

Hong Kong Physics Olympiad 2019

2019 年香港物理奧林匹克競賽

Organisers 合辦機構

Education Bureau

教育局

The Hong Kong Academy for Gifted Education

香港資優教育學苑

The Hong Kong University of Science and Technology

香港科技大學

Advisory Organisations 顧問機構

The Physical Society of Hong Kong

香港物理學會

Hong Kong Physics Olympiad Committee

香港物理奧林匹克委員會

5 May, 2019

2019 年 5 月 5 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English, but only ONE language should be used throughout the whole paper.

所有題目均為中英對照。你可選擇以中文或英文作答，惟全卷必須以單一語言作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

選擇題的答題紙將於比賽後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. On the cover of the answer book and the multiple-choice answer sheet, please write your 8-digit Contestant Number, English Name, and Seat Number.

在答題簿封面及選擇題答題紙上，請填上你的 8 位數字參賽者號碼、英文姓名、及座位號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆**完全塗黑**。

5. The open-ended problems are quite long. Please read the whole problem first before attempting to solve them. If there are parts that you cannot solve, you are allowed to treat the answer as a known answer to solve the other parts.

開放式問答题較長，請將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來做其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

除非特別注明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.8 m s^{-2}
Gravitational constant 重力常數	G	$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$

Trigonometric identities:

三角學恆等式：

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos x - \cos y = 2 \sin \frac{x + y}{2} \sin \frac{y - x}{2}$$

$$\sin x \sin y = \frac{1}{2} \cos(x - y) - \frac{1}{2} \cos(x + y)$$

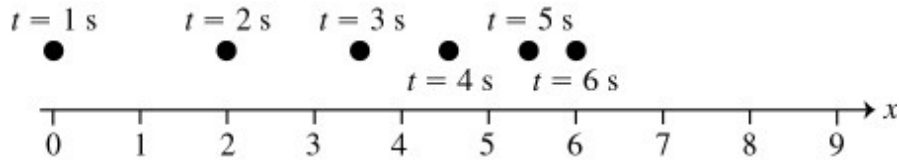
$$\cos x \cos y = \frac{1}{2} \cos(x - y) + \frac{1}{2} \cos(x + y)$$

$$\sin x \cos y = \frac{1}{2} \sin(x + y) + \frac{1}{2} \sin(x - y)$$

Multiple Choice Questions (2 Marks Each) 選擇題(每題 2 分)

1. The figure shows the position of an object (moving along a straight line) as a function of time. Assume two significant figures in each number. Which of the following statements about this object is true over the interval shown?

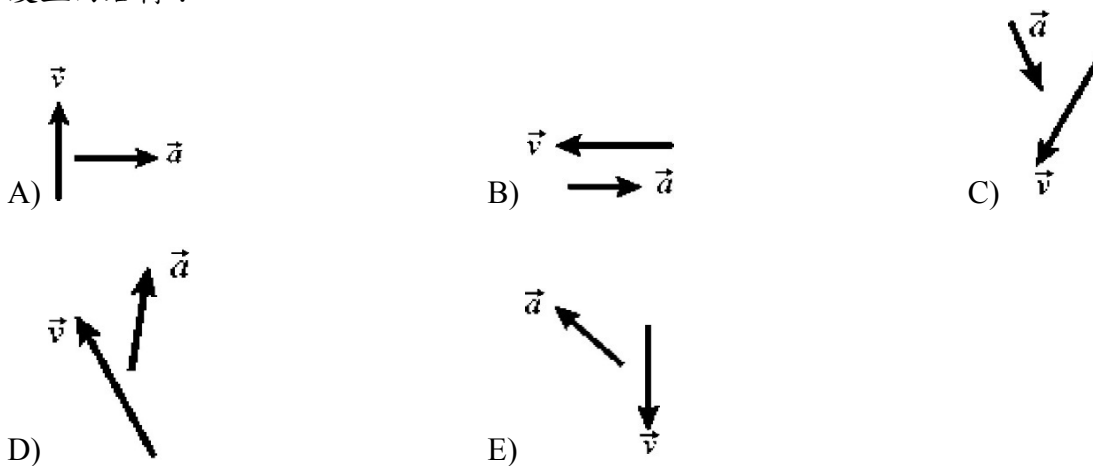
1. 圖中顯示了一物體的位置表示為時間的函數（沿直線移動）。假設每個數字中有兩個有效數字。在顯示的時間間隔內，以下哪些關於此物體的陳述是正確的？



- A) The object is accelerating to the right. 物體向右加速。
- B) The average speed of the object is 1.0 m/s. 物體的平均速率是 1.0 m/s。
- C) The acceleration of the object is in the same direction as its velocity. 物體的加速度與其速度方向相同。
- D) The object is accelerating to the left. 物體向左加速。
- E) The acceleration of the object is in the same direction as its displacement. 物體的加速度與其位移方向相同。

2. Shown below are the velocity and acceleration vectors for a person in several different types of motion. In which case is the person slowing down and turning to his right?

2. 下面顯示的是幾種不同類型運動中人的速度和加速度矢量。在哪種情況下，這個人放慢速度並向右轉？



3. You are standing in a moving bus, facing forward, and you suddenly step forward as the bus comes to an immediate stop. The force acting on you that causes you to step forward is

3. 你站在一輛移動的巴士上，面向前方，當巴士剎停時你突然向前摔倒。作用在你身上並讓你向前摔倒的力是

- A) the force due to static friction between you and the floor of the bus
- B) the force due to kinetic friction between you and the floor of the bus
- C) the force of gravity
- D) the normal force due to your contact with the floor of the bus
- E) No forces acting on you to cause you to fall.

- A) 由您和巴士地板之間的靜摩擦力所產生的力
- B) 由您和巴士地板之間的動摩擦力所產生的力
- C) 引力
- D) 由於您與巴士地板接觸而產生的法向力
- E) 沒有任何力令你向前摔倒

4. A worker at the local grocery store has a job consisting of the following five segments:

- (1) picking up boxes of tomatoes from the stockroom floor
- (2) accelerating to a comfortable speed
- (3) carrying the boxes to the tomato display shelf at constant speed
- (4) decelerating to a stop
- (5) lowering the boxes slowly to the floor

During which of the five segments of the job does the stock person do positive work on the boxes?

4. 一名雜貨店工人的工作包括以下五個部分：

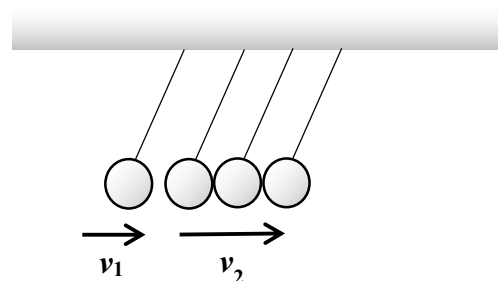
- (1) 從儲藏室的地板上撿起一盒番茄
- (2) 加速到舒適的速率
- (3) 將盒子以恆定速率運送到番茄陳列架
- (4) 減速到停止
- (5) 慢慢地將盒子放到地板上。

在工作的五個部分中，哪部分工人對盒子做了正功？

- A) (1) only
- B) (1) and (5)
- C) (1), (2), (4) and (5)
- D) (1) and (2)
- E) (2) and (3)

5. A Newton's cradle consists of four solid balls of equal mass hung on a bar. When the left ball undergoes an elastic collision with the other balls, it is traveling at the velocity v_1 and the other three balls are traveling at the velocity v_2 . Here, $v_1 > v_2$. What is the velocity of the right ball after the collision?

牛頓擺由四個等質量的實心球組成。當左球與其他球發生彈性碰撞時，它以速度 v_1 前進而其他三個球以速度 v_2 前進，其中 $v_1 > v_2$ 。碰撞後右球的速度是多少？



- A) v_1
- B) v_2
- C) $v_1 - v_2$
- D) $v_1 + v_2$
- E) none of the above 以上皆不是

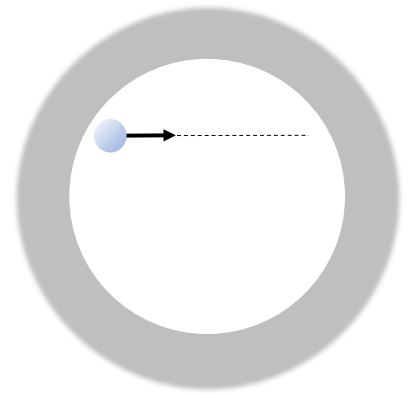
6. A bunch of bananas hangs from the end of a rope that passes over a light, frictionless pulley hung from a fixed tree. A monkey of mass equal to the mass of bananas hangs from the other end of the rope. The monkey and the bananas are initially balanced and at rest. Now the monkey starts to climb up the rope, moving away from the ground with speed v . What happens to the bananas?

6. 一串香蕉掛在一根繩子的末端，繩子從一個固定在樹上的滑輪穿過。滑輪很輕，而且沒有摩擦。質量等同於香蕉質量的猴子懸在繩子的另一端。猴子和香蕉最初是平衡和靜止不動。現在猴子開始爬上繩子，以速度 v 離開地面。請問香蕉會怎樣？

- A) They move up at speed $2v$. 它們以速率 $2v$ 上升。
- B) They move downward at speed v . 它們以速率 v 下降。
- C) They move up at speed $v/2$. 它們以速率 $v/2$ 上升。
- D) They remain stationary. 它們維持靜止不動。
- E) They move up at speed v . 它們以速率 v 上升。

7. Suppose the death star in Star Wars series can be modelled as a huge space station with the shape of a hollow sphere, a uniform spherical shell in the empty space. Now, you stand against the inner wall and fire a bullet to the other side of the inner wall as illustrated. The dashed line is its firing direction in a straight line.

7. 假設“星球大戰”系列中的死星可被模擬為一個巨大的太空站，其形狀為空心球體，球形外殼均勻。現在發生內戰，你向內壁的另一側發射子彈，如圖所示。虛線是其直線射擊方向。



Which of the following will describe the motion of the bullet.

- A) The bullet bends upwards and accelerates with larger speed.
- B) The bullet travels with constant velocity along the dashed line.
- C) The bullet decelerates and accelerates back to the original velocity when it hits the other side of wall
- D) The bullet accelerates and decelerates back to the original velocity when it hits the other side of wall
- E) None of the above is correct.

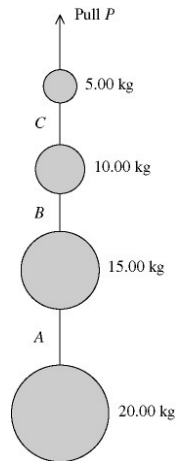
以下哪一項將描述子彈的運動？

- A) 子彈向上彎並加速至更快的速度。
- B) 子彈沿著虛線以恆定速度行進。
- C) 子彈先減速，後加速回初速度並擊中牆的另一側。
- D) 子彈先加速，後減速回初速度並擊中牆的另一側。
- E) 以上皆不正確。

8. A series of weights connected by very light cords are given a vertically upward acceleration of 4.00 m/s^2 by a pull P , as shown in the figure. A , B , and C are the tensions in the connecting cords. The **SMALLEST** of the three tensions, A , B , and C , is closest to

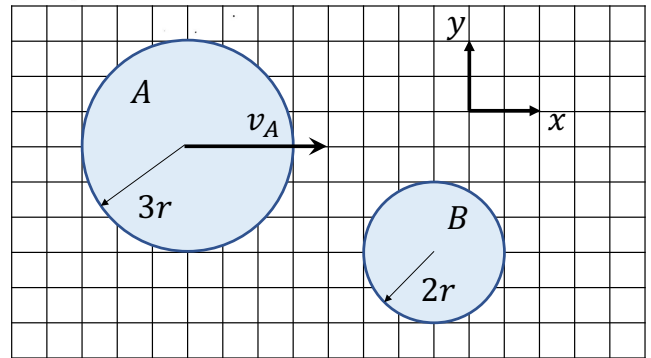
8. 通過非常輕的繩索連接的一系列配重通過拉力 P 給出 4.00 m/s^2 的垂直向上加速度，如圖所示。 A ， B 和 C 是連接繩索中的張力。三個張力 A ， B 和 C 中的最小值最接近

- A) 80.0 N. B) 196 N. C) 621 N.
D) 483 N. E) 276 N.



9. Two discs A and B of the same density and height with radii $3r$ and $2r$ respectively are placed on a smooth horizontal table. Disk A moves with the speed $v_A = 20 \text{ m/s}$ in a direction shown in the figure toward disk B which is at rest. If the friction between two discs is negligible, and the collision is perfectly elastic, what is the speed of disc B after collision:

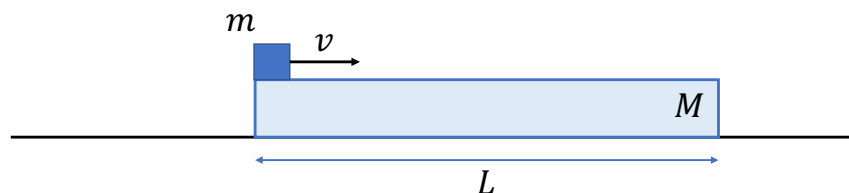
9. 在光滑水平面上放置兩個密度和高度相同的圓盤， A 和 B ，其半徑分別為 $3r$ 和 $2r$ 。 A 圓盤沿著圖示方向，以速率 $v_A = 20 \text{ m/s}$ 朝向靜止的 B 圓盤運動。若兩圓盤之間的摩擦力可以忽略不計，而且碰撞為完全彈性， B 圓盤被碰撞後的速率為：



- A) 22.15 m/s B) 16.62 m/s C) 27.69 m/s
D) 19.20 m/s E) 14.40 m/s

10. As shown in the figure below, a rectangular block of mass M and length L is placed on a smooth table. A small cube of mass m with negligible length is placed on the left end of the block with horizontal initial velocity v sliding to the right. If the coefficient of kinetic friction between the block and the cube is μ , and the cube does not fall from the right end of the block, the maximum value of initial velocity v of the cube is:

10. 如下圖所示，在一光滑水平上，靜置有一質量為 M ，長度為 L 的長方體。一質量為 m ，長度可忽略的小木塊放在長方體上，由左端沿中線以水平初速 v 開始向右滑行。若木塊與長方體之間的動摩擦係數為 μ ，且木塊不會由長方體右端掉落，則木塊初速 v 之最大值為：



- A) $\sqrt{2\mu gL}$ B) $\sqrt{\mu gL \left(1 + \frac{m}{M}\right)}$ C) $\sqrt{2\mu gL \left(1 + \frac{m}{M}\right)}$
D) $\sqrt{2\mu gL \left(1 + \frac{m^2}{M^2}\right)}$ E) $\sqrt{2\mu gL \left(1 + \frac{2m}{M}\right)}$

11. Following the previous question. What is the displacement of the rectangular block when the cube stops sliding on the rectangular block?

11. 承上題，當木塊在長方體上停止滑行時，長方體的位移是多少？

- A) 0
 B) $\frac{L}{2}$
 C) $\left(\frac{m}{m+M}\right)^2 L$
 D) $\left(\frac{M}{m+M}\right) L$
 E) $\left(\frac{m}{m+M}\right) L$

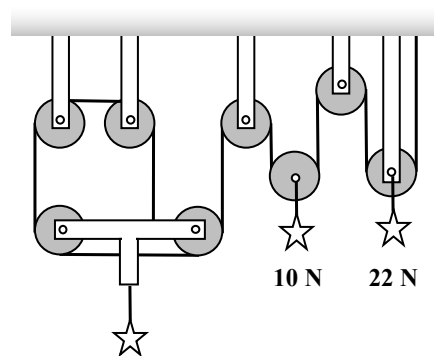
12. Train stations A and B are separated by a distance of 100 km. At 12 pm train T_A starts from station A and runs at a speed of 50 km per hour along a railway towards station B. The railway between A and B is a straight line. At the same time train T_B starts from B and runs at the same speed along the same railway towards station A. When T_A starts, a bird flies forward from T_A at a speed of 70 km per hour. When the bird meets T_B , it turns around and flies towards T_A with the same speed as it flew before. When the bird meets T_A again, the time is

12. 火車站 A 和 B 相隔 100 公里。中午 12 時，火車 T_A 從 A 站開出，沿著鐵路向 B 站以每小時 50 公里的速率行駛。A 和 B 之間的鐵路是直線。同一時間，列車 T_B 從 B 出發，沿同一鐵路以相同的速率向 A 站行駛。當 T_A 出發時，一隻小鳥以每小時 70 公里的速率從 T_A 向前飛。當小鳥與 T_B 相遇時，它轉身並以牠以前飛過的速率飛回 T_A 。當小鳥回到 T_A 時，時間是

- A) 12:52 pm
 B) 12:54 pm
 C) 12:56 pm
 D) 12:58 pm
 E) 1:00 pm

13. Three star medals hang from the (stationary) assembly of massless pulleys and cords as shown in the figure. One long cord runs from the ceiling at the right to the T-shaped structure at the left, looping around all the pulleys. The weights of two medals are given. What is the weight of the medal on the left?

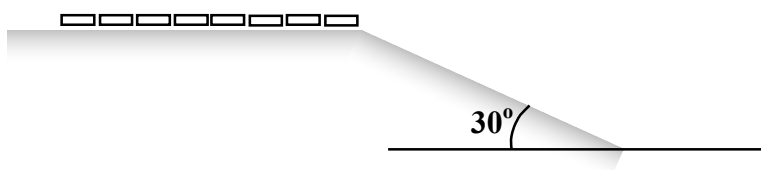
13. 如圖所示，三個星形獎章懸掛在無質量滑輪和繩索的（固定）組件上。一根長繩從右邊的天花板延伸到左邊的 T 形結構，環繞著所有滑輪。現給出其中兩枚獎章的重量。左邊獎章的重量是多少？



- A) 5 N
 B) 15 N
 C) 24 N
 D) 36
 E) 48 N

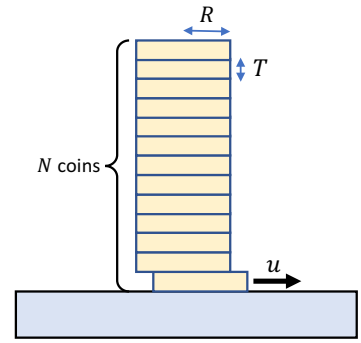
14. As shown in the figure, a train with a length of 100 m rests on a horizontal rail, and the front carriage of the train rests at the edge of a rail on a slope inclined at an angle of 30° . When the front carriage is tipped from rest onto the slope by a very small force, the train rolls freely down. What is the speed of the train when half of the carriages have rolled down the slope?

14. 如圖所示，長度為 100 米的列車停在水平軌道上，列車的前車廂位於傾斜角度為 30° 的斜坡上的軌道邊緣。當前車廂被非常小的力從靜止推下斜坡時，火車自由地向下滑動。當一半車廂滾下斜坡時，列車的速度是多少？



- A. 5.5 ms^{-1} B. 7.8 ms^{-1} C. 11 ms^{-1} D. 16 ms^{-1} E. 22 ms^{-1}

15. A pile of N one-dollar coins rest on a table. The mass, radius and thickness of each coin are M , R and T respectively. The coefficient of kinetic friction between the coin surfaces is μ . An external agent strikes the coin at the bottom such that it is removed from the pile with horizontal speed u . The removal speed u is so high that the pile of coins effectively remains vertical after the removal. What is the horizontal displacement of the pile of coins after the strike?



15. 一疊 N 個硬幣靜止在桌上。每個硬幣的質量、半徑和厚度分別為 M , R 和 T 。硬幣表面之間的動摩擦係數是 μ 。一人從底部撞擊硬幣，使得最底之硬幣以水平速度 u 從硬幣堆中飛出。飛出的速度非常高，以致硬幣飛出後堆疊的硬幣仍然有效地保持垂直。請問撞擊後這一疊硬幣的水平位移是多少？

- A) $\frac{2R\sqrt{2gT}}{(N-1)\mu u}$ B) $\frac{2(N-1)R\sqrt{2gT}}{N\mu u}$ C) $\frac{2\mu R\sqrt{2gT}}{u}$ D) $\frac{2(N-1)\mu R\sqrt{2gT}}{u}$ E) $\frac{2N\mu R\sqrt{2gT}}{u}$

16. A pendulum is formed by a particle of mass $m = 0.1$ kg and a massless string of length $l = 0.5$ m hung from a fixed point in the ceiling. The pendulum is swung from rest at an initial angle of $\theta_0 = 30^\circ$ with the vertical. Calculate the tension of the string when the angle of the string with the vertical is $\theta = 15^\circ$.

16. 一物塊從天花板的固定點以無質量的弦線懸掛，組成單擺。物塊的質量為 $m = 0.1$ kg，弦線的長度為 $l = 0.5$ m。單擺從靜止狀態開始擺動，與垂直方向的初始夾角為 $\theta_0 = 30^\circ$ 。當弦線與垂直方向的夾角為 $\theta = 15^\circ$ 時，求弦線的張力。

- A) 0.10 N B) 0.20 N C) 0.22 N D) 1.14 N E) 1.24 N

17. A bullet of mass 0.01 kg is fired towards a rest block of wood with mass 10 kg on a smooth table. What is the maximum fraction of the initial kinetic energy of the bullet that can be turned into heat when it hits the rest mass?

17. 一質量為 0.01 kg 的子彈射向一塊放在光滑桌面上質量為 10 kg 的靜止木塊。求子彈的初始動能在撞擊靜止質量後轉變為熱量的最大分數比是多少？

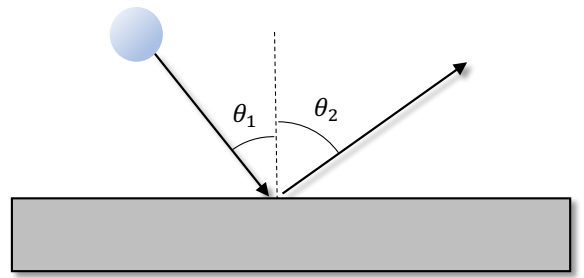
- A. 1 B. $\frac{1000}{1001}$ C. $\frac{999}{1000}$
D. $\frac{4000}{1002001}$ E. Not enough information 資料不足

18. The orbiting period of Moon is T_M (27 days), orbiting at a height of $(r_M - R_E)$ above the Earth of radius R_E . A geostationary satellite is a satellite that always stays above the same location on the Equator. It stays at a height of $(r_S - R_E)$ above the Earth with an orbiting period of T_S (1 day). What is the ratio r_M/r_S in terms of T_M and T_S ?

18. 月球的軌道周期是 T_M (27 天)，從地面高度 $(r_M - R_E)$ 圍繞半徑為 R_E 的地球運行。地球同步衛星一直停留在赤道上的同一位置，位於地面高度 $(r_S - R_E)$ ，軌道周期為 T_S (1 天)。求 r_M/r_S 的比，以 T_M 和 T_S 來表示。

- A. $(T_M/T_S)^{1/2}$ B. T_M/T_S C. T_S/T_M D. $(T_M/T_S)^{2/3}$ E. None of the above

19. A ball, with mass 1 kg, is bouncing off a frictionless rectangular block at incoming angle $\theta_1 = 30^\circ$ and exit angle $\theta_2 = 60^\circ$ measured from the normal direction of the block, as shown in the figure. The rectangular block can also freely move. What is the mass of the block? Elastic collision is assumed.

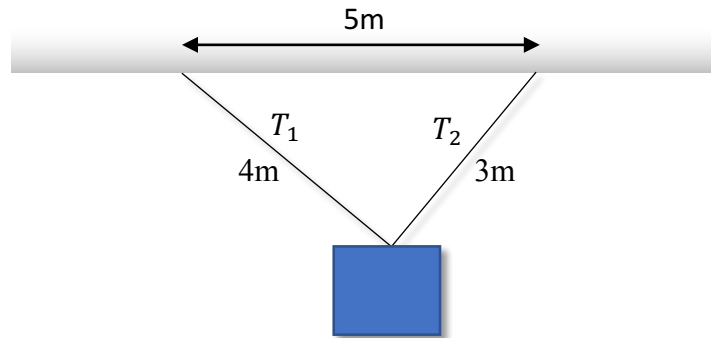


19. 質量為 1 kg 的球從一個無摩擦的矩形塊以入射角 $\theta_1 = 30^\circ$ 射入，並以出射角 $\theta_2 = 60^\circ$ 反彈。所有角度均由矩形塊的法線方向測量，見附圖。矩形塊可以自由移動，而且碰撞是彈性的。求矩形塊的質量。

- A. 1kg B. 2kg C. 3kg D. 5kg E. Infinite mass

20. A mass of 5 kg is hung by two strings attached to the same position of the mass, one with 4 m long and tension T_1 and another with 3 m long and tension T_2 , which are connected to the ceiling at two locations with 5 m separation. What are the tensions T_1 and T_2 of the two strings?

20. 質量為 5 kg 的物塊由兩根繩子從物塊的同一位置懸掛著，兩根繩子分別連接到天花板兩個位置上，間隔為 5 m。繩子 1 長 4 m、張力 T_1 ，繩子 2 長 3 m、張力 T_2 。求兩根繩子的張力 T_1 和 T_2 。

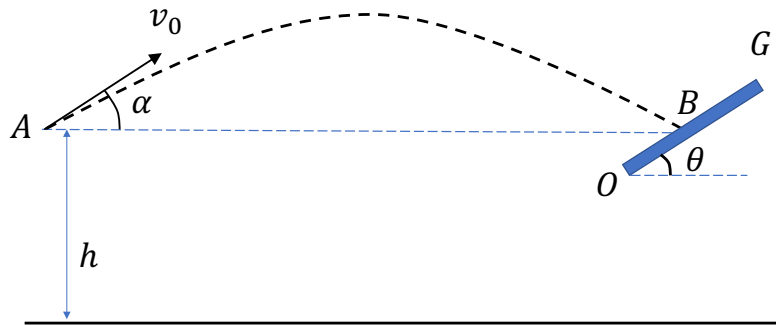


- A. $T_1 = 4 \times 9.8 \text{ N}$, $T_2 = 3 \times 9.8 \text{ N}$ B. $T_1 = 5 \times 9.8 \text{ N}$, $T_2 = 3 \times 9.8 \text{ N}$
 C. $T_1 = 3 \times 9.8 \text{ N}$, $T_2 = 4 \times 9.8 \text{ N}$ D. $T_1 = 4 \times 9.8 \text{ N}$, $T_2 = 5 \times 9.8 \text{ N}$
 E. $T_1 = 5 \times 9.8 \text{ N}$, $T_2 = 4 \times 9.8 \text{ N}$

Open Problems (15 Marks Each) 開放題(每題 15 分)

1. From a fixed point A at a height h above the ground, a ball X is thrown at a speed v_0 with launching angle α ($0 < \alpha < \frac{\pi}{2}$). A very massive plate OG is placed in front of point A and the ball hits at the point B on the plate OG which is above the ground at the same level as the point A.

從離地面的高度為 h 的固定點 A 將球 X 以速度 v_0 拋出，拋射角為 α ($0 < \alpha < \frac{\pi}{2}$)。在 A 點前方適當位置放一質量非常大的平板 OG ，讓球 X 撞擊平板 OG 上的 B 點，並使碰撞點 B 與 A 點等高。



a) What is the horizontal distance AB ?

a) AB 的水平距離是多少？

b) If the collision between the ball X and the plate is perfectly elastic. Find all the values of the inclination angle θ ($0 < \theta < \frac{\pi}{2}$) of the plate such that the ball X will return to the initial position A after the collision.

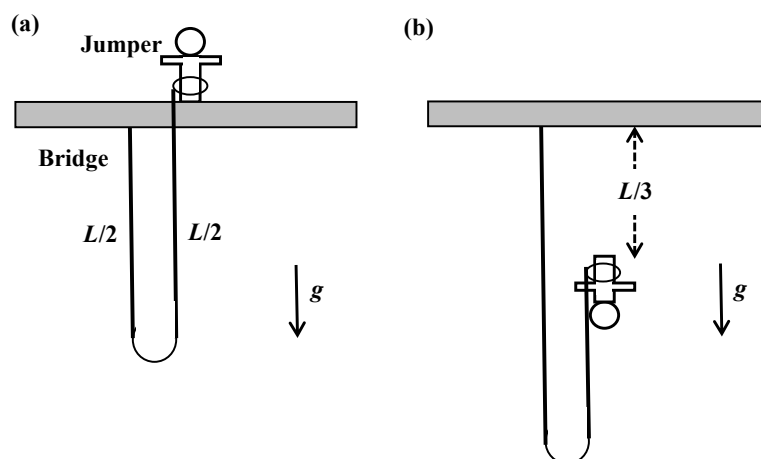
b) 如果球 X 與平板做完全彈性碰撞。求所有平板的傾角 θ ($0 < \theta < \frac{\pi}{2}$) 之值，使球 X 在碰撞後恰好能回到 A 點。

c) There is another small ball Y. At the instant when the ball X is thrown from point A, it falls from rest freely from point A and makes a perfectly elastic collision with the ground. Find all the values of h such that the ball Y will return to the initial position A with ball X at the same instant.

c) 另有一小球 Y，在球 X 自 A 點拋出的同時，它從 A 點自由落下，並與地面完全彈性碰撞。如果球 Y 與球 X 在同一時間返回初始位置 A，求所有 h 之值。

2. Bungee jump is a sport for adventurous people. As shown in Fig. 1(a), a bungee jumper of mass $m = 60$ kg is initially standing on a bridge with a rope of mass $M = 120$ kg, length $L = 30$ m and of uniform density. One end of the rope is fastened to the bridge and the other end is tied to the jumper's body, and the rope is hanging freely below the bridge, with its midpoint furthest down.

笨豬跳是一項適合喜歡冒險的人的運動。如圖 1(a) 所示，質量 $m = 60$ kg 的笨豬跳高手最初站在橋上，繩索的質量 $M = 120$ kg，長度 $L = 30$ m，密度均勻。繩索的一端固定在橋上，另一端繫在高手身上，繩索在橋下自由懸掛，中點在最下方。



In the following calculations, you may assume that friction and air resistance are negligible, and the gravitational acceleration is $g = 9.8 \text{ m/s}^2$. You may further assume that the rope has no stretching until the entire rope becomes hanging downward.

在下面的計算中，摩擦和空氣阻力可忽略不計，重力加速度為 $g = 9.8 \text{ m/s}^2$ 。您可以進一步假設繩索在整個繩索向下懸垂之前沒有伸展。

(a)-(b) Consider the instant that the jumper has fallen a distance $L/3$,

(a)-(b) 考慮高手下降至距離 $L/3$ 的瞬間，

(a) What is the change in the potential energy of the rope compared with the initial state?

(a) 與初始狀態相比，繩索的勢能變化是什麼？

(b) Calculate the speed of the jumper when the jumper has fallen a distance $L/3$.

(b) 當高手下降至距離 $L/3$ 時，計算高手的速率。

(c) Calculate the speed of the jumper when the entire rope becomes hanging downward.

(c) 計算當整個繩索向下懸垂時高手的速率。

(d) Jumpers claim that they enjoy the thrill of reaching a speed faster than free fall. Calculate the speed of the jumper if the jumper falls from the bridge through the same distance as in part (c) during a free fall.

(d) 高手聲稱他們享受比自由落體更快的速度刺激感。假設高手從橋上以自由落體方式下降至與 (c) 部分相同的距離時，計算高手的速率。

(e)-(f) After the entire rope becomes hanging downward, the jumper remains tied to the rope but continues to fall further downward due to the elastic nature of the rope. Let the force constant of the rope be $k = 500 \text{ N/m}$.

(e)-(f) 在整個繩索向下懸垂後，高手仍然繫在繩索上，但由於繩索的彈性，他會繼續向下墜落。假設繩索的力常數為 $k = 500 \text{ N/m}$ 。

(e) Calculate the maximum distance the jumper falls as measured from the bridge.

(e) 計算高手從橋上下降的最大距離。

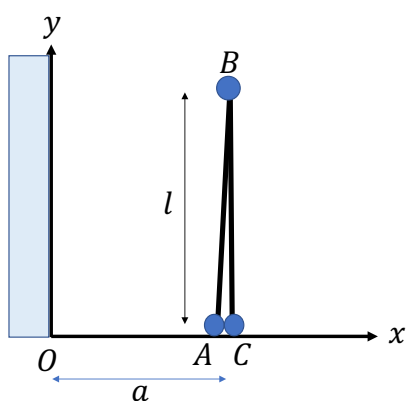
(f) After the motion of the jumper stops, what is the distance of the jumper below the bridge?

(f) 當高手的運動停止後，高手在橋下的距離是多少？

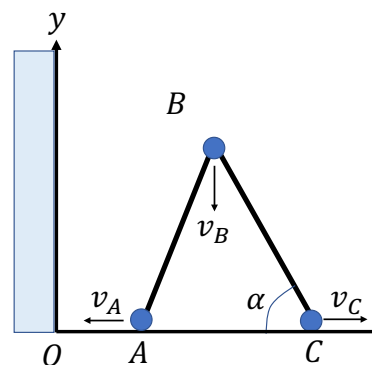
3. Three identical steel balls A, B, and C are connected by two light, hard rods AB and BC, each of length l . A and C lie on a horizontal surface along the x axis, and B is supported by the rods to stand in a vertical direction parallel to the y axis. As shown in Figure I, there is a vertical wall at distance $a = \frac{1}{\sqrt{2}}l$ from the rod. Due to small disturbance, two rods slide to two sides and ball B falls (Figure II).

In this problem, all friction is negligible, and the diameter of each ball is much smaller than l .

3. 三個相同的鋼球 A, B, C 由兩根輕質的長為 l 的硬杆 AB 和 BC 連接, A 和 C 處於水平面的 x 軸上, B 則由 AB 和 BC 兩杆支持著, 豎立在垂直面上, 與 y 軸平行, 如圖 I 所示。距杆 $a = \frac{1}{\sqrt{2}}l$ 處有一面豎直牆, 因受微小擾動, 兩杆分別向兩邊滑動, 使 B 球下降(圖 II)。在這問題中, 所有摩擦可忽略不計, 各球直徑都比 l 小很多。



(I)



(II)

(a) What is the horizontal distance of the center of mass (CM) of the system from the origin O when $\angle ACB = \alpha$?

(a) 當 $\angle ACB = \alpha$ 時, 求系統質心 (CM) 與原點 O 的水平距離是多少?

(b) What is the ratio of the speed v_A/v_C of two balls A and C when $\angle ACB = \alpha$?

(b) 當 $\angle ACB = \alpha$ 時, 求球 A 和 C 的速率比例 v_A/v_C 。

(c) What is the angle α when ball A hits the wall?

(c) 當球 A 撞到牆壁時的角度 α 是多少?

(d) What is the ratio v_B/v_C immediately before this instant?

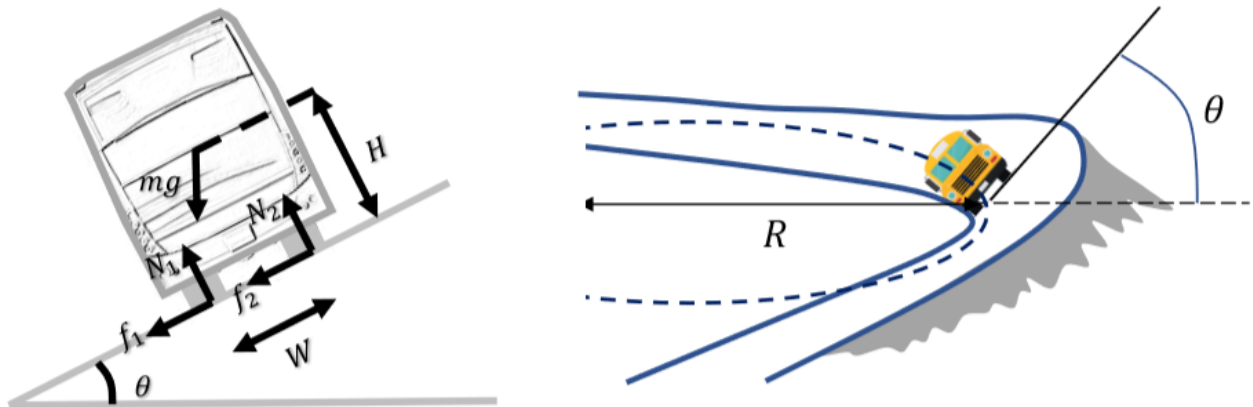
(d) 求在這瞬間前 v_B/v_C 的比例。

(e) What is the velocities of three balls at this moment?

(e) 在這瞬間前三球的速度分別是多少?

4. A bus is going around a banked turn, as shown in the figure. The horizontal radius of the turn is $R = 60$ m and the inclination angle of the bank is $\theta = 30^\circ$. The figure also shows the cross-section of the bus, having a width of $W = 3$ m between the inner and outer wheels and a center of mass (CM) at a height of $H = 2.5$ m. We also label several forces: N_1 and N_2 are the normal forces acting on the inner and outer wheels while f_1 and f_2 are the friction action on the inner and outer wheels. mg is the weight of the bus.

4. 如圖所示，一輛巴士正在轉彎處轉彎。轉彎處的水平半徑是 $R = 60$ m，傾斜角度是 $\theta = 30^\circ$ 。圖中還畫出了一架巴士的橫截面，其內輪和外輪之間的寬度為 $W = 3$ m，質心的高度是 $H = 2.5$ m。我們還標記了幾種力： N_1 和 N_2 是內輪和外輪上的法向力， f_1 和 f_2 是內輪和外輪上的摩擦力。 mg 是巴士的重量。



a) Suppose the static coefficient of friction of the bank is $\mu = 0.5$, which is a bit slippery in a rainy day. Determine the valid range of speed v for the bus so that its center-of-mass motion remains in circular shape. For simplicity, we can start by considering the total friction as $f = f_1 + f_2$, the total normal force as $N = N_1 + N_2$ and neglect the internal distribution between f_1 and f_2 , i.e. we can assume $|f/N| \leq \mu$. Note that the radius of turn is measured from center of the turn to the CM.

a) 假設在一個下雨天，路面比較濕滑，轉彎處的靜摩擦係數為 $\mu = 0.5$ 。求巴士的速率 v 的有效範圍，使巴士的質心運動維持圓形。為簡單起見，我們可以考慮總摩擦力為 $f = f_1 + f_2$ ，總法向力為 $N = N_1 + N_2$ ，忽略摩擦力 f_1 和 f_2 的內部分佈，即我們可假設 $|f/N| \leq \mu$ 。注意，轉彎處的水平半徑是指從轉彎中心到質心位置的距離。

b) By considering the moment of the bus about its CM, determine the valid range of the speed v for the bus so that the bus will not topple. Toppling means the bus overturns.

b) 通過考慮巴士相對於其質心 CM 的力矩，求巴士的有效速度範圍，以使巴士不會傾倒。傾倒意味著巴士翻車。

END OF PAPER (全卷完)