Hong Kong Physics Olympiad 2019
2019 年香港物理奧林匹克競賽

Organisers 合辦機構

Education Bureau
教育局
The Hong Kong Academy for Gifted Education
香港資優教育學苑
The Hong Kong University of Science and Technology
香港科技大學

Advisory Organisations 顧問機構

The Physical Society of Hong Kong
香港物理學會
Hong Kong Physics Olympiad Committee
香港物理奧林匹克委員會

5 May, 2019
2019 年 5 月 5 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English, but only ONE language should be used throughout the whole paper.

   所有題目均為中英對照。你可選擇以中文或英文作答，惟全卷必須以單一語言作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將於比賽後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. On the cover of the answer book and the multiple-choice answer sheet, please write your 8-digit Contestant Number, English Name, and Seat Number.

   在答題簿封面及選擇題答題紙上，請填上你的 8 位數字參賽者號碼、英文姓名、及座位號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆**完全塗黑**。

5. The open-ended problems are quite long. Please read the whole problem first before attempting to solve them. If there are parts that you cannot solve, you are allowed to treat the answer as a known answer to solve the other parts.

   開放式問答題較長，請將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來做其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別注明，否則本卷將使用下列符號和常數 ：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	$9.8\ \text{m s}^{-2}$
Gravitational constant 重力常數	G	$6.67\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$

Trigonometric identities:

三角學恆等式 ：

$$\sin(x+y)=\sin x\cos y+\cos x\sin y$$

$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$$

$$\sin x+\sin y=2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x+\cos y=2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x-\cos y=2\sin\frac{x+y}{2}\sin\frac{y-x}{2}$$

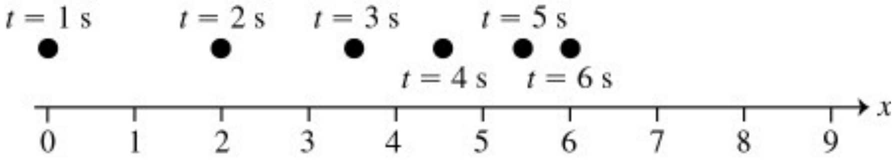
$$\sin x\sin y=\frac{1}{2}\cos(x-y)-\frac{1}{2}\cos(x+y)$$

$$\cos x\cos y=\frac{1}{2}\cos(x-y)+\frac{1}{2}\cos(x+y)$$

$$\sin x\cos y=\frac{1}{2}\sin(x+y)+\frac{1}{2}\sin(x-y)$$

Multiple Choice Questions (2 Marks Each) 選擇題(每題 2 分)

1. The figure shows the position of an object (moving along a straight line) as a function of time. Assume two significant figures in each number. Which of the following statements about this object is true over the interval shown?
1. 圖中顯示了一物體的位置表示為時間的函數 （沿直線移動） 。假設每個數字中有兩個有效數字。在顯示的時間間隔內，以下哪些關於此物體的陳述是正確的？



A) The object is accelerating to the right. 物體向右加速。

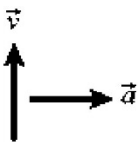
B) The average speed of the object is 1.0 m/s. 物體的平均速率是1.0 m/s。

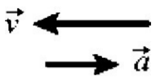
C) The acceleration of the object is in the same direction as its velocity. 物體的加速度與其速度方向相同。

D) The object is accelerating to the left. 物體向左加速。

E) The acceleration of the object is in the same direction as its displacement. 物體的加速度與其位移方向相同。

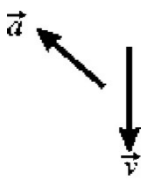
2. Shown below are the velocity and acceleration vectors for a person in several different types of motion. In which case is the person slowing down and turning to his right?
2. 下面顯示的是幾種不同類型運動中人的速度和加速度矢量。在哪種情況下，這個人放慢速度並向右轉？

A) 

B) 

C) 

D) 

E) 

3. You are standing in a moving bus, facing forward, and you suddenly step forward as the bus comes to an immediate stop. The force acting on you that causes you to step forward is

你站在一輛移動的巴士上，面向前方，當巴士剎停時你突然向前摔倒。作用在你身上並讓你向前摔倒的力是

A) the force due to static friction between you and the floor of the bus 由您和巴士地板之間的靜摩擦力所產生的力

B) the force due to kinetic friction between you and the floor of the bus
B)由您和巴士地板之間的動摩擦力所產生的力

C) the force of gravity
C) 引力

D) the normal force due to your contact with the floor of the bus
D) 由於您與巴士地板接觸而產生的法向力

E) No forces acting on you to cause you to fall.
E) 沒有任何力令你向前摔倒

4. A worker at the local grocery store has a job consisting of the following five segments:
(1) picking up boxes of tomatoes from the stockroom floor
(2) accelerating to a comfortable speed
(3) carrying the boxes to the tomato display shelf at constant speed
(4) decelerating to a stop
(5) lowering the boxes slowly to the floor
During which of the five segments of the job does the stock person do positive work on the boxes?

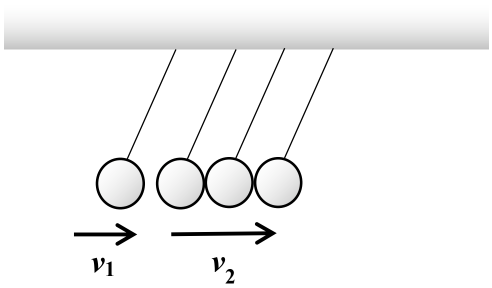
4. 一名雜貨店工人的工作包括以下五個部分:
(1) 從儲藏室的地板上撿起一盒番茄
(2) 加速到舒適的速率
(3) 將盒子以恆定速率運送到番茄陳列架
(4) 減速到停止
(5) 慢慢地將盒子放到地板上。
在工作的五個部分中，哪部分工人對盒子做了正功？

A) (1) only
B) (1) and (5)
C) (1), (2), (4) and (5)
D) (1) and (2)
E) (2) and (3)

5. A Newton's cradle consists of four solid balls of equal mass hung on a bar. When the left ball undergoes an elastic collision with the other balls, it is traveling at the velocity v_1 and the other three balls are traveling at the velocity v_2. Here, $v_1 > v_2$. What is the velocity of the right ball after the collision?
牛頓擺由四個等質量的實心球組成。當左球與其他球發生彈性碰撞時，它以速度v_1前進而其他三個球以速度v_2前進，其中$v_1 > v_2$。碰撞後右球的速度是多少？

A. v_1 B. v_2 C. $v_1 - v_2$

D. $v_1 + v_2$ E. none of the above 以上皆不是



Consider the reference frame traveling with velocity v_2 relative to the lab frame. The velocity of the left ball is $v_1 - v_2$ and the 3 balls are stationary. Due to the conservation of momentum and energy, the velocity of the right ball is $v_1 - v_2$ and the other 3 balls are stationary.
Converting to the lab frame, velocity of the right ball is $(v_1 - v_2) + v_2 = v_1$.
Answer: A.

6. A bunch of bananas hangs from the end of a rope that passes over a light, frictionless pulley hung from a fixed tree. A monkey of mass equal to the mass of bananas hangs from the other end of the rope. The monkey and the bananas are initially balanced and at rest. Now the monkey starts to climb up the rope, moving away from the ground with speed v. What happens to the bananas?
6. 一串香蕉掛在一根繩子的末端，繩子從一個固定在樹上的滑輪穿過。滑輪很輕，而且沒有摩擦。質量等同於香蕉質量的猴子懸在繩子的另一端。猴子和香蕉最初是平衡和靜止不動。現在猴子開始爬上繩子，以速度v離開地面。請問香蕉會怎樣？

A. They move up at speed $2v$. 它們以速率 $2v$ 上升。

B. They move downward at speed v. 它們以速率 v 下降。

C. They move up at speed $v/2$. 它們以速率 $v/2$ 上升。

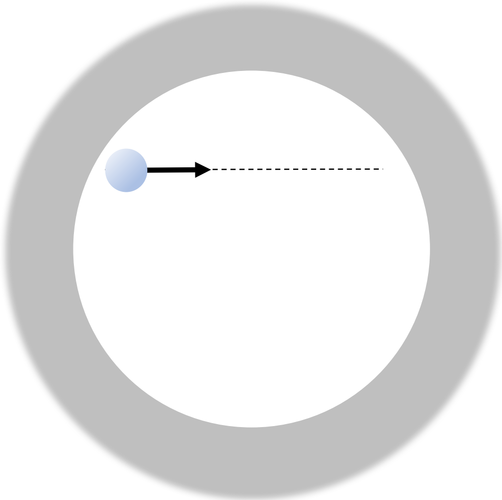
D. They remain stationary. 它們維持靜止不動。

E. They move up at speed v. 它們以速率 v 上升。

When the monkey starts to move up the tree, its momentum increases in the upward direction. This momentum change is provided by an impulse due to the change in the tension of the rope. This impulse is transmitted to the bananas and causes the bananas to move upward with the same speed.
Answer: E.

7. Suppose the death star in Star Wars series can be modelled as a huge space station with the shape of a hollow sphere, a uniform spherical shell in the empty space. Now, you stand against the inner war and fire a bullet to the other side of the inner wall as illustrated. The dashed line is its firing direction in a straight line.

7. 假設“星球大戰”系列中的死星可被模擬為一個巨大的太空站，其形狀為空心球體，球形外殼均勻。現在發生內戰，你向內壁的另一側發射子彈，如圖所示。虛線是其直線射擊方向。



Which of the following will describe the motion of the bullet.
A. The bullet bends upwards and accelerates with larger speed.

B. The bullet travels with constant velocity along the dashed line.

C. The bullet decelerates and accelerates back to the original velocity when it hits the other side of wall

D. The bullet accelerates and decelerates back to the original velocity when it hits the other side of wall

E. None of the above is correct.

以下哪一項將描述子彈的運動?

A. 子彈向上彎並加速至更快的速度 。

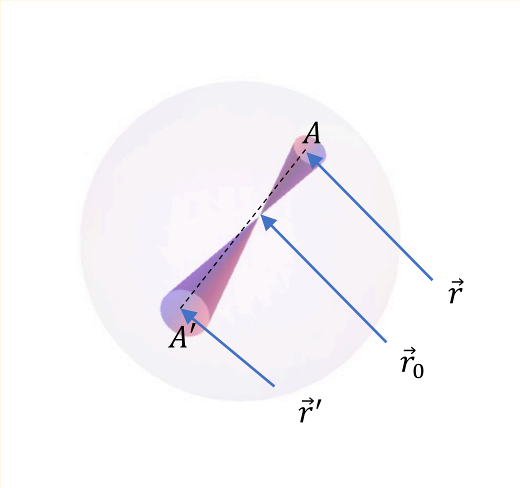
B. 子彈沿著虛線以恆定速度行進。

C. 子彈先減速，後加速回初速度並擊中牆的另一側。

D. 子彈先加速，後減速回初速度並擊中牆的另一側。

E. 以上皆不正確。

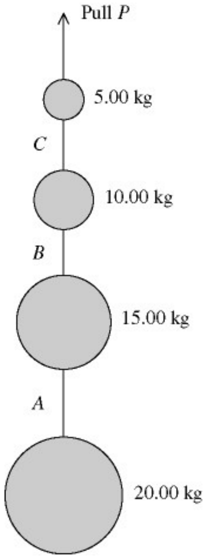
Ans:



Small area = distance^2×solid angle. For an arbitrary point $\vec{r}_0$ enclosed by the shell, a small area A on the shell is projected to its counterpart area A' through the fixed point $\vec{r}_0$. A and A' have the same solid angle and therefore $A/|\vec{r}-\vec{r}_0|^2 = A'/|\vec{r}'-\vec{r}_0|^2$. As a result of this equality, the gravitational force from mass on area A cancels the gravitational force from mass on area A' at $\vec{r}_0$. This cancellation happens for all areas on the shell. Therefore, no gravity is being felt within the shell. The situation is equivalent to just an empty space and a bullet travels along a straight line.

8. A series of weights connected by very light cords are given a vertically upward acceleration of 4.00m/s^2 by a pull *P*, as shown in the figure. *A*, *B*, and *C* are the tensions in the connecting cords. The **SMALLEST** of the three tensions, *A*, *B*, and *C*, is closest to

8. 通過非常輕的繩索連接的一系列配重通過拉力P給出 4.00 m/s^2的垂直向上加速度，如圖所示。 A，B和C是連接繩索中的張力。三個張力A，B和C中的**最小值最接近**



A) 80.0 N.　　B) 196 N.　　C) 621 N.

D) 483 N.　　E) 276 N.

Solution:

We take $g = 9.8m/s^2$,

$$P-(5+10+15+20)g=(5+10+15+20)a \Rightarrow P=50(a+g)=690\text{ N}$$

The tension C is

$$P-C-5g=5a \Rightarrow C=700-5*14=621\ N$$

The tension B is

$$C-B-10g=10a \Rightarrow B=C-10*14=483\text{ N}$$

The tension A is

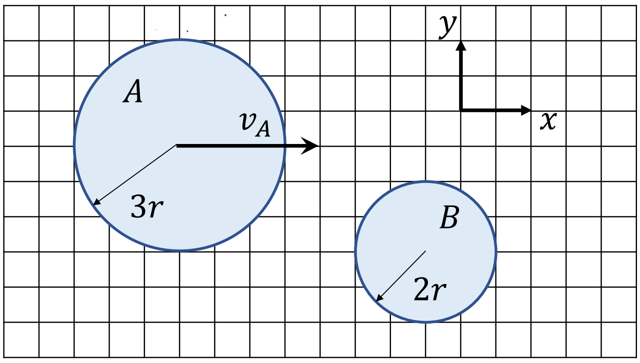
$$B-A-15g=15a \Rightarrow A=B-15*14=276\text{ N}$$

Alternatively,

$$A-20g=20a \Rightarrow A=20*14=276\text{ N}$$

9. Two discs A and B of the same density and height with radii $3r$ and $2r$ respectively are placed on a smooth horizontal table. Disk A moves with the speed $v_A = 20\ m/s$ in a direction shown in the figure toward disk B which is at rest. If the friction between two discs is negligible, and the collision is perfectly elastic, what is the speed of disc B after collision:

9. 在光滑水平面上放置兩個密度和高度相同的圓盤，A和B，其半徑分別為$3r$ 和$2r$。A圓盤沿著圖示方向，以速率$v_A = 20\ m/s$朝向靜止的B圓盤運動。若兩圓盤之間的摩擦力可以忽略不計，而且碰撞為完全彈性，B圓盤被碰撞後的速率為:



A. 22.15 m/s
B. 16.62 m/s
C. 27.69 m/s
D. 19.20 m/s
E. 14.40 m/s

Solution:

At the collision, the angle with the horizontal axis is

$$\tan\theta = \frac{3}{4}$$

and the velocity of A along the collision axis is

$$u_A = v\cos\theta = \frac{4}{5}v_A = 16\text{ m/s}$$

The masses of and disc A and B are $m_A = 9m_0$ and $m_B = 4m_0$ respectively.

By the conservation of momentum and energy,

$$m_A u_A = m_A u_A' + m_B u_B'$$

and

$$\frac{1}{2}m_A u_A^2 = \frac{1}{2}m_A {u_A'}^2 + \frac{1}{2}m_B {u_B'}^2$$

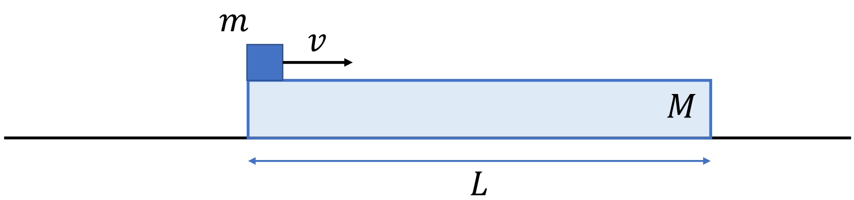
By simplification,

$$9u_A^2 = 9\left(u_A - \frac{4}{9}u_B'\right)^2 + 4{u_B'}^2$$

$$\Rightarrow u_B' = 22.15 \text{ m/s } or \ \ 0 \text{ (rejected)}$$

10. As shown in the figure below, a rectangular block of mass M and length L is placed on a smooth table. A small cube of mass m with negligible length is placed on the left end of the block with horizontal initial velocity v sliding to the right. If the coefficient of kinetic friction between the block and the cube is μ, and the cube does not fall from the right end of the block, the maximum value of initial velocity v of the cube is:

10. 如下圖所示，在一光滑水平上，靜置有一質量為M，長度為L的長方體。一質量為m，長度可忽略的小木塊放在長方體上，由左端沿中線以水平初速v開始向右滑行。若木塊與長方體之間的動摩擦係數為μ，且木塊不會由長方體右端掉落，則木塊初速v之最大值為:



A) $\sqrt{2\mu g L}$

B) $\sqrt{\mu g L\left(1+\frac{m}{M}\right)}$

C) $\sqrt{2\mu g L\left(1+\frac{m}{M}\right)}$

D) $\sqrt{2\mu g L\left(1+\frac{m^2}{M^2}\right)}$

E) $\sqrt{2\mu g L\left(1+\frac{2m}{M}\right)}$

11. Following the previous question. What is the displacement of the rectangular block when the cube stops sliding on the rectangular block?

11. 承上題，當木塊在長方體上停止滑行時， 長方體的位移是多少？

A) 0

B) $\frac{L}{2}$

C) $\left(\frac{m}{m+M}\right)^2 L$

D) $\left(\frac{M}{m+M}\right) L$

E) $\left(\frac{m}{m+M}\right) L$

Solution:

Initially, the velocity of the cube and rectangle block are v and 0 respectively.

The kinetic friction is $f_k = \mu m g$ and the acceleration on the cube and the block are $a_m = -\mu g$ and $a_M = \frac{\mu m g}{M}$.

$$v_m = v - \mu g t$$

$$v_M = \frac{\mu m g t}{M}$$

They will accelerate until they have the same velocity when the cube is at the right end, i.e.,

$$v-\mu gt=\frac{\mu mgt}{M}\Rightarrow\left(\frac{m}{M}+1\right)\mu gt=v\Rightarrow t=\frac{v}{\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)$$

At this time the position of the cube and the right end of the block are:

$$x_m=vt+\frac{1}{2}a_m t^2=\frac{v^2}{\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)-\frac{1}{2}\frac{v^2}{\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)^2=\frac{v^2}{\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)\left(1-\frac{1}{2}\left(\frac{1}{1+\frac{m}{M}}\right)\right)$$

$$x_m=\frac{v^2}{\mu g}\left(\frac{\frac{1}{2}+\frac{m}{M}}{\left(1+\frac{m}{M}\right)^2}\right)$$

$$x_M=L+\frac{1}{2}a_M t^2=L+\frac{1}{2}\frac{m}{M}\frac{v^2}{\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)^2$$

If they coincide at time t,

$$x_m=x_M$$

$$\Rightarrow\frac{v^2}{\mu g}\frac{1}{\left(1+\frac{m}{M}\right)^2}\left(\frac{1}{2}+\frac{m}{M}-\frac{1}{2}\frac{m}{M}\right)=L$$

$$\Rightarrow\frac{v^2}{2\mu g}=L\left(1+\frac{m}{M}\right)\Rightarrow v=\sqrt{2\mu gL\left(1+\frac{m}{M}\right)}$$

Note that in the limit when $\frac{m}{M}\to 0$, the rectangular block doesn't move ($a_M\to 0$) and by the work-energy theorem,

$$\frac{1}{2}mv^2=\mu mgL\to v=\sqrt{2\mu gL}$$

Hence we know $v\to\sqrt{2\mu gL}$ as $\frac{m}{M}\to 0$.

The displacement of the block is

$$d=\frac{1}{2}a_M t^2=\frac{1}{2}\frac{m}{M}\frac{v^2}{\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)^2=\frac{m}{2M\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)^2\times 2\mu gL\left(1+\frac{m}{M}\right)$$

$$d=\frac{L\frac{m}{M}}{1+\frac{m}{M}}=\left(\frac{m}{m+M}\right)L$$

(Method 2) When the cube stops at the right end relative to the rectangle, we have $v_m=v_M$. Conservation of momentum gives

$$mv=mv_m+Mv_M\Rightarrow v_M=\frac{m}{m+M}v$$

The displacement of the rectangular block is

$$v_M^2-0^2=2a_M d\Rightarrow d=\left(\frac{m}{m+M}\right)^2\frac{v^2}{2\frac{\mu mg}{M}}=\frac{v^2}{2\mu g}\left(\frac{mM}{(m+M)^2}\right)$$

$$\Rightarrow d=L\left(1+\frac{m}{M}\right)\left(\frac{mM}{(m+M)^2}\right)=\frac{m}{m+M}L$$

12. Train stations A and B are separated by a distance of 100 km. At 12 pm train T_A starts from station A and runs at a speed of 50 km per hour along a railway towards station B. The railway between A and B is a straight line. At the same time train T_B starts from B and runs at the same speed along the same railway towards station A. When T_A starts, a bird flies forward from T_A at a speed of 70 km per hour. When the bird meets T_B, it turns around and flies towards T_A with the same speed as it flew before. When the bird meets T_A again, the time is

12. 火車站 A 和 B 相隔 100 公里。中午 12 時，火車 T_A 從 A 站開出，沿著鐵路向 B 站以每小時 50 公里的速率行駛。 A 和 B 之間的鐵路是直線。同一時間，列車 T_B 從 B 出發，沿同一鐵路以相同的速率向 A 站行駛。當 T_A 出發時，一隻小鳥以每小時 70 公里的速率從 T_A 向前飛。當小鳥與 T_B 相遇時，它轉身並以牠以前飛過的速率飛回 T_A。當小鳥回到 T_A 時，時間是

A. 12:52 pm B. 12:54 pm C. 12:56 pm
D. 12:58 pm E. 1:00 pm

The displacement of T_A, T_B and the bird are respectively given by

$$y = \frac{50}{60}t,$$

$$y = 100 - \frac{50}{60}t.$$

The units of y and t are km and minutes respectively. When the bird flies forward,

$$y = \frac{70}{60}t.$$

When the bird meets T_B,

$$\frac{70}{60}t = 100 - \frac{50}{60}t,$$

$$t = 50 \text{ and } y = \frac{175}{3}.$$

When the bird flies backward,

$$y = \frac{175}{3} - \frac{70}{60}(t - 50).$$

When the bird meets T_A again,

$$\frac{50}{60}t = \frac{175}{3} - \frac{70}{60}(t - 50),$$

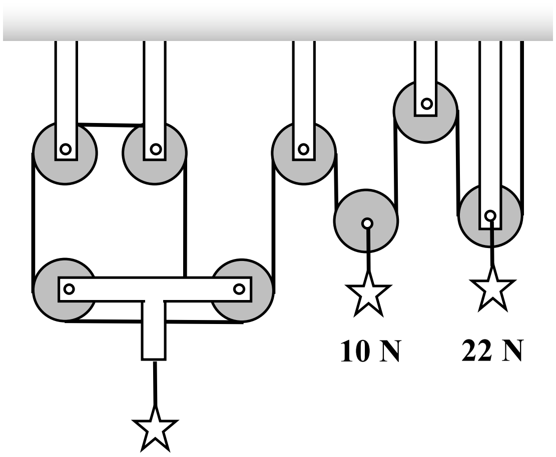
$$t = \frac{175}{3} = 58.3.$$

Answer: D.

13. Three star medals hang from the (stationary) assembly of massless pulleys and cords as shown in the figure. One long cord runs from the ceiling at the right to the T-shaped structure at the left, looping around all the pulleys. The weights of two medals are given. What is the weight of the medal on the left?

13. 如圖所示，三個星形獎章懸掛在無質量滑輪和繩索的（固定）組件上。一根長繩從右邊的天花板延伸到左邊的 T 形結構，環繞著所有滑輪。現給出其中兩枚獎章的重量。左邊獎章的重量是多少？

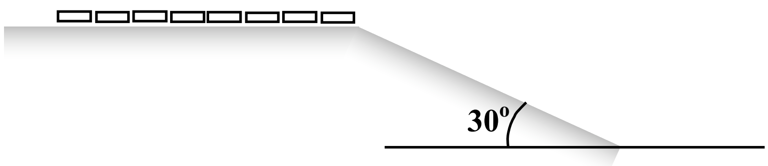
A. 5 N B. 15 N C. 24 N D. 36 N E. 48 N



Let T = tension of the cord.
Consider forces acting on the medal at the center. $2T = 10\text{ N} \Rightarrow T = 5$ N.
Consider forces acting on the medal at the left. $W = 3T = 15$ N.
Answer: B.

14. As shown in the figure, a train with a length of 100 m rests on a horizontal rail, and the front carriage of the train rests at the edge of a rail on a slope inclined at an angle of 30°. When the front carriage is tipped from rest onto the slope by a very small force, the train rolls freely down. What is the speed of the train when half of the carriages have rolled down the slope?
14. 如圖所示，長度為 100 米的列車停在水平軌道上，列車的前車廂位於傾斜角度為 30°的斜坡上的軌道邊緣。當前車廂被非常小的力從靜止推下斜坡時，火車自由地向下滑動。當一半車廂滾下斜坡時，列車的速度是多少？

A. 5.5 ms^{-1} B. 7.8 ms^{-1} C. 11 ms^{-1} D. 16 ms^{-1} E. 22 ms^{-1}

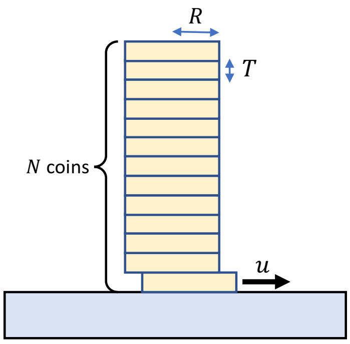


Let x = length of the train that has rolled down the slope
Potential energy of the train: $U = \frac{Mx}{L} g \left(-\frac{x}{2}\sin\theta\right) = -\frac{Mgx^2}{2L}\sin\theta$
Kinetic energy of the train: $K = \frac{1}{2}Mv^2$
Conservation of energy: $\frac{1}{2}Mv^2 - \frac{Mgx^2}{2L}\sin\theta = 0 \Rightarrow v = x\sqrt{\frac{g}{L}\sin\theta} = 50\sqrt{\frac{9.8}{100}\sin 30^{\circ}} = 11\ \text{ms}^{-1}$

Answer: C.

15. A pile of N one-dollar coins rest on a table. The mass, radius and thickness of each coin are M, R and T respectively. The coefficient of kinetic friction between the coin surfaces is μ. An external agent strikes the coin at the bottom such that it is removed from the pile with horizontal speed u. The removal speed u is so high that the pile of coins effectively remains vertical after the removal. What is the horizontal displacement of the pile of coins after the strike?

15. 一疊N個硬幣靜止在桌上。每個硬幣的質量、半徑和厚度分別為M, R 和 T。硬幣表面之間的動摩擦係數是μ。一人從底部撞擊硬幣，使得最底之硬幣以水平速度u從硬幣堆中飛出。飛出的速度非常高，以致硬幣飛出後堆疊的硬幣仍然有效地保持垂直。請問撞擊後這一疊硬幣的水平位移是多少？



A. $\frac{2R\sqrt{2gT}}{(N-1)\mu u}$ B. $\frac{2(N-1)R\sqrt{2gT}}{N\mu u}$ C. $\frac{2\mu R\sqrt{2gT}}{u}$ D. $\frac{2(N-1)\mu R\sqrt{2gT}}{u}$ E. $\frac{2N\mu R\sqrt{2gT}}{u}$

Time of contact between the bottom coin and the pile of coins during the strike $= 2R/u$.

Impulse acting on the pile of coins during the strike $= \mu(N-1)Mg2R/u$.

Using the impulse-momentum theorem, horizontal velocity of the pile $= \frac{\mu(N-1)Mg2R}{(N-1)Mu} = \frac{\mu g2R}{u}$.

After the strike, the pile falls down a height T.

Time of fall: $T = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2T}{g}}$.

Horizontal displacement of the pile: $= \frac{\mu g2R}{u}\sqrt{\frac{2T}{g}} = \frac{2\mu R\sqrt{2gT}}{u}$.

Answer: C.

16. A pendulum is formed by a particle of mass $m = 0.1$ kg and a massless string of length $l = 0.5$ m hung from a fixed point in the ceiling. The pendulum is swung from rest at an initial angle of $\theta_0 = 30^o$ with the vertical. Calculate the tension of the string when the angle of the string with the vertical is $\theta = 15^o$.

16. 一物塊從天花板的固定點以無質量的弦線懸掛，組成單擺。物塊的質量為$m = 0.1$ kg，弦線的長度為 $l = 0.5$ m。單擺從靜止狀態開始擺動，與垂直方向的初始夾角為 $\theta_0 = 30^o$。當弦線與垂直方向的夾角為$\theta = 15^\circ$時，求弦線的張力。

A. 0.10 N B. 0.20 N C. 0.22 N D. 1.14 N E. 1.24 N

Using the conversation of energy,

$$-mgl\cos\theta_0 = -mgl\cos\theta + \frac{1}{2}mv^2.$$

$$v^2 = 2gl(\cos\theta - \cos\theta_0).$$

Using Newton's second law,

$$T - mg\cos\theta = \frac{mv^2}{l} = 2mg(\cos\theta - \cos\theta_0).$$
$$T = mg(3\cos\theta - 2\cos\theta_0)$$
$$= (0.1)(9.8)(3\cos 15^o - 2\cos 30^o) = 1.1\text{ N}.$$

Answer: D.

17. A bullet of mass 0.01 kg is fired towards a rest block of wood with mass 10 kg on a smooth table. What is the maximum fraction of the initial kinetic energy of the bullet that can be turned into heat when it hits the rest mass?

17. 一質量為0.01 kg的子彈射向一塊放在光滑桌面上質量為10 kg 的靜止木塊。求子彈的初始動能在撞擊靜止質量後轉變為熱量的最大分數比是多少？

A. 1 B. $\frac{1000}{1001}$ C. $\frac{999}{1000}$

D. $\frac{4000}{1002001}$ E. Not enough information 資料不足

Ans:

Conservation of momentum reads

$$m_1 v_1 = m_1 v_1' + m_2 v_2'$$

The energy being turned to heat is therefore

$$Q = \frac{1}{2}m_1(v_1^2 - v_1'^2) - \frac{1}{2}m_2 v_2'^2 = \frac{1}{2}m_1(v_1^2 - v_1'^2) - \frac{1}{2}\frac{m_1^2}{m_2}(v_1 - v_1')^2$$
$$= \frac{m_1}{2m_2}\left((m_2 - m_1)v_1^2 + 2m_1 v_1 v_1' - (m_1 + m_2)v_1'^2\right)$$

It is a quadratic function with maximum occurring at

$$\frac{dQ}{dv_1'} = 2(m_1 v_1 - (m_1 + m_2)v_1') = 0$$
$$v_1' = \frac{m_1 v_1}{m_1 + m_2} = v_2'$$

Therefore, $v_{1,after} = v_{2,after}$ is the situation when absorbed heat is maximum.

$$m_1 v_1 = m_1 v + m_2 v$$
$$v = \frac{m_1 v_1}{m_1 + m_2}$$

Fraction of converted energy = $1 - \frac{m_1 v^2 + m_2 v^2}{m_1 v_1^2} = 1 - \frac{m_1 + m_2}{m_1}\left(\frac{m_1}{m_1 + m_2}\right)^2 = \frac{m_2}{m_1 + m_2} = \frac{10}{10.01}$

18. The orbiting period of Moon is T_M (27 days), orbiting at a height of $(r_M - R_E)$ above the Earth of radius R_E. A geostationary satellite is a satellite that always stays above the same location on the Equator. It stays at a height of $(r_S - R_E)$ above the Earth with an orbiting period of T_S (1 day). What is the ratio r_M/r_S in terms of T_M and T_S?

18. 月球的軌道周期是T_M（27 天），從地面高度$(r_M - R_E)$圍繞半徑為R_E的地球運行。地球同步衛星一直停留在赤道上的同一位置，位於地面高度$(r_S - R_E)$，軌道周期為T_S（1 天）。求r_M/r_S的比，以T_M和T_S來表示。

A. $(T_M/T_S)^{1/2}$ B. T_M/T_S C. T_S/T_M D. $(T_M/T_S)^{2/3}$ E. None of the above

Ans:
Centripetal force = Gravitational force:

$$\frac{GMm}{r^2} = m\frac{4\pi^2}{T^2}r \Rightarrow r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$$

For the moon, we have

$$r_M = \left(\frac{GMT_M^2}{4\pi^2}\right)^{1/3}$$

For the satellite, we have

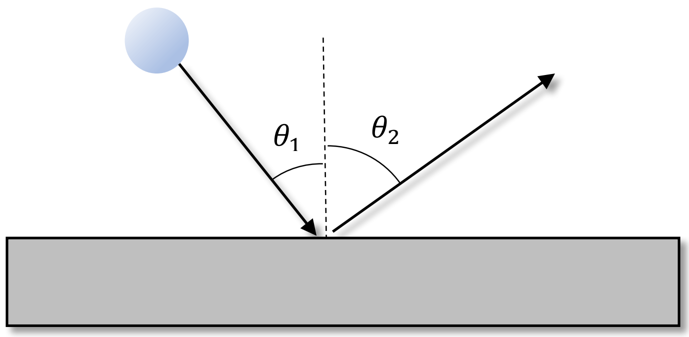
$$r_S = \left(\frac{GMT_S^2}{4\pi^2}\right)^{1/3}$$

Therefore, we have

$$\frac{r_M}{r_S} = \left(\frac{GMT_M^2}{4\pi^2}\right)^{1/3} / \left(\frac{GMT_S^2}{4\pi^2}\right)^{1/3} = \left(\frac{T_M}{T_S}\right)^{2/3}$$

19. A ball, with mass 1 kg, l is bouncing off a frictionless rectangular block at incoming angle $\theta_1 = 30°$ and exit angle $\theta_2 = 60°$ measured from the normal direction of the block, as shown in the figure. The rectangular block can also freely move. What is the mass of the block? Elastic collision is assumed.

19. 質量為 1 kg 的球從一個無摩擦的矩形塊以入射角$\theta_1 = 30°$射入，並以出射角$\theta_2 = 60°$反彈。所有角度均由矩形塊的法線方向測量，見附圖。矩形塊可以自由移動，而且碰撞是彈性的。求矩形塊的質量。



A. 1kg B. 2kg C. 3kg D. 5kg E. Infinite mass

Ans:
Let v_1 and v_2 be the speed of the ball before and after collision, v_w as the downward vertical speed of the wall. As there is no friction to change the horizontal momentum of either the ball or the wall, we have

$$v_1 \sin\theta_1 = v_2 \sin\theta_2 \Rightarrow v_2 = v_1\frac{1}{\sqrt{3}}$$

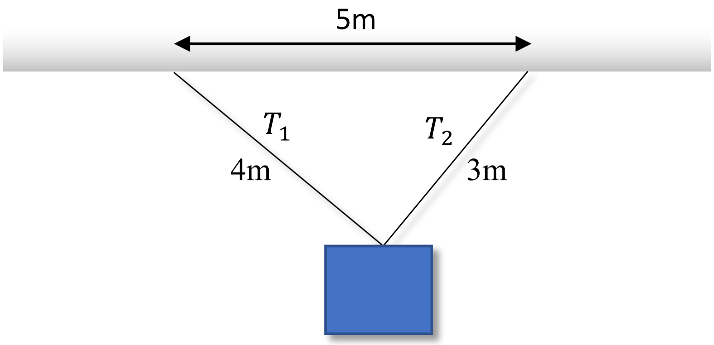
The conservation of vertical momentum gives

$$mv_1\cos\theta_1 = Mv_w - mv_2\cos\theta_2 \Rightarrow m\frac{v_1\sqrt{3}}{2} = Mv_w - m\frac{1}{2}v_1\frac{1}{\sqrt{3}} \Rightarrow \frac{v_1}{v_w} = \frac{\sqrt{3}}{2}\frac{M}{m}$$

The conservation of energy due to elastic collision gives

$$mv_1^2 = mv_2^2 + Mv_w^2$$
$$\Rightarrow mv_1^2\left(1 - \frac{1}{3}\right) = Mv_w^2$$
$$\Rightarrow \frac{M}{m} = \frac{2v_1^2}{3v_w^2} = \frac{2}{3}\frac{3}{4}\frac{M^2}{m^2} = \frac{1}{2}\frac{M^2}{m^2} = 2$$

20. A mass of 5 kg is hung by two strings attached to the same position of the mass, one with 4 m long and tension T_1 and another with 3 m long and tension T_2, which are connected to the ceiling at two locations with 5 m separation. What are the tensions T_1 and T_2 of the two strings?
20. 質量為 5 kg 的物塊由兩根繩子從物塊的同一位置懸掛著，兩根繩子分別連接到天花板兩個位置上，間隔為5 m。繩子 1 長4 m，繩子 2 長3 m。求兩根繩子的張力T_1和T_2。



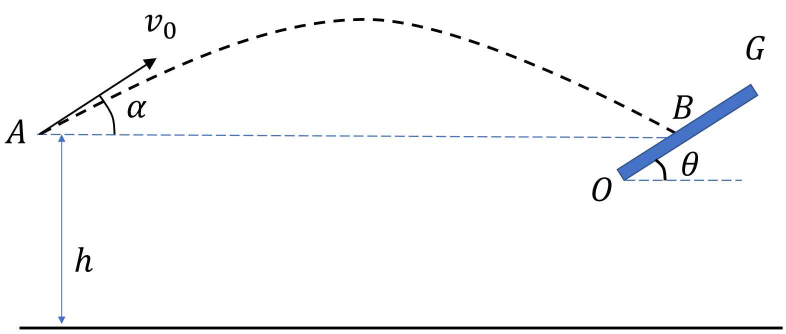
A. $T_1 = 4\times9.8N$, $T_2 = 3\times9.8N$
B. $T_1 = 5\times9.8\ N$, $T_2 = 3\times9.8N$
C. $T_1 = 3\times9.8N$, $T_2 = 4\times9.8N$
D. $T_1 = 4\times9.8N$, $T_2 = 5\times9.8N$
E. $T_1 = 5\times9.8N$, $T_2 = 4\times9.8N$

Ans:
We note that it forms a right-angle triangle. Then,
The horizontal forces balance by
$$T_1 4/5 = T_2 3/5$$
while the vertical forces balance by
$$T_1 3/5 + T_2 4/5 = (5\text{kg})\times g$$

giving $T_1 = (3\text{kg})\times g$ and $T_2 = (4\text{kg})\times g$

Open Problems (15 Marks Each) 開放題(每題 15 分)

1. From a fixed point A at a height h above the ground, a ball X is thrown at a speed v_0 with launching angle $\alpha\left(0<\alpha<\frac{\pi}{2}\right)$. A very massive plate OG is placed in front of point A and the ball hits at the point B on the plate OG which is above the ground at the same level as the point A.
從離地面的高度為 h 的固定點 A 將球 X 以速度 v_0 拋出，拋射角為 $\alpha\left(0<\alpha<\frac{\pi}{2}\right)$。在 A 點前方適當位置放一質量非常大的平板 OG，讓球 X 撞擊平板OG上的B點，並使碰撞點 B 與 A 點等高。

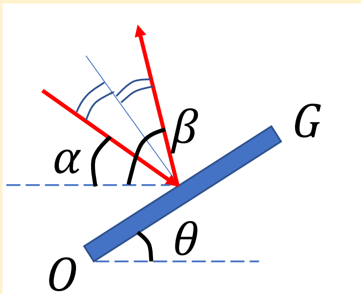


a) What is the horizontal distance AB? [3 pt]
a) AB 的水平距離是多少？

Solution:
The time t to hit on the plate is:
$0=v_0\sin\alpha\, t-\frac{1}{2}gt^2 \Rightarrow t=0$ (neglected) or $\frac{2v_0}{g}\sin\alpha$ [2 pt]
The horizontal distance AB is:
$AB=v_0\cos\alpha\, t=\frac{2v_0^2}{g}\sin\alpha\cos\alpha=\frac{v_0^2}{g}\sin 2\alpha$ [1 pt]

b) If the collision between the ball X and the plate is perfectly elastic. What is the values of the inclination angle $\theta\left(0<\theta<\frac{\pi}{2}\right)$ of the plate such that the ball X will return to the initial position A after the collision? [6 pt]
b) 如果球 X 與平板做完全彈性碰撞。求平板的傾角 $\theta\left(0<\theta<\frac{\pi}{2}\right)$，使球 X 在碰撞後恰好能回到 A 點。

Solution:



Assuming the launching angle at point B after the collision is β. In order to return to the initial position A, we have

$$AB = \frac{v_0^2}{g}\sin 2\alpha = \frac{v_0^2}{g}\sin 2\beta$$

$$\Rightarrow \beta = \alpha \quad \text{or} \quad \beta = 90^\circ - \alpha$$ [2+2 pt]

So we have two possible answers.

Case 1:
If $\beta = \alpha$, ball X hits on the plate along the normal direction, we have
$\theta + \alpha = 90^\circ \Rightarrow \theta = 90^\circ - \alpha$ [1 pt]

Case 2:
If $\beta = 90^\circ - \alpha$, we have

$$\frac{\beta - \alpha}{2} = 90 - \theta - \alpha$$

$\Rightarrow \theta = 45^\circ$ [1 pt]

c) There is another small ball Y. At the instant when the ball X is thrown from point A, it falls from rest freely from point A and makes a perfectly elastic collision with the ground. What are the values of h if the ball Y will return to the initial position A with ball X at the same instant? [6 pt]
c) 另有一小球 Y，在球 X 自 A 點拋出的同時，它從 A 點自由落下，並與地面完全彈性碰撞。如果球 Y 與球 X 在同一時間返回初始位置 A，求所有h之值。

Solution:
We need to consider two cases separately.

Case 1: If $\theta = 90^\circ - \alpha$, the trajectory of the ball X will return to point A along the same trajectory. The time required is

$$T = 2t = \frac{4v_0}{g}\sin\alpha$$

For ball Y, it will hit the ground at time t':

$$-h = -\frac{1}{2}gt'^2 \Rightarrow t' = \sqrt{\frac{2h}{g}}$$

Since the collision is elastic, it will return to point A at
$T = 2t' = 2\sqrt{\frac{2h}{g}}$ [1 pt]
Hence, they will arrive at the same time if
$2\sqrt{\frac{2h}{g}} = \frac{4v_0}{g}\sin\alpha \Rightarrow \frac{2h}{g} = \left(\frac{2v_0}{g}\sin\alpha\right)^2$ [1 pt]
$\Rightarrow h = \frac{2v_0^2}{g}\sin^2\alpha$ [1 pt]

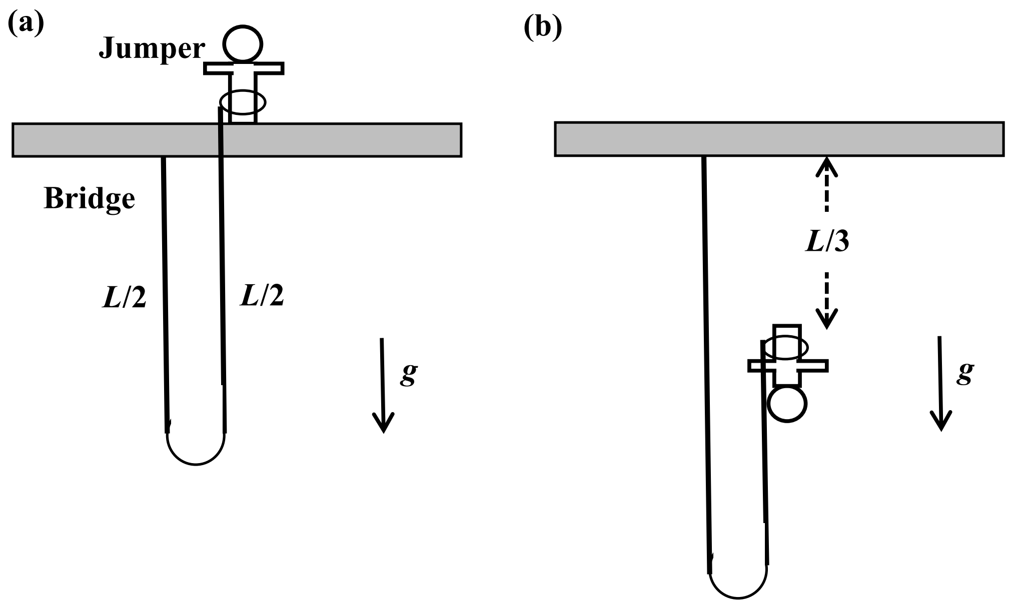
Case 2: if $\theta = 45^\circ$,
The time of ball X to return to the initial position A is
$T = \frac{2v_0}{g}\sin\alpha + \frac{2v_0}{g}\sin\beta = \frac{2v_0}{g}(\sin\alpha + \cos\alpha) = 2\sqrt{2}\frac{v_0}{g}\sin(\alpha + 45^\circ)$ [1 pt]

We have

$2\sqrt{\frac{2h}{g}} = 2\sqrt{2}\frac{v_0}{g}\sin(\alpha + 45°)$ [1 pt]

$\Rightarrow h = \frac{v_0^2}{g}\sin^2(\alpha + 45°)$ [1 pt]

2. Bungee jump is a sport for adventurous people. As shown in Fig. 1(a), a bungee jumper of mass m = 60 kg is initially standing on a bridge with a rope of mass M = 120 kg, length L = 30 m and of uniform density. One end of the rope is fastened to the bridge and the other end is tied to the jumper's body, and the rope is hanging freely below the bridge, with its midpoint furthest down. [15 pt]
笨豬跳是一項適合喜歡冒險的人的運動。如圖 1（a）所示，質量m = 60 kg 的笨豬跳高手最初站在橋上，繩索的質量M = 120 kg，長度L = 30 m，密度均勻。繩索的一端固定在橋上，另一端繫在高手身上，繩索在橋下自由懸掛，中點在最下方。



In the following calculations, you may assume that friction and air resistance are negligible, and the gravitational acceleration is g = 9.8 m. You may further assume that the rope has no stretching until the entire rope becomes hanging downward.
在下面的計算中，摩擦和空氣阻力可忽略不計，重力加速度為g = 9.8 m。您可以進一步假設繩索在整個繩索向下懸垂之前沒有伸展。

(a)-(b) Consider the instant that the jumper has fallen a distance $L/3$,
(a)-(b) 考慮高手下降至距離L / 3的瞬間，

(a) What is the change in the potential energy of the rope compared with the initial state?
(a) 與初始狀態相比，繩索的勢能變化是什麼？[3 pt]

Initially, the center of mass of the rope is $L/4$ below the bridge. When the jumper has fallen a distance $L/3$, the center of mass of the rope is

$\left(\frac{2}{3}\right)\left(\frac{L}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{L}{3} + \frac{L}{6}\right) = \frac{7}{18}L$ [1 pt]

below the bridge.

Change in the potential energy of the rope $= Mg\left(-\frac{7L}{18}\right) - Mg\left(-\frac{L}{4}\right) = -\frac{5}{36}MgL$ [1 pt]

$= -\frac{5}{36}(120)(9.8)(30) = -4900\text{ J}$ [1 pt]

(b) Calculate the speed of the jumper when the jumper has fallen a distance $L/3$. [3 pt]
(b) 當高手下降至距離 L / 3 時，計算高手的速率。

Using the conservation of energy,

$-\frac{5}{36}MgL - mg\frac{L}{3} + \frac{1}{2}\left(\frac{M}{3}\right)v^2 + \frac{1}{2}mv^2 = 0,$ [1 pt]

$\left(\frac{M}{6} + \frac{m}{2}\right)v^2 = \left(\frac{5M}{36} + \frac{m}{3}\right)gL,$

$v^2 = \left(\frac{\frac{5M}{36}+\frac{m}{3}}{\frac{M}{6}+\frac{m}{2}}\right)gL = \left[\frac{\frac{(5)(120)}{36}+\frac{(60)}{3}}{\frac{120}{6}+\frac{60}{2}}\right](9.8)(30) = \left(\frac{11}{15}\right)(9.8)(30) = 215.6,$ [1 pt]

$v = 14.7\text{ ms}^{-1}.$ [1 pt]

(c) Calculate the speed of the jumper when the entire rope becomes hanging downward. [2 pt]
(c) 計算當整個繩索向下懸垂時高手的速率。

Using the conservation of energy,

$-Mg\frac{L}{4} = -Mg\frac{L}{2} - mgL + \frac{1}{2}mv^2,$ [1 pt]

$v^2 = \left(\frac{M}{4m} + 1\right)2gL = \left[\frac{120}{(4)(60)} + 1\right](2)(9.8)(30) = 882$

$v = 29.7\text{ ms}^{-1}$ [1 pt]

(d) Jumpers claim that they enjoy the thrill of reaching a speed faster than free fall. Calculate the speed of the jumper if the jumper falls from the bridge through the same distance as in part (c) during a free fall. [2 pt]
(d) 高手聲稱他們享受比自由落體更快的速度刺激感。假設高手從橋上以自由落體方式下降至與（c）部分相同的距離時，計算高手的速率。

Using the conservation of energy,

$0 = -mgL + \frac{1}{2}mv^2,$ [1 pt]

$v^2 = 2gL = (2)(9.8)(30) = 588$

$v = 24.2\text{ ms}^{-1}$ [1 pt]

(e)-(f) After the entire rope becomes hanging downward, the jumper remains tied to the rope but continues to fall further downward due to the elastic nature of the rope. Let the force constant of the rope be $k = 500\text{ Nm}^{-1}$.
(e)-(f) 在整個繩索向下懸垂後，高手仍然繫在繩索上，但由於繩索的彈性，他會繼續向下墜落。假設繩索的力常數為 $k = 500\text{ Nm}^{-1}$。

(e) Calculate the maximum distance the jumper falls as measured from the bridge. [3 pt]

(e) 計算高手從橋上下降的最大距離。

When the jumper reaches the maximum distance, the speed is zero.
Let x = extension of the rope.
Using the conservation of energy,

$$-Mg\frac{L}{4} = -Mg\frac{L+x}{2} - mg(L+x) + \frac{1}{2}kx^2,$$ [1 pt]

$$\left(\frac{M}{4}+m\right)gL = -\left(\frac{M}{2}+m\right)gx + \frac{1}{2}kx^2,$$

$$kx^2 - (M+2m)gx - \left(\frac{M}{2}+2m\right)gL = 0,$$

$$500x^2 - [120+(2)(60)](9.8)x - \left[\frac{120}{2}+(2)(60)\right](9.8)(30) = 0,$$

$$500x^2 - 2352x - 52920 = 0,$$ [1 pt]

$$x = 12.9 \text{ m.}$$

The maximum distance the jumper falls is 30 + 12.9 = 42.9 m. [1 pt]

(f) After the motion of the jumper stops, what is the distance of the jumper below the bridge? [2 pt]
(f) 當高手的動態停止後，高手在橋下的距離是多少？

Using Hooke's law,

$$(M+m)g = kx,$$

$$x = \frac{(M+m)g}{k} = \frac{(120+60)(9.8)}{500} = 3.53 \text{ m.}$$ [1 pt]

Hence the distance of the jumper below the bridge is 30 + 3.53 m = 33.5 m. [1 pt]

Remark: The above calculation is not exact. In fact, the tension in the rope is mg at the lower end and increases linearly to $(M + m)g$ at the upper end. Hence the extension is given by

$$\left(\frac{M}{2}+m\right)g = kx,$$

$$x = \frac{(M/2+m)g}{k} = \frac{(120/2+60)(9.8)}{500} = 2.35 \text{ m.}$$

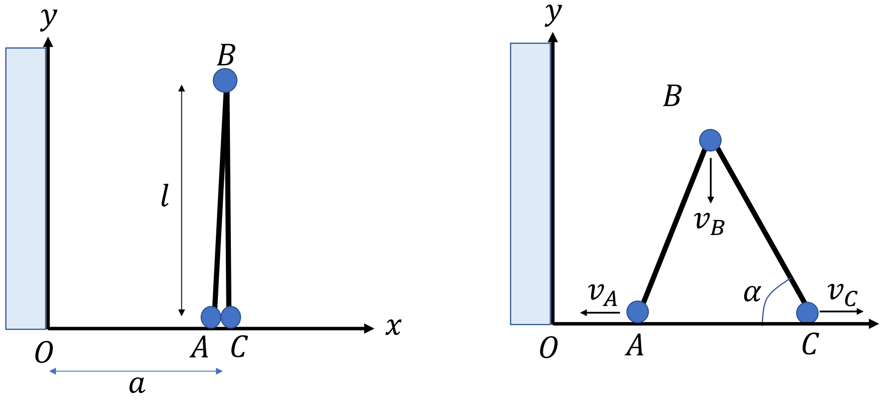
Hence the distance of the jumper below the bridge is 30 + 2.35 m = 32.4 m.

For the purpose of this competition, full marks will be given to both answers. Half marks will be given to the answer excluding M.

3. Three identical steel balls A, B, and C are connected by two light, hard rods AB and BC, each of length l. A and C lies on a horizontal surface along the x axis, and B is supported by the rods to stand in a vertical direction parallel to the y axis. As shown in Figure I, there is a vertical wall at distance $a = \frac{1}{\sqrt{2}}l$ from the rod. Due to small disturbance, two rods slide to two sides and ball B

falls (Figure II). In this problem, all friction is negligible, and the diameter of each ball is much smaller than l.

3. 三個相同的鋼球 A，B，C 由兩根輕質的長為 l 的硬杆 AB 和 BC 連接，A 和 C 處於水平面的黎波里 x 軸上，B 則由 AB 和 BC 兩杆支持著，豎立在垂直面上，與 y 軸平行，如圖 I 所示。距杆 $a = \frac{1}{\sqrt{2}} l$ 處有一面豎直牆，因受微小擾動，兩杆分別向兩邊滑動，使 B 球下降(圖 II)。在這問題中，所有摩擦可忽略不計，各球直徑都比 l 小很多。



(I) (II)

(a) What is the horizontal distance of the center of mass (CM) of the system from the origin O when $\angle ACB = \alpha$? [2 pts]

(a) 當 $\angle ACB = \alpha$ 時，求系統質心（CM）與原點 O 的水平距離是多少？

Since there is no external horizontal forces act on the system, the horizontal position of the center of mass doesn't change during the motion.
The horizontal distance from the origin O is a. [2 pts]

(b) What is the ratio of the speed v_A/v_C of two balls A and C when $\angle ACB = \alpha$? [2 pts]

(b) 當 $\angle ACB = \alpha$ 時，求球 A 和 C 的速率比例 v_A/v_C。

Because of the symmetry of the system, balls A and C will move with the same speed.
$\frac{v_A}{v_C} = 1$ [2 pts]

(c) What is the angle α when ball A hits the wall? [3 pts]

(c) 當球 A 撞到牆壁時的角度 α 是多少？

When ball A hits the wall, the horizontal position of the CM is
$x_{CM} = \frac{m\times 0 + ml\cos\alpha + m2l\cos\alpha}{3m} = a$ [1 pts]
$\Rightarrow l\cos\alpha = a$
$\Rightarrow \cos\alpha = \frac{1}{\sqrt{2}}$
$\Rightarrow \alpha = 45°$ [2 pts]

(d) What is the ratio v_B/v_C immediately before this instant? [5 pts]

(d) 求在這瞬間前 v_B/v_C 的比例。

The two ends of an incompressible rod should move with the same tangential velocity. Hence, we have

$v_{BC} = v_C \cos\alpha = \frac{v_C}{\sqrt{2}}$ [1 pts]

$v_{BA} = v_A \cos\alpha = \frac{v_A}{\sqrt{2}}$ [1 pts]

Since $\angle ABC = 90°$,

$v_B = \sqrt{v_{BC}^2 + v_{BA}^2} = \frac{\sqrt{v_A^2+v_C^2}}{\sqrt{2}} = \sqrt{\frac{2}{2}} v_C = v_C$ [2 pts]

$\Rightarrow \frac{v_B}{v_C} = 1$ [1 pts]

(e) What is the velocities of three balls at this moment? [3 pts]

(e) 在這瞬間前三球的速度分別是多少？

By the conservation of energy,

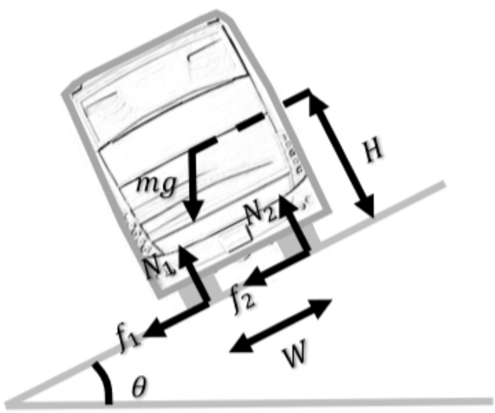
$mg(l - l\sin\alpha) = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2 = \frac{3}{2}mv_C^2$ [1 pts]

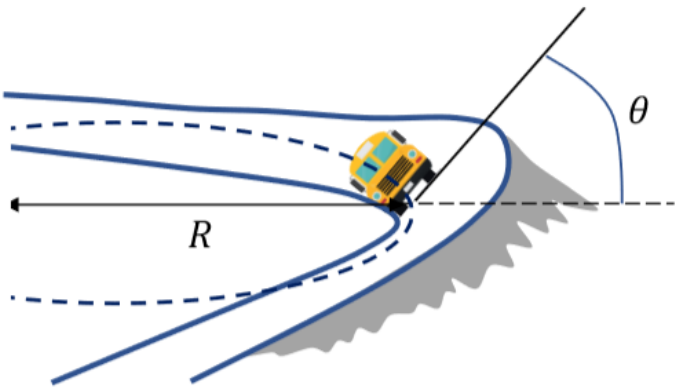
$\Rightarrow gl\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{3}{2}v_C^2$ [1 pts]

$\Rightarrow v_A = v_B = v_C = \sqrt{\frac{2}{3}\left(1 - \frac{1}{\sqrt{2}}\right)gl}$ [1 pts]

4. A bus is going around a banked turn, as shown in the figure. The horizontal radius of the turn is $R = 60$ m and the inclination angle of the bank is $\theta = 30°$. The figure also shows the cross-section of the bus, having a width of $W = 3$ m between the inner and outer wheels and a center of mass (CM) at a height of $H = 2.5$ m. We also label several forces: N_1 and N_2 are the normal forces acting on the inner and outer wheels while f_1 and f_2 are the friction action on the inner and outer wheels. mg is the weight of the bus.

4. 如圖所示，一輛巴士正在轉彎處轉彎。轉彎處的水平半徑是$R = 60$ m，傾斜角度是 $\theta = 30°$ 。圖中還畫出了一架巴士的橫截面，其內輪和外輪之間的寬度為 $W = 3$ m ，質心的高度是 $H = 2.5$ m。我們還標記了幾種力：N_1 和 N_2 是內輪和外輪上的法向力，f_1 和 f_2是內輪和外輪上的摩擦力。mg 是巴士的重量。





a) Suppose the static coefficient of friction of the bank is μ =0.5, which is a bit slippery in a rainy day. Determine the valid range of speed v for the bus so that its center-of-mass motion remains in circular shape. For simplicity, we can start by considering the total friction as $f = f_1 + f_2$, the total normal force as $N = N_1 + N_2$ and neglect the internal distribution between f_1 and f_2, i.e. $|f/N| \leq \mu$. Note that the radius of turn is measured from center of the turn to the CM. [8 pts]

a) 假設在一個下雨天，路面比較濕滑，轉彎處的靜摩擦係數為$\mu = 0.5$。求巴士的速率 v 的有效範圍，使巴士的質心運動維持圓形。為簡單起見，我們可以考慮總摩擦力為$f = f_1 + f_2$，總法向力為$N = N_1 + N_2$，忽略摩擦力 f_1 和 f_2的內部分佈，即 $|f/N| \leq \mu$。注意，轉彎處的水平半徑是指從轉彎中心到質心位置的距離。

Answer:

Let

$$f = f_1 + f_2, \qquad N = N_1 + N_2$$

Resultant force in components along and perpendicular to surface:

$\frac{mv^2}{R}\cos\theta = f + mg\sin\theta$ [1 pt]

$\frac{mv^2}{R}\sin\theta = N - mg\cos\theta$ [1 pt]

It can be rewritten as

$f = mg\sqrt{\frac{v^4}{g^2R^2} + 1}\sin\left(\tan^{-1}\frac{v^2}{gR} - \theta\right)$ [1 pt]

$N = mg\sqrt{\frac{v^4}{g^2R^2} + 1}\cos\left(\tan^{-1}\frac{v^2}{gR} - \theta\right)$ [1 pt]

and therefore we have

$\tan^{-1}\frac{f}{N} = \tan^{-1}\frac{v^2}{gR} - \theta$ [1 pt]

If the bus remains in circular motion, we require

$\left|\tan^{-1}\frac{f}{N}\right| = \left|\tan^{-1}\frac{v^2}{gR} - \theta\right| \leq \tan^{-1}\mu$ [1 pt]

$0.06 = \theta - \tan^{-1}\mu \leq \tan^{-1}\frac{v^2}{gR} \leq \theta + \tan^{-1}\mu = 0.987$ [1 pt]

$0.06 \leq \frac{v^2}{gR} \leq 1.515$

Specifically,

$$\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta} < \frac{v^2}{gR} < \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$$

The valid range of the speed is

$5.94\, ms^{-1} \leq v \leq 29.84 ms^{-1}$ [1 pt]

That is between 21.38 km/h and 107.42km/h or between 13.29mph and 66.75 mph

b) By considering the moment of the bus about its CM, determine the valid range of the speed v for the bus so that the bus will not topple. Toppling means the bus overturns. [7 pts]

b) 通過考慮巴士相對於其質心 CM 的力矩，求巴士的有效速度範圍，以使巴士不會傾倒。傾倒意味著巴士翻車。

Answer:

Without toppling, the bus has zero moment:
$fH + N_1 W/2 = N_2 W/2$ [1 pt]

From part (a), for a shallow bank with $\theta < \tan^{-1}\frac{v^2}{gR}$, we have $f > 0$. Therefore $N_2 > N_1$, the bus starts to topple (towards the outer side) when v is larger than the v at condition $N_1 = 0$: [1 pt]

$\tan^{-1}\frac{W}{2H} = \tan^{-1}\frac{f}{N} = \tan^{-1}\frac{v^2}{gR} - \theta$ [1 pt]

For a steeper bank with $\theta > \tan^{-1}\frac{v^2}{gR}$, we have $f < 0$ and $N_2 < N_1$. The bus starts to topple (towards the inner side) when v is smaller than the v at condition $N_2 = 0$: [1 pt]

$-\tan^{-1}\frac{W}{2H} = \tan^{-1}\frac{f}{N} = \tan^{-1}\frac{v^2}{gR} - \theta$ [1 pt]

Combining the two cases give the condition the bus to topple when

$\tan^{-1}\frac{W}{2H} < \left|\tan^{-1}\frac{v^2}{gR} - \theta\right|$ [1 pt]

The valid range of speed v that the bus will not topple is

$$-0.017 = \theta - \tan^{-1}\frac{W}{2H} \leq \tan^{-1}\frac{v^2}{gR} \leq \theta + \tan^{-1}\frac{W}{2H} = 1.064$$

$v \leq 32.55\ \text{ms}^{-1}$ [1 pt]

Additional information:

We recall in part (a) that the bus deviates from circular motion (R starts to become larger) when

$$\tan^{-1}\mu < \left|\tan^{-1}\frac{v^2}{gR} - \theta\right|$$

Therefore, if $\mu < W/(2H)$, the bus will not topple. In fact, this condition inherits from the case without circular motion. In the current case, the bus slides before toppling.

END OF PAPER（全卷完）