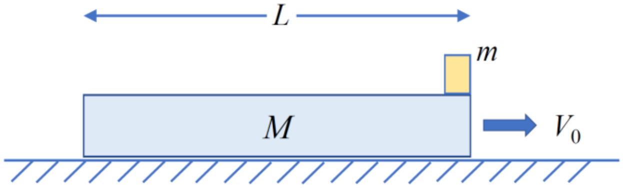
$$v_{30}^2 = v_0^2 + 2gh\left[30-\frac{M}{m}\left(\frac{1}{\frac{M}{m}+30}+\cdots\frac{1}{\frac{M}{m}+1}\right)\right] \approx v_0^2 + 2gh\left[30-100\ln\left(1+\frac{30}{100.5}\right)\right]$$

$$v_{30}^2 = v_0^2 + 2gh(3.878)$$

$$v_{30} = 6.76 \text{ m s}^{-1}$$

In contrast to the free fall, $mg(18) = 0.5mv^2$ -> $v = 18.78$ m s^{-1}, the velocity is lower.

4. A block of mass M and length L is sliding on the frictionless table and moves with constant velocity V_0 to the right. Suddenly, a small mass m is put on the right end of the block. The mass m slides relative to the block and fall on the left end of the block. Let the coefficient of friction between the block M and mass m be μ.
質量為 M 、長度為 L 的木板，在光滑水平面上以速度向右作勻速正線運動。突然，在長木板右端放上一個質量為 m 的小物塊。小物塊產生相對滑動，並從木板左端滑出。設 M 和 m 之間的動摩擦系數為μ。



(a) What is the loss of the total mechanical energy during the process?
問此過程中損失的總機械能是多少?

(b) What is the final velocity of the mass m and the total travelled time of the mass m before it falls on the left end?
問小物塊脫離木板時的速度和小物塊在本板上運動的時間是多少?

(c) What is the minimum value of V_0?

問 V_0 的最小值是多少？

The friction acts on the mass *m* (-ve direction)

$$f = -\mu m g = m a$$

Let *V* and *v* be the velocities of the block *M* and the mass *m* when the mass *m* leaves the block.

$$v = \mu g t$$

By the conservation of momentum

$$M V_0 = m v + M V$$

and the work-energy theorem,

$$-fL = \left(\frac{1}{2}MV^2 + \frac{1}{2}mv^2\right) - \frac{1}{2}MV_0^2$$

(a) The loss of total mechanical energy $\Delta E = fL = \mu m g L$

(b) By elimination, we have

$$(Mm + m^2)v^2 - 2mMV_0 v + 2\mu m M g L = 0$$

$$v = \frac{1}{M+m}\left(MV_0 \pm \sqrt{M^2V_0^2 - 2\mu g M L(M+m)}\right)$$

Since the velocity of the mass *m* must less than *V*, we have $v \le \frac{1}{M+m}V_0$ and hence

$$v = \frac{1}{M+m}\left(MV_0 - \sqrt{M^2V_0^2 - 2\mu g M L(M+m)}\right)$$

and $t = \frac{v}{\mu g} = \frac{1}{\mu g(M+m)}\left(MV_0 - \sqrt{M^2V_0^2 - 2\mu g M L(M+m)}\right)$

(c) For v to be real, we much have $M^2V_0^2 - 2\mu g M L(M+m) \ge 0$

$$V_0 \ge \sqrt{\frac{2\mu g L(M+m)}{M}}$$

5. A space station orbits around the Earth at the height *h* above the Earth's surface. A small satellite of mass *m* is launched from the space station to arrive at a geosynchronous orbit, such that the satellite is constantly above a fixed location on Earth's surface. Find the energy required to launch the satellite. The Earth has mass *M*, radius *R*, and angular velocity ω. The gravitational constant is *G*.
一太空站位於環繞地球的軌道上，離地面高度為 *h*。從太空站發射一個質量為 *m* 的小人造衛星至一個地球同步軌道，令衛星恆常處於地球同一地點的上空。求發射該衛星時所需要提供的能量。設 *M*、 *R* 和ω分別為地球的質量、半徑和自轉角速度。萬有引力常數為 *G*。

The satellite is in uniform circular motion, gravitational force = centripetal force:

$$G\frac{Mm}{(R+h)^2} = m\frac{v^2}{R+h} \quad \text{or} \quad v^2 = \frac{GM}{R+h},$$

and therefore the kinetic energy of the satellite (in the space station) is

$$E_K = \frac{1}{2}mv^2 = \frac{GMm}{2(R+h)}.$$

The gravitational potential energy of the satellite (in the space station) is

$$E_P = -\frac{GMm}{R+h}.$$

Thus the mechanical energy of the satellite (in the space station) is

$$E_1 = E_K + E_P = -\frac{GMm}{2(R+h)}.$$

When the satellite moves to the geosynchronous orbit,

$$\frac{GMm}{r^2} = mr\omega^2 \quad \text{or} \quad r = \sqrt[3]{\frac{GM}{\omega^2}}.$$

Thus the mechanical energy of the satellite in the geosynchronous orbit is

$$E_2 = -\frac{GMm}{2r} = -\frac{1}{2}m\sqrt[3]{G^2M^2\omega^2}.$$

Let E_{K0} be the initial kinetic energy of the satellite when it leaves the Space Station. By conservation of mechanical energy for the satellite,

$$E_1' = E_2$$

$$E_{K0} + \left(-\frac{GMm}{2(R+h)}\right) = -\frac{1}{2}m\sqrt[3]{G^2M^2\omega^2}$$

$$\text{or} \quad E_{K0} = -\frac{1}{2}m\sqrt[3]{G^2M^2\omega^2} + \frac{GMm}{2(R+h)}.$$

(Remark: After the competition, the explanation of the coefficient of restitution was added in Open Problem 2. This change does not affect the correctness of the solution and the marking standard.)

(註：比賽後在開放題 2 加入了恢復係數的解釋。改動並不影響答案的正確性和評分標準。)