Hong Kong Physics Olympiad 2021 – Paper 1
2021 年香港物理奧林匹克競賽 – 卷一

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12 September, 2021
2021 年 9 月 12 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English, but only ONE language should be used throughout the whole paper.

   所有題目均為中英對照。你可選擇以中文或英文作答，惟全卷必須以單一語言作答。

2. On the first page of the answer books, please write your 3-digit Contestant Number and English Name.

   在答題簿的第一頁上，請填上你的 3 位數字參賽者號碼及英文姓名。

3. The open-ended problems are quite long. Please read the whole problem first before attempting to solve them. If there are parts that you cannot solve, you are allowed to treat the answer as a known answer to solve the other parts.

   開放式問答題較長，請將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來做其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別注明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.8 m s^{-2}
Gravitational constant 重力常數	G	6.67×10^{-11} $m^3kg^{-1}s^{-2}$

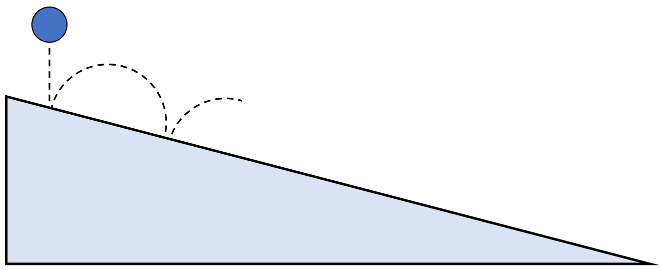
Multiple Choice Questions (2 Marks Each) 選擇題(每題 2 分)

1. A mass m hangs from a massless spring connected to the roof of a lift. When the lift is stationary, the mass-spring system oscillates vertically with angular frequency ω. If the lift is moving downward with a constant velocity, how will the angular frequency change?
質量 m 的物塊懸掛在連接到電梯頂部的無質量彈簧上。當升降機靜止時，物塊以角頻率 ω 垂直振動。如果電梯以恆定速度向下移動，角頻率將如何變化？

A. ω will be unchanged. ω 將不變。
B. ω will increase. ω 將增加。
C. ω will decrease. ω 將減少。
D. Oscillations are impossible under these conditions. 在這些條件下不可能發生振盪。
E. ω will increase or decrease depending on the values of m and the spring constant of the massless spring. ω 將根據 m和無質量彈簧的彈簧常數的值而增加或減少。

Answer: a
Newton's 1st law.

2. A ball is released from rest above an inclined plane and bounces elastically down the plane. As the ball progresses down the plane, the time and the distance between each collision will
一個球體在斜面上方從靜止狀態釋放，並沿斜面彈性彈跳。隨著球沿斜面向下移動，每次碰撞之間的時間和距離將



	The time 時間	The distance 距離
A.	Unchanged 不變	Increase 增加
B.	Increase 增加	Unchanged 不變
C.	Decrease 減少	Increase 增加
D.	Decrease 減少	Unchanged 不變
E.	Unchanged 不變	Unchanged 不變

Answer: A
Work in the reference frame with x-axis along the plane and y-axis perpendicular to the plane. Then in the y-direction, the ball simply bounces regularly up and down, so the collisions are uniformly spaced in time. However, gravity provides an acceleration in the x-direction, so the distance increases.

3. How many of the following statements about an action-reaction force pair is/are true?
I. The sum of the impulses due to the two forces is always zero.
II. The sum of the torques due to the two forces is always zero.
III. The sum of the torques due to the two forces depends on the choice of reference point with respect to which torques are measured.
IV. The sum of the works done by the two force is always zero.

下列關於一對作用-反作用力的陳述，其中多少項為正確？
I. 兩力之衝量和永為零
II. 兩力產生之力矩之和永為零
III. 兩力產生之力矩之和依賴於所選取之計算力矩之參考點
IV. 兩力所作之功總和永為零

A. 0
B. 1
C. 2
D. 3
E. 4

Solution: C
I is true because $\vec{F}_{12}\Delta t + \vec{F}_{21}\Delta t = (\vec{F}_{12} + \vec{F}_{21})\Delta t = \vec{0}$.
II is true because by strong form they have same line of action and hence

$$\vec{r}_1 \times \vec{F}_{12} + \vec{r}_2 \times \vec{F}_{21} = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_{12} = (\vec{r}_2 - \vec{r}_1) \times \vec{F}_{21} = \vec{0}$$

III is not true because when the total force is zero, the torque is reference-point independent:

$$\sum_i \vec{r}_i \times \vec{F}_i = \vec{d} \times \sum_i \vec{F}_i + \sum_i \vec{r}_i \times \vec{F}_i = \sum_i (\vec{r}_i + \vec{d}) \times \vec{F}_i$$

IV is not true because in general the two displacements of the points of action of the two forces are different and hence

$$\vec{F}_{12} \cdot \Delta\vec{r}_1 + \vec{F}_{21} \cdot \Delta\vec{r}_2 = \vec{F}_{12} \cdot (\Delta\vec{r}_1 - \Delta\vec{r}_2) = \vec{F}_{21} \cdot (\Delta\vec{r}_2 - \Delta\vec{r}_1) \neq 0$$

in general.

4. A geosynchronous satellite is an artificial satellite in circular orbit around the Earth with the same period as the self-rotation period of the Earth. A geostationary satellite is an artificial satellite that appears stationary in the sky when observed by a ground observer. Which of the following statements is/are correct?
地球同步衛星為軌道週期與地球自轉週期相同之人造衛星。地球靜止軌道衛星為從地面觀察者角度固定靜止於天空中之人造衛星。下列哪個陳述為正確？

I. All geosynchronous satellites are geostationary
II. All geostationary satellites are geosynchronous
III. It is possible to have a geostationary satellite at the zenith (directly overhead in the sky) of the local sky of Hong Kong.
I. 所有地球同步衛星皆為地球靜止軌道衛星
II. 所有地球靜止軌道衛星皆為地球同步衛星
III. 一地球靜止軌道衛星能固定位於香港天空之天頂(正上方之天空位置)

A. I only
B. II only
C. III only

D. I and II only
E. I, II, and III

Solution: B
The only geostationary orbit is the geosynchronous orbit directly above the equator. Hence I is wrong and II is correct. III is wrong because this orbit does not centered at the Earth's center.

5. A circular ring and a circular disk have the same mass and radius. Consider the following three motions:
   I. The ring rotating about an axis passing through the center of the ring and perpendicular to the plane on which the ring lies
   II. The ring rotating about its diameter
   III. The disk rotating about its diameter

   The three rotational speeds are the same. Rank the kinetic energies due to the rotation, from smallest to largest.

   一圓環與一圓盤之質量與半徑均相同。考慮以下運動:
   I. 環繞通過其中心並垂直於環所在平面之軸轉動
   II. 環繞其直徑轉動
   III. 盤繞其直徑轉動

   三者之轉動速率相同。將各動能從小到大排序。

   A. I, II, III
   B. I, III, II
   C. II, III, I
   D. III, I, II
   E. III, II, I

Solution: E
Let the angular speed be ω and the radius be R. KE of I > KE of II because in I the whole ring has speed ωR while in II some mass elements have speeds lower than ωR due to the smaller distances from the rotational axis. KE of II > KE of III. Compare a small arc in II subtending a small angle at the center and a small sector in III subtending the same angle. Some mass elements in the sector have speeds slower than the maximum speed at the rim, which is also the speed of the arc.

6. An object is launched vertically upwards somewhere on the Earth's surface not at the north and south pole. Regarding the minimum initial vertical velocity the object needs to have so that it can escape the Earth's gravity, and taking the Earth's self-rotation into account, how does it compare to the escape velocity at the surface of the Earth?
   在地球上除南北極外某處將一物體垂直向上射出。為使其能逃離地球之引力，物體之初始速度必須不低於某最小值。關於物體逃脫地球引力所需的最小初始垂直速度，並考慮到地球的自轉，它與地球表面的逃逸速度相比如何？

   A. It is slightly smaller than the escape velocity. 它略小於逃逸速度
   B. It is equal to the escape velocity. 它等於逃逸速度
   C. It is slightly larger than the escape velocity. 它略大於逃逸速度
   D. Which of A, B, C being correct depends on the longitude of the location. A、B、C 中何者正確取決於位置之經度。

E. Which of A, B, C being correct depends on the latitude of the location. A、B、C 中何者正確取決於位置之緯度。

Solution: A
Let the vertical velocity be v and the horizontal velocity due to the Earth's self-rotation be V. Then the speed of the object is

$$\sqrt{v^2 + V^2} \geq v_e$$

where v_e is the escape velocity of the Earth. The minimum vertical velocity is hence

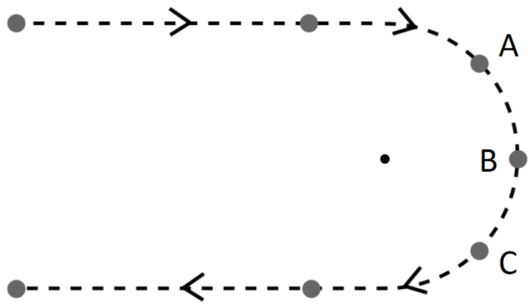
$$v_{min} = \sqrt{v_e^2 - V^2}$$

It is given that the object is not at the north and south pole and therefore $V^2 > 0$. So

$$v_{min} < v_e$$

The above holds at all longitudes and latitudes except at the two poles.

7. A car makes a U-turn along a semi-circular curve as shown below. Moving from A to C, the car is slowing down with a constant rate. What is the direction of the net force acting on the car when the car is at point B?
汽車沿著半圓形曲線掉頭，如下圖所示。從 A 到 C，汽車以恆定的比率減速。當汽車在 B 點時，作用在汽車上的合力的方向是什麼？



A. ←
B. ↑
C. ↖
D. ↗
E. ↙

Solution: C
The net force should provide the centripetal acceleration toward the center of the curve and the deceleration backward.

8. An ice is floating on water in a cup. At the beginning, part of the ice submerges under the water surface. Finally the ice is completely melted. Ignoring evaporation and thermal expansion. Which statement below is true?
冰塊漂浮在杯子裡的水面上。在開始時，冰塊的一部分淹沒在水面之下。最後冰塊完全融化。忽略蒸發和熱膨脹。下面哪個說法是正確的？

A. The water level rises after the ice is melted. 冰塊融化後水位上升。
B. The water level drops after the ice is melted. 冰塊融化後水位下降。
C. The water level remains unchanged after the ice is melted. 冰塊融化後水位保持不變。

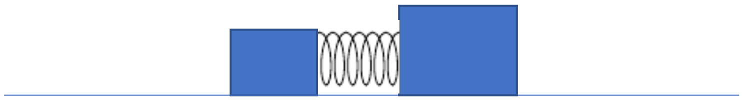
D. The ice could not melt completely as no heat is supplied. 由於沒有提供熱量，冰塊無法完全融化。
E. Further information is needed to determine what could happen. 需要更多信息來確定可能發生的情況。

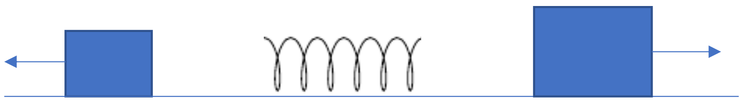
Solution: C
Consider the ice has mass m, as both ice and water. As ice, the buoyancy by the water is $\rho_w g V_i$ where V_i is the volume of the ice submerged below water surface. V_i is also the volume added to the water. Since this buoyancy equal the weight,

$$\rho_w g\, V_i = mg \rightarrow V_i = m/\rho_w$$

If we melt the ice into water, it will have a volume of $V_w = \frac{m}{\rho_w} = V_i$. Therefore, the volume occupying the water by the ice in either ice form or water form is the same.

9. A 1.0-kg block and a 2.0-kg block are pressed together on a horizontal frictionless surface with a compressed very light spring between them. They are not attached to the spring. After they are released and have both moved away from the spring
一個 1.0-kg 的物塊和一個 2.0-kg 的物塊被壓在一起放在一水平無摩擦的桁面上，它們之間有一個壓縮的非常輕的彈簧，但它們並沒有連接到彈簧上。當它們被釋放並且離開彈簧之後



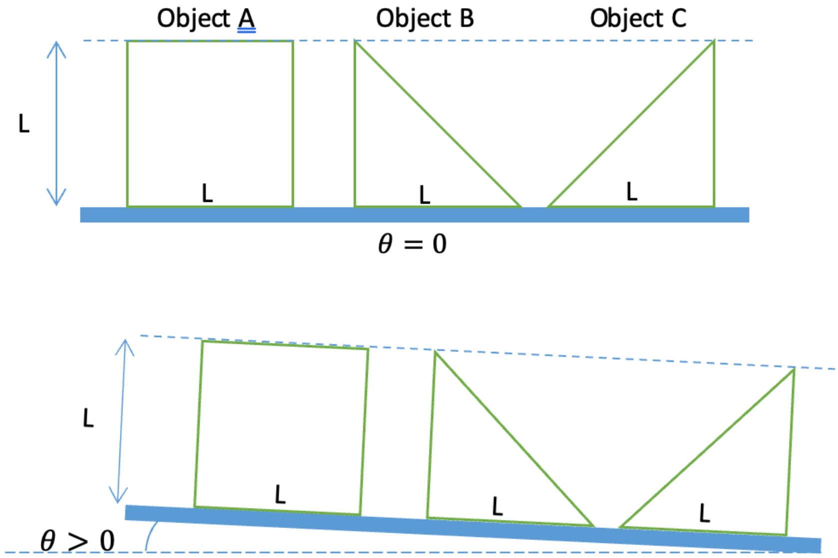


A. both blocks will have the same amount of kinetic energy. 兩個物塊將具有相同的動能。
B. both blocks will have equal speeds. 兩個物塊將具有相同的速率。
C. the lighter block will have more kinetic energy than the heavier block. 較輕的物塊比較重的物塊有更多的動能。
D. the magnitude of the momentum of the heavier block will be greater than the magnitude of the momentum of the lighter block. 較重的物塊的動量大小將大於較輕的物塊。
E. the heavier block will have more kinetic energy than the lighter block.. 較重的物塊比較輕的物塊有更多的動能。

Solution: C
Magnitudes of the momentum are the same in both block due to conservation of momentum. But energy is $E = \frac{1}{2}mv^2 = \frac{p^2}{m}$. Therefore, kinetic energy is inversely proportional to the mass of the block.

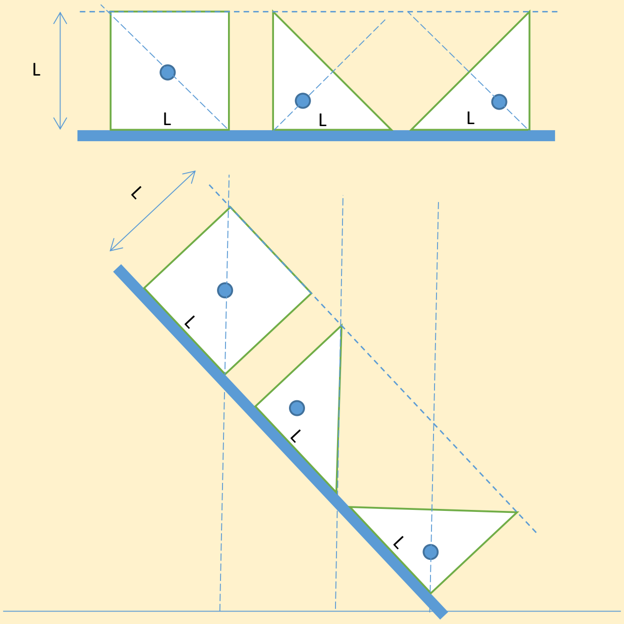
10. Three uniform objects are placed on a frictional plane so they will not slide. The plane is being tilted from horizontal. As the angle θ gradually increases from 0^o, determine which object(s) will start to tip over at the smallest angle and which object(s) will start to tip over at a largest angle.

三個均勻的物體放置在摩擦平面上，因此它們不會滑動。平面從水平方向開始傾斜。隨著角度 θ 從 0° 逐漸增加，找出哪些物體將以最小角度開始傾翻，哪些物體又將以最大角度開始傾翻。



	以最小角度滾動 Tip over at smallest angle	以最大 角度滾動 Tip over at largest angle
A.	Object C	Object B
B	Object C	Object A
C	Object B and C	Object A
D	Object A and C	Object B
E	Object A	Object C

Solution: D
The objects will tip over when it center of mass lies beyond the vertical line drawn from its edge. For symmetrical objects, the center of the mass (blue dot) must lies on the symmetric axis as shown in the diagram. When the table is tilted beyond 45 degree, the CM of object A and C move beyond their edges. Therefore, they will both tip over at the same angle, while object B will still be stationary.

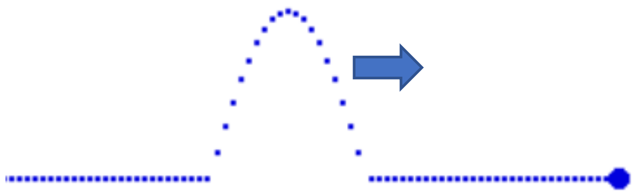


11. A particle sliding on a curved track (similar to surface of a bowl) is under the net force consisting of gravity, normal force and friction. The total mechanical energy is defined as the sum of the kinetic energy and the gravitational potential energy. Which of the following statement is incorrect?
一個粒子在彎曲的軌道（類似於碗的表面）上滑動，受到由重力、法向力和摩擦力組成的合力影響。總機械能定義為動能和重力勢能的總和。以下哪個說法是不正確的？

A. The change in mechanical energy equals work done by friction and gravity. 機械能的變化等於摩擦力和重力所做的功。
B. The change in mechanical energy equals work done by friction only. 機械能的變化僅等於摩擦力所做的功。
C. The change in kinetic energy equals work done by friction and gravity. 動能的變化等於摩擦和重力所做的功。
D. Normal force does not do any work. 法向力不做任何功。
E. Work done by friction must be negative. 摩擦力做的功一定是負的。

Solution: A
According to work energy theorem, change of kinetic energy equals to total work done on the system. In this system, the normal force is perpendicular to the motion; thus $W_{normal} = 0$ (D is correct).
The total work done on the system is only due to gravity and friction. (C is correct).
While friction is always opposite to the direction of motion, work done by friction is negative. (E is correct)
If one includes the potential energy and consider mechanical energy, the work-energy theorem becomes:
Change of total mechanical energy (E=KE+PE) = total work done by non-conservative forces
In this case, the change in mechanical energy equals work done by friction only as gravity is a conservative force. (A is incorrect and B is correct.)

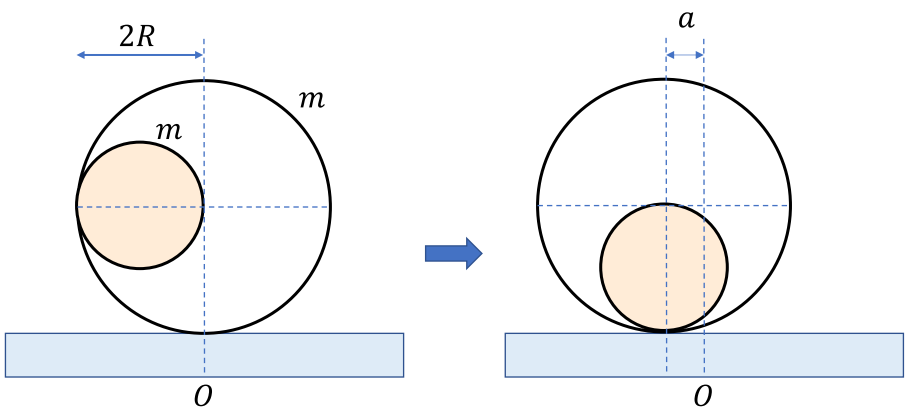
12. Consider a pulse with an upward displacement of a half-cosine pulse shape is travelling to the right on a string with a fixed end on the right (fixed on a wall with no displacement allowed). Which of the following in describing the pulse after it hits the fixed end is correct?
考慮一個半餘弦脈衝波形狀向上位移的脈沖在右端固定的弦上向右行進（固定在牆上，不允許位移）。 下列哪項描述脈衝波到達固定端后是正確的?



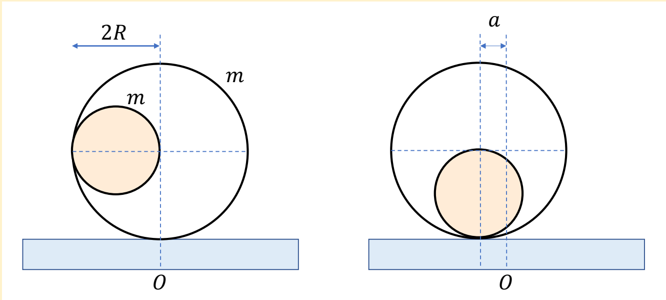
A. The pulse gets reflected with the pulse width halved. 脈衝波被反射，脈衝波的寬度減半。
B. The pulse gets reflected with upward displacement of the same shape. 脈衝波以相同形狀的向上位移被反射。
C. The pulse gets reflected with downward displacement of the same shape. 脈衝波以相同形狀的向下位移被反射。
D. The pulse is absorbed by the fixed end. 脈衝波被固定端吸收。
E. None of the above. 以上都不是。

Solution: C
First the pulse should get reflected as the fixed end cannot absorb the energy due to its zero displacement at the end. To have zero displacement at the fixed end, we use the so-called method of image. We can imagine a wave of the same shape with downward displacement is travelling to the left in a mirroring location about the fixed end as if there is a string beyond the fixed end. These two waves travel with opposite velocities and always create a zero displacement at the location of the fixed in the whole duration. Therefore, the correct answer is (c).

13. A ball of radius R and mass m is put inside a cylindrical shell of the same mass and radius $2R$. The system is at rest on a horizontal frictionless surface initially. When the ball is released inside the shell, it moves along the shell and eventually stops at the bottom of the shell. What is the horizontal displacement a does the shell move from its initial contact point with the surface?
一個半徑為 R、質量為 m 的球放在一個質量相同、半徑為 $2R$ 的薄圓柱殼內。系統最初靜止在水平無摩擦枱面上。當球在殼內釋放時，它沿著殼運動並最終停在殼的底部。 問圓柱殼從其與枱面的初始接觸點的水平位移 a 是多少？
A. R
B. $R/2$
C. $R/4$
D. $3R/8$
E. $R/8$



Solution: B



Since there are no external horizontal force acting on the system, the horizontal position of the center of mass x_{cm} is fixed.

Initially,

$$x_{cm} = \frac{m(-R)}{2m} = -\frac{R}{2}$$

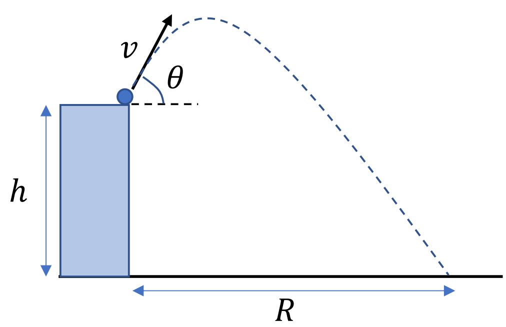
When the ball stops at the bottom of the shell

$$x_{cm} = \frac{m(-a) + m(-a)}{2m} = -a$$

$$\Rightarrow a = \frac{R}{2}$$

14. A projectile is launched with speed v off the edge of a cliff of height $h > 0$, at an angle θ from the horizontal. Air friction is negligible. To maximum the horizontal range R of the projectile, θ should satisfy

炮彈以速度 v 從高度為 $h > 0$ 的懸崖邊緣發射，與水平面成 θ 角。空氣摩擦可以忽略不計。為了使炮彈的水平射程 R 最大化，θ 應滿足



A. $45° < \theta < 90°$
B. $\theta = 45°$
C. $0° < \theta < 45°$
D. $\theta = 0°$
E. Insufficient information. Depending on the values of h and v.

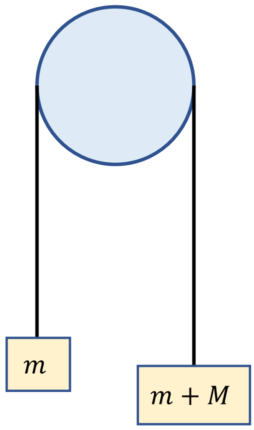
Answer: C
1. When $h \to 0$, we know $\theta \to 45°$ in order to have maximum horizontal range.

2. In the limit $h \to \infty$, the stone should be thrown horizontally ($\theta \to 0°$) in order to have maximum displacement.

3. For finite h, we expect $0 < \theta < 45°$.

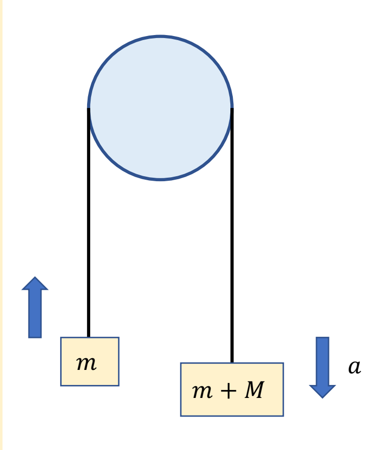
15. A massless rope passes over a frictionless pulley. Particles of mass m and $m + M$ are suspended from the two different ends of the rope and they move under the gravity. If $M = 0$, the tension T in the rope is mg. If the value of M increases to infinity, the value of the tension
一條無質量的繩索通過無摩擦的滑輪。質量為 m 和 $m + M$ 的物體懸掛在繩索的兩端，它們在重力的作用下移動。如果 $M = 0$，則繩索中的張力 T 為 mg。如果 M 的值增加到無窮大，則張力的值

A. Stays constant 維持不變
B. Decreases to a nonzero constant 減少為非零的常數
C. Decreases to zero 減少到零
D. Increases to a finite constant 增加到有限大的常數
E. Increases to infinity 增加到無窮大



Answer: D
Method 1:
Two masses will have the same acceleration.
As $m \to \infty$, we expect the free fall motion for the mass $m + M$. By inspecting the free body diagram of the mass M, we have
$$T - Mg = Mg \Rightarrow T = 2Mg$$

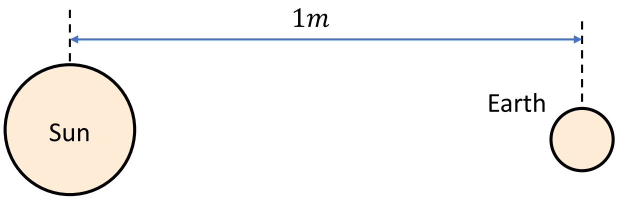
Method 2:



From Newton's 2nd law,

$$(M+m)g - T = (M+m)a$$
$$T - mg = ma$$
$$\Rightarrow T - mg = m\left(g - \frac{T}{M+m}\right)$$
$$\Rightarrow \frac{2m+M}{m+M}T = 2mg$$
$$\Rightarrow T = \left(\frac{m+M}{2m+M}\right)2mg \to 2mg \quad as\ M \to \infty$$

16. If the solar system scales down such that the average distance between the earth and the sun is $1m$, what is the period of the earth around the sun? Assuming that the density of the object does not change.
如果太陽系按比例縮小，當地球和太陽的平均距離為 $1m$ 的時候，地球圍繞太陽的周期為多少？假設物體密度不變。
A. 0.25 year
B. 0.5 year
C. 1 year
D. 2 years
E. 4 years



Solution: C
From the circular motion, we have

$$\frac{GmM}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = mr\frac{4\pi^2}{T^2}$$

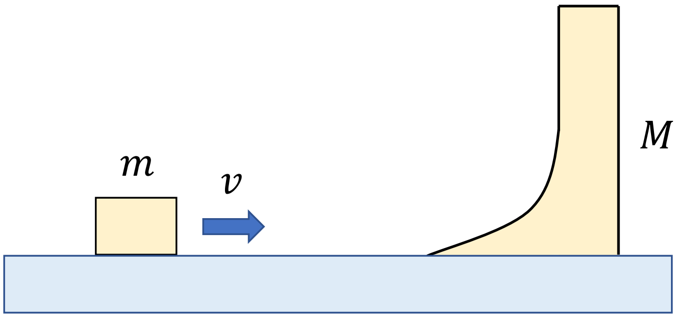
And the mass of the sun can be rewritten as (R is the radius of the sun)

$$M = \frac{4}{3}\pi R^3 \rho$$
$$\Rightarrow T = \sqrt{\frac{3\pi}{G\rho}\left(\frac{r}{R}\right)^3}$$

We can see that when both r and R scale down equally, the period will not change.
Answer is (c).

17. A block of mass m is launched horizontally onto a curved wedge of mass M at a velocity v. What is the maximum height reached by the block after it shoots off the vertical segment of the wedge? Assume all surfaces are frictionless; both the block and the curved wedge are free to move.
質量為 m 的物塊以速度 v 水平發射到質量為 M 的曲面體上。問物塊從曲面體的垂直部分射出後所達到的最大高度是多少？假設所有表面都是無摩擦的；物塊和曲面體都可以自由移動。

A. $\frac{v^2}{2g}$ B. $\left(\frac{m}{m+M}\right)^2\frac{v^2}{2g}$ C. $\left(\frac{M}{m+M}\right)^2\frac{v^2}{2g}$ D. $\frac{m}{m+M}\frac{v^2}{2g}$ E. $\frac{M}{m+M}\frac{v^2}{2g}$



Solution: E
(Method 1) we can first consider the (perfectly inelastic) collision between the block and the wedge.

$$mv = (m+M)V$$
$$\Rightarrow V = \frac{m}{m+M}v$$

During the inelastic collision, the missing energy is converted to the (relative) kinetic energy of the block m.

$$\Rightarrow mgh = \frac{1}{2}mv^2 - \frac{1}{2}(m+M)V^2 = \frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2}{m+M}v^2$$
$$\Rightarrow h = \frac{v^2}{2g}\left(\frac{M}{m+M}\right)$$

(Method 2)
We consider the moment before the collision and the moment when the mass m reaches its maximum height. Since there is no external horizontal force, the horizontal momentum ($p = mv$) must be conserved. The loss of the kinetic energy of the system,

$$\Delta E = \frac{p^2}{2m} - \frac{p^2}{2(M+m)} = \frac{Mmv^2}{2(M+m)}$$

Therefore, the energy conservation gives

$$\Delta E = mgh$$
$$\Rightarrow h = \frac{M}{M+m}\frac{v^2}{2g}$$

The answer is (e).

18. Two small balls, A and B, are attached to the two ends of a rigid rod with length 2. At a certain moment, it is observed that ball A is located at $(x_A,\ y_A)$ and moving with velocity $(v_{Ax},\ v_{Ay})$ while ball B is located at $(x_B,\ y_B)$ and moving with velocity $(v_{Bx},\ v_{By})$. Which of the following is/are impossible?
兩個小球，A 和 B ，連接到長度為 2 的剛性棒的兩端。 在某一時刻，觀察到球 A 位於 $(x_A,\ y_A)$ 並以速度 $(v_{Ax},\ v_{Ay})$ 移動；球 B 位於 $(x_B,\ y_B)$ 並以速度 $(v_{Bx},\ v_{By})$ 移動。以下哪項為不可能？

I. $(x_A,\ y_A) = (0,1), (x_B,\ y_B) = (0,-1), (v_{Ax},\ v_{Ay}) = (1,0), (v_{Bx},\ v_{By}) = (2,0)$
II. $(x_A,\ y_A) = (0,1), (x_B,\ y_B) = (0,-1), (v_{Ax},\ v_{Ay}) = (1,1), (v_{Bx},\ v_{By}) = (1,2)$
I. $(x_A,\ y_A) = \left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right), (x_B,\ y_B) = \left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right), (v_{Ax},\ v_{Ay}) = (2,0), (v_{Bx},\ v_{By}) = (1,1)$

A. I only
B. II only
C. III only
D. I and III
E. II and III

Solution: B
The position of A relative to B is

$$\vec{r}_{AB} = (x_A, y_A) - (x_B, y_B)$$

The velocity of A relative to B is

$$\vec{v}_{AB} = (v_{Ax}, v_{Ay}) - (v_{Bx}, v_{By})$$

Since the rod is rigid, $\vec{v}_{AB}$ cannot have radial component along $\vec{r}_{AB}$:

$$\vec{v}_{AB} \cdot \vec{r}_{AB} = 0$$

I.

$$\vec{r}_{AB} = (x_A, y_A) - (x_B, y_B) = (0,1) - (0,-1) = (0,2)$$

$$\vec{v}_{AB} = (v_{Ax}, v_{Ay}) - (v_{Bx}, v_{By}) = (1,0) - (2,0) = (-1,0)$$

$$\vec{v}_{AB} \cdot \vec{r}_{AB} = 0$$

II.

$$\vec{r}_{AB} = (x_A, y_A) - (x_B, y_B) = (0,1) - (0,-1) = (0,2)$$

$$\vec{v}_{AB} = (v_{Ax}, v_{Ay}) - (v_{Bx}, v_{By}) = (1,1) - (1,2) = (0,-1)$$

$$\vec{v}_{AB} \cdot \vec{r}_{AB} = -2 \neq 0$$

III.

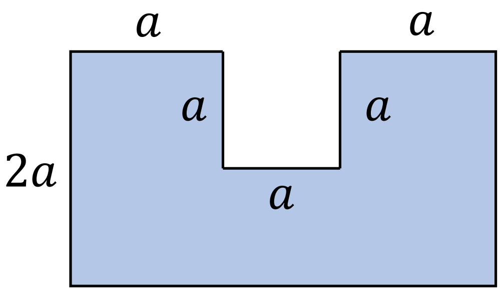
$$\vec{r}_{AB} = (x_A, y_A) - (x_B, y_B) = (1/\sqrt{2}, 1/\sqrt{2}) - (-1/\sqrt{2}, -1/\sqrt{2}) = (\sqrt{2}, \sqrt{2})$$

$$\vec{v}_{AB} = (v_{Ax}, v_{Ay}) - (v_{Bx}, v_{By}) = (2,0) - (1,1) = (1,-1)$$

$$\vec{v}_{AB} \cdot \vec{r}_{AB} = 0$$

19. Consider the following U-shaped figure with uniform mass density. Find the distance of the center of mass from the bottom.

考慮以下具有均勻質量密度之 U 形圖形。求質心和底部的距離。



A. 0.5 a
B. 0.6 a
C. 0.7 a
D. 0.8 a
E. 0.9 a

Solution: E
Consider the uniform U-shaped figure as a uniform rectangular figure less a uniform square, all with the same mass density.
The CM of the rectangle is obviously at distance a from the bottom. The CM of the square is at distance $1.5a$ from the bottom.
Let the mass of the square be m, then the mass of the rectangle is $6m$ and the mass of the U-shaped figure is $5m$.
Let the distance of the CM of the U-shaped figure from the bottom be h.
Then

$$a = \frac{5m \times h + m \times 1.5a}{5m + m}$$
$$a = \frac{5h + 1.5a}{6}$$
$$h = 0.9a$$

20. If gravitational force between two masses is given by $F_g = \frac{GMm}{r}$ instead of $F_g = \frac{GMm}{r^2}$, Kepler's third law will become: (where r is the radius and T is the period of a circular orbit)
如果兩個質量之間的引力由 $F_g = \frac{GMm}{r^2}$ 變為 $F_g = \frac{GMm}{r}$，則開普勒第三定律將變為：（其中 r 是半徑，T 是圓形軌道的周期）

A. $r \propto T$
B. $r \propto T^2$
C. $r^2 \propto T$
D. $r^3 \propto T^2$
E. $r^2 \propto T^3$

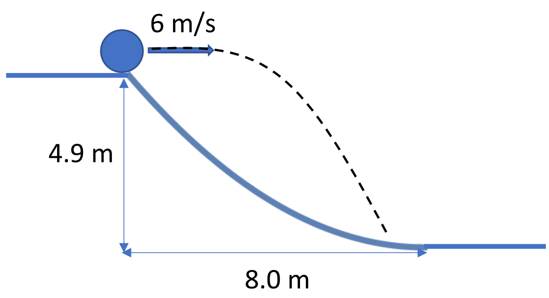
Solution: A
Newton's 2nd law gives:

$$\frac{GMm}{r} = \frac{mv^2}{r}$$
$$\Rightarrow GM = v^2$$

$$\Rightarrow \frac{2\pi r}{T} = v = \sqrt{GM} = \text{constant}$$

So $r \propto T$

21. A slope of parabolic in shape has a height of 4.9 m and a width of 8 m so that the slope goes back to zero just before connecting to a flat ground. Initially at the height of 4.9m, the ball is pushed to have a horizontal velocity of 6 m/s and with a zero vertical velocity. The ball is acted upon gravity. Find the time elapsed when the ball hits again the parabolic slope.
抛物線形的斜坡高 4.9m，寬 8m，以便在連接到平坦地面之前斜率回到零。 最初在 4.9m 的高度，球被推動以具有 6 m/s 的水平速度和零垂直速度。 球受重力作用。 求球再次擊中抛物線斜率所用的時間。



A. 1.92 s
B. 1.00 s
C. 0.96 s
D. 0.70 s
E. 0.48 s

Solution: C
Let the initial location of the ball is at (x,y) = (0, 4.9). SI unit of length and time is assumed and missed out in the following expression for brevity. The parabolic slope follows a curve of

$$y = 4.9\left(\frac{x}{8} - 1\right)^2$$

The ball follows trajectory:

$$x = 6t$$

$$y = 4.9 - \frac{1}{2}9.8\,t^2 = 4.9(1 - t^2)$$

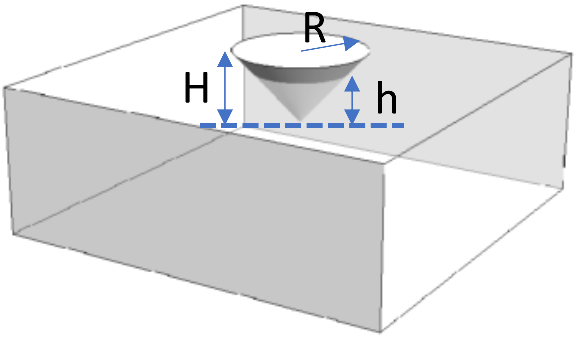
Combining the above equations, we obtain

$$4.9\left(\frac{6t}{8} - 1\right)^2 = 4.9(1 - t^2)$$

$$(3t - 4)^2 = 16 - 16t^2$$
$$t = 0.96s$$

22. Supposed a solid inverted cone, with base radius R, height H and density ρ_c, can be stably submerged in water with a larger density ρ_w for a depth of h inside water, being less than H, as the following figure shows. Express h in terms of the other defined variables, assuming no external force, except those by the gravity (standard gravity g) and the fluid, is applied on the solid.

假設一個底半徑為 R、高度為H、密度為 ρ_c 的實心倒錐體，可以穩定地浸沒在密度 ρ_w 較大的水中，深度為 h (倒錐體水中高度)，小於 H，如下圖所示。 用其他已定義的變量表達 h，假設除了重力（標準重力 g）和流體之外沒有外力施加在倒錐體上。



A. $\left(\frac{\rho_c}{\rho_w}\right)^{1/2} H$

B. $\left(\frac{\rho_w}{\rho_c}\right)^{1/2} H$

C. $\left(\frac{\rho_c}{\rho_w}\right)^{1/2} R$

D. $\left(\frac{\rho_c}{\rho_w}\right)^{1/3} R$

E. $\left(\frac{\rho_c}{\rho_w}\right)^{1/3} H$

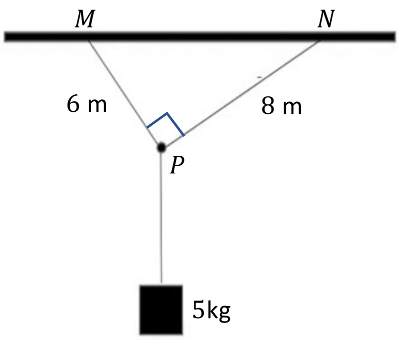
Solution: E

Let the radius of the cone at the water level be $r = Rh/H$.

The buoyance force is $\rho_w \frac{\pi r^2 h}{3} g = \rho_c \frac{\pi R^2 H}{3} g$. Therefore, we have

$$h = (\rho_c/\rho_w)^{\frac{1}{3}} H$$

23. Two ropes suspended from the roof at points M and N are joint at point P with a right angle. The length of the rope joining MP is 6 m and the length of rope joining NP is 8 m. A wooden block of mass 5 kg hangs from point P through a third rope. Find the tension on the rope joining NP.
兩根繩索在 M 點和 N 點懸掛在屋頂上，在 P 點連接成直角。 MP 的繩索連接長度為 6 m，NP 的繩索連接長度為 8 m。 質量為 5 kg 的木塊通過第三根繩子從 P 點懸掛下來。 求連接 NP 的繩索的張力。



A. 39.2 N
B. 29.4 N
C. 28.0 N
D. 24.5 N
E. 21.0 N

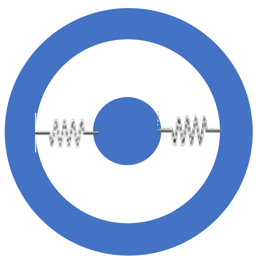
Solution: B
The T_1 be the tension on rope joining MP and T_2 be the tension on rope joining NP. Balancing the horizontal and vertical forces at point P gives

$$T_1(6/10) = T_2(8/10)$$
$$T_1(8/10) + T_2(6/10) = 5 \times 9.8$$

And therefore

$$T_1 = 39.2N, \qquad T_2 = 29.4N$$

24. Inside a rigid spherical shell of mass M = 10 kg, a rigid sphere of mass m = 2 kg is connected by two identical springs of spring constant k = 0.5 N/m, as shown in figure. What is the resonating frequency when you shake the structure sideway along the spring direction?
在質量為 M = 10 kg 的剛性球殼內，質量為 m = 2 kg 的剛性球體由兩個彈簧常數 k = 0.5 N/m 的相同彈簧連接 (如圖所示)。 沿彈簧方向橫向搖晃結構時的共振頻率是多少？



A. 0.707 Hz
B. 0.316 Hz
C. 0.113 Hz
D. 0.056 Hz
E. 0.046 Hz

Solution: C
The resonating frequency is about the resonance of internal motion of the sphere within the shell. It will be

$$f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2 \times 0.5}{2}} = 0.113\text{ Hz}$$

25. Hubble's law states that the galaxies are moving away from each other at distance r at a velocity given by the Hubble's law: $v = H\,r$ where H is called the Hubble's constant. By treating the universe as a giant sphere of homogeneous density and galaxies escaping away from the edge of universe, find out the critical density of the universe, ρ_c ,so that the velocity of the galaxies moving away from us. According to the Hubble's law, it is exactly the escape velocity on such a giant sphere.
哈勃定律指出，星系以哈勃定律給出的速度在距離 r 處相互遠離：$v = Hr$ 其中 H 稱為哈勃常數。 把宇宙看成一個密度均勻的巨大球體，星係從宇宙邊緣逃逸，求出宇宙的臨界密度ρ_c。根據哈勃定律，星系遠離我們的速度正好是在如此巨大的球體上的逃逸速度。

A. $H^2/(4\pi G)$
B. $3H^2/(4\pi G)$
C. $H^2/(8\pi G)$
D. $3H^2/(8\pi G)$
E. None of the above. 以上都不是。

Solution: D
The escape velocity of such a giant sphere of universe is given by

$$\sqrt{\frac{2GM}{r}} = \sqrt{\frac{2G}{r}\frac{4\pi}{3}r^3\rho_c} = Hr$$

Therefore,

$$\frac{2G}{r}\frac{4\pi}{3}r^3\rho_c = H^2r^2$$

$$\rho_c = \frac{3H^2}{8\pi G}$$

Hong Kong Physics Olympiad 2021 – Paper 2
2021 年香港物理奧林匹克競賽 – 卷二

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12 September, 2021

2021 年 9 月 12 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English, but only ONE language should be used throughout the whole paper.
   所有題目均為中英對照。你可選擇以中文或英文作答，惟全卷必須以單一語言作答。

2. On the first page of the answer books, please write your 3-digit Contestant Number and English Name.
   在答題簿的第一頁上，請填上你的 3 位數字參賽者號碼及英文姓名。

3. The open-ended problems are quite long. Please read the whole problem first before attempting to solve them. If there are parts that you cannot solve, you are allowed to treat the answer as a known answer to solve the other parts.
   開放式問答題較長，請將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來做其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別注明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.80 ms^{-2}
Gravitational constant 重力常數	G	6.67×10^{-11} $m^3kg^{-1}s^{-2}$

Open Problems 開放題

1. (15 points) A bullet with mass 10 g was fired against a stationary and fixed wooden block with mass 1kg. It has an initial velocity of 800 m/s and penetrated the block into depth 10 cm. Assume the wood resists the movement of the bullet steadily.
1. (15 分) 一顆質量為 10 g 的子彈射向一個質量為 1 kg 的靜止且固定的木塊。 它的初始速度為 800 m/s，並在進入木塊 10 cm 的深度停下來。 假設木頭穩定地抵抗子彈的運動。

(a) (4 points) What is the resistance force in the wooden block against the bullet?
(4 分) 木塊對子彈的阻力是多少？

Now the wooden block is movable freely on a table with negligible friction between the block and the table and the bullet is now fired horizontally to the wooden block and is finally stopped inside the block.
現在木塊可以在水平和平坦的桌子上自由移動，木塊和桌子之間的摩擦可以忽略不計，子彈現在沿水平方向射向木塊，最後停在木塊內。

(b) (3 points) What is the final speed of the wooden block with bullet?
(3 分) 裝有子彈的木塊的最終速度是多少？

(c) (4 points) What is the depth into which the bullet has penetrated when the block is movable?
(4 分) 當木塊可移動時，子彈穿透的深度是多少？

(d) (4 points) How long does the bullet take to be stopped after it has entered the block?
(4 分) 子彈進入方塊後需要多長時間停下來？

Solution:

(a) Let u_0 be the initial speed of the bullet, h_0 be the depth of penetration, m as the mass of bullet and M as the mass of the wooden block. We also let f as the resistance force. The conservation of energy is expressed as

$$\frac{1}{2}mu_0^2 = Fh_0 \Rightarrow F = \frac{1}{2h_0}mu_0^2 = \frac{1}{2\times 0.1}0.01\times 800^2 = 32000N$$

(b) Let the final velocity be v. By conservation of momentum, we have

$$mu_0 + 0 = (M+m)v \Rightarrow v = \frac{mu_0}{M+m} = \frac{0.01\times 800}{1+0.01} = 7.921\ ms^{-1}$$

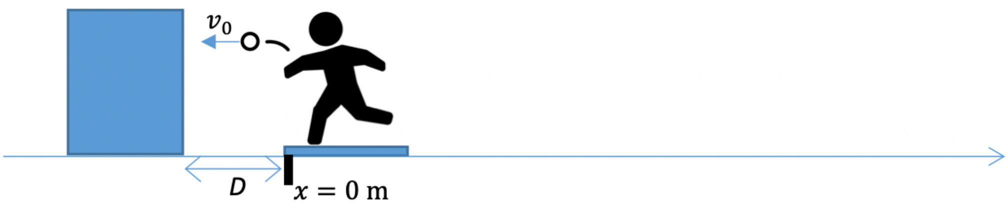
(c) Let the penetration distance be h. By conservation of energy, we have

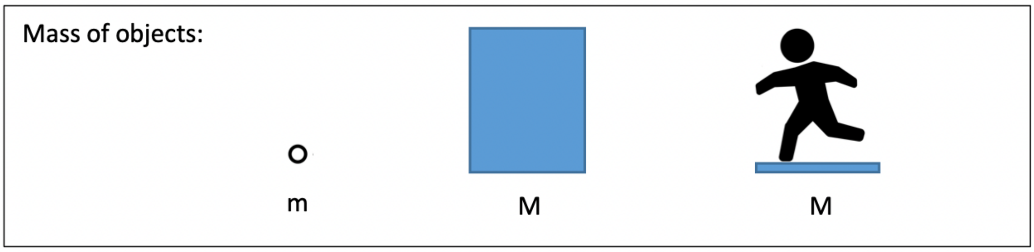
$$\begin{aligned}
&\frac{1}{2}mu_0^2 + 0 = Fh + \frac{1}{2}(M+m)v^2 \\
&\Rightarrow \frac{1}{2}mu_0^2 = Fh + \frac{m^2u_0^2}{2(M+m)} \\
&\frac{1}{2}mu_0^2\frac{M}{M+m} = \frac{1}{2h_0}mu_0^2 h \\
&h = \frac{M}{M+m}h_0 = \frac{1}{1+0.01}0.1 = 9.9\ cm
\end{aligned}$$

(d)

$$\frac{du}{dt} = -\frac{F}{m} \Rightarrow T = \frac{u_0 - v}{F/m} = \frac{Mu_0}{M+m}\frac{m}{F} = \frac{M}{M+m}\frac{2h_0}{u_0} = \frac{1}{1.01}\frac{2\times 0.1}{800} = 0.248\,ms$$

2. (15 points) A person is sitting on a frictionless board holding a ball. A block is placed separately and at rest in front of the person. The person wants to set himself moving by throwing the ball towards the block. However, in order to catch the ball again after the first throw, the ball must return back to the person after bouncing from the block.
2. (15 分) 一個人坐在一塊無摩擦的板上，手裡拿著一個球。一個箱子被靜止放置在人的面前。該人希望通過將球投向箱子來使自己移動。但是，為了在第一次投擲後再次接住球，球必須在從箱子反彈並回到該人。





Suppose the total mass of the person and the board system is M, the block has a mass M and the ball has mass m. Assume the collision between the ball and the block is elastic; and ignore all frictions and gravity.
假設人和無摩擦板的總質量為 M，箱子的質量為 M，球的質量為 m。假設球和箱子之間的碰撞是彈性的；並忽略所有摩擦和重力。

(a) (3 points) The person is initially at rest. If the ball is thrown at speed v_0 with respect to ground. What is the speed of the person and the board after his throwing?
(3 分) 人最初處於靜止狀態。如果球以相對於地面的速度 v_0 拋出。投擲後人與板的速度是多少？

(b) (3 points) Find the kinetic energy transferred to the block by the ball after the collision.
(3 分) 求球在碰撞後傳給箱子的動能。

(c) (3 points) Find the conditions, if any, for the ratio of the masses $r = m/M$ and for the initial throwing speed v_0 such that the person can catch the ball back after bouncing from the block.
(3 分) 找出質量比 $r = m/M$ 和初始投擲速度 v_0 的條件（如果有），以使人可以接住從箱子反彈回來的球。

(d) (3 points) If $r = \frac{m}{M} = 0.1$ and $D = 1\text{m}$, find the distance the person travelled between throwing the ball and catching it again.
(3 分) 如果 $r = \frac{m}{M} = 0.1$ 且 $D = 1\text{m}$，求出人在投球和再次接球之間的移動距離。

(e) (3 points) After catching the ball, the person wants to stop by throwing the ball away (this time he does not need to catch it back). Describe at which direction and at what speed he should throw the ball again . Answer with the speed of the ball with respect to the ground.
(3 分) 接球後，人希望利用再次用投球的方法來停止運動（這次他並不需要接回球）。請描述他應該向哪個方向和以什麼速度投球。請用球相對於地面的速度作答。

Solution:

(a) By conservation of momentum, $mv_0 = MV$. The speed of the person is $V = \frac{m}{M}v_0$.

(b) Let v_1 be the speed of the ball after the bouncing from the block
v_B be the speed of the block after the ball bouncing.
For elastic collision, we have

$$mv_0 = Mv_B - mv_1 \qquad \text{--- (1)}$$

$$\text{and } -v_0 = -(v_1 - (-v_B))$$

$$\Rightarrow v_B = v_0 - v_1 \qquad \text{--- (2)}$$

Combining the two we have:

$$v_1 = \frac{M-m}{M+m}v_0$$

$$v_B = \frac{2m}{M+m}v_0$$

The energy transferred to the block is

$$K_B = \frac{1}{2}Mv_B^2 = \frac{2Mm^2}{(M+m)^2}v_0^2$$

(c) To catch the ball again, the ball must have a faster speed then the person. $v_1 > V$
Therefore, $\frac{M-m}{M+m}v_0 > \frac{m}{M}v_0$

$$\Longrightarrow m^2 + 2Mm - M^2 < 0$$

$$\Longrightarrow r^2 + 2r - 1 < 0$$

$$r < \sqrt{2} - 1 \sim 0.4142$$

But there is no conditions on v_0 !!!

(d) Distance the person travels $= d = V *$ (Time between throwing and catching)

$$\text{Time between throwing and catching} = T = t_1 + t_2$$

$$t_1 = \textit{time the ball is moving to the block} = D/v_o$$

$$t_2 = \textit{time the ball is going back to the person} = (D+d)/v_1$$

We have

$$d = \frac{m}{M}v_0\left[\frac{D}{v_0} + \frac{D+d}{v_0}\frac{M+m}{M-m}\right]$$

Solving for d, $d(r) = 2D\frac{r}{1-2r-r^2}$

$$d(0.1) = 0.253\text{m}$$

(e) Catching the ball is a completely inelastic collision. After catching the ball, the person moves at V'

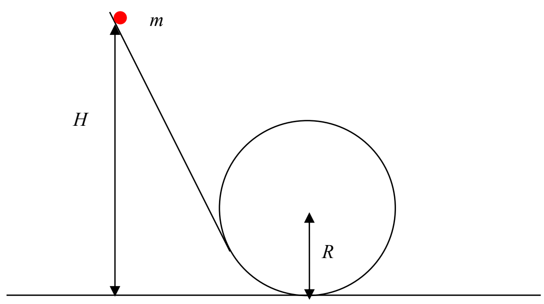
$$m\frac{M-m}{M+m}v_0 + mv_0 = (M+m)V'$$
$$V' = \frac{2Mm}{(M+m)^2}v_0$$

The total momentum of the system (person and ball) is now $\frac{2Mm}{(M+m)}v_0$. Therefore the person need to throw the ball towards right (+x) with a speed $v_f = \frac{2M}{(M+m)}v_0$.

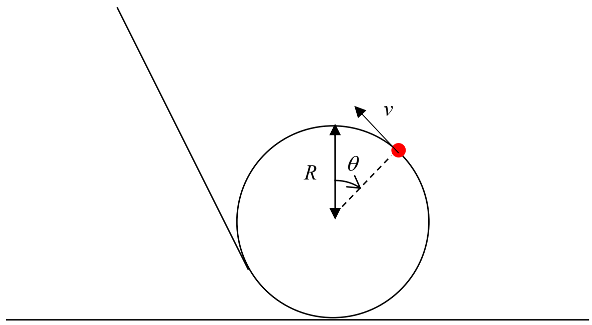
If $r = \frac{m}{M} = 0.1$, *we have*

$$v_f = \frac{2}{1.1}v_0.$$

3. (20 points) In this question, it is assumed that all frictional forces can be ignored.
A smooth track consists of two parts. The left part is straight and is connected to the right circular part at a point at which the tangent of the circle has the same slope as the straight track. The radius of the circular part is *R*. A point object with mass *m* is initially at rest on the left straight track at a height of *H* above ground, as shown in the figure below. The object falls under gravity to move along the track.
3. (20 分) 在此題中，假設所有摩擦力皆可以忽略。
一平滑的軌道由兩部分組成。左側部分是直的，並在某點接上右側的圓形部分。在接點直線軌道與圓形切線斜率相同。圓形部分半徑為 *R*。一個質量為 *m* 的質點最初靜止在離地面高度 *H* 處的左側直線軌道上，如下圖所示。物體在重力作用下下落並沿軌道移動。



(a) (5 points) Find the speed of the object *v* in terms of H, R, and θ when it reaches a point on the circular track with an angular position of θ measured from the center of the track, as shown in the figure below.
(5 分) 如下圖所示，當物體到達圓軌道上一個相對軌道中心角位置為 θ 的點時，求以 H、R 及 θ 表示之物體速率。



(b) (5 points) Find the normal reaction of the track at this moment in terms of $H, R, \theta,$ and m.

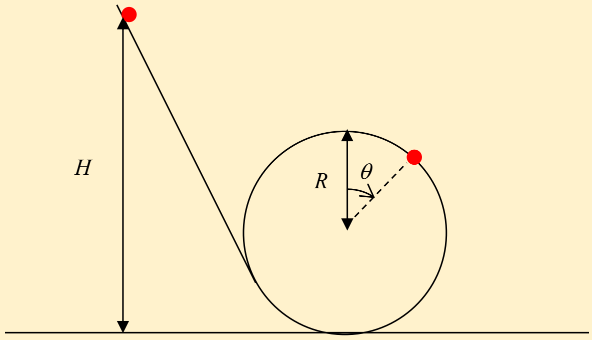
(5 分) 求此刻以 H、R、θ 及 m 表示之軌道正向支持力。

(c) (5 points) Find, in terms of R, the critical initial height H_c below which the object cannot complete the whole journey along the circular track.
(5 分) 若初始高度低於某臨界高度H_c，物體就不能完成整個沿圓形軌道之運動。求以 R表示之臨界初始高度 。

(d) (5 points) A point target is located at the center of the circular track. The object released at a certain initial height H will hit the target. Find H in terms of R.
(5 分) 一點靶物被置於圓形軌道中心。在某個初始高度 H 釋放的物體將擊中靶物。求以 R 表示之 H。

Solution:

(a) Measure the angular position of the object by the angle θ defined in the figure below.



By conservation of energy, the speed v is determined by

$$\frac{1}{2}mv^2 + mgR(1+\cos\theta) = mgH$$

$$v = \sqrt{2g\left[H - R(1+\cos\theta)\right]}$$

(b) Considering the centripetal force, we have

$$N + mg\cos\theta = \frac{mv^2}{R}$$

where N is the normal reaction of the track.

$$N = -mg\cos\theta + \frac{2mgH - 2mgR(1+\cos\theta)}{R} = \frac{2mgH}{R} - mg(3\cos\theta + 2)$$

(c) The object will leave the track when $N = 0$. One can see that this is possible only when $-90^\circ \le \theta \le 90^\circ$.

Hence the angular position of the point the object loses contact is determined by

$$\frac{2mgH}{R} - mg(3\cos\theta + 2) = 0 \Rightarrow \cos\theta = \frac{2}{3}\left(\frac{H}{R} - 1\right),$$

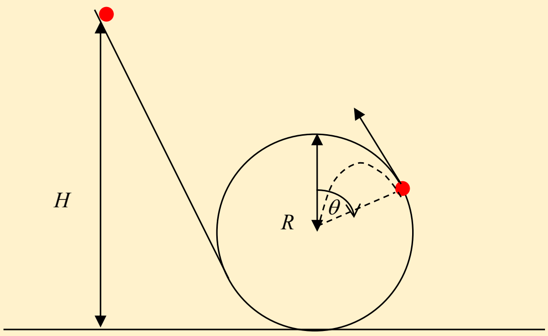
where $-90^\circ \le \theta \le 90^\circ \Rightarrow \cos\theta \ge 0$.

One can see that there will be no solution when

$$\frac{2}{3}\left(\frac{H}{R}-1\right)>1 \Rightarrow H>\frac{5}{2}R.$$

Hence $H_c = 5R/2$. If $H < H_c$, the object will lose contact at some point.

(d) After the object leaves the track, it will perform projectile motion. By geometry, the launching angle is θ.



Hence the trajectory is

$$y = R(1+\cos\theta) + x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}.$$

The coordinate of the center is $(R\sin\theta, R)$.
If the center is on the trajectory

$$R = R(1+\cos\theta) + R\sin\theta\tan\theta - \frac{gR^2\sin^2\theta}{2v^2\cos^2\theta}$$

$$R\cos\theta + R\frac{\sin^2\theta}{\cos\theta} - \frac{gR^2\sin^2\theta}{2v^2\cos^2\theta} = 0$$

$$\cos^2\theta + \sin^2\theta - \frac{gR\sin^2\theta}{2v^2\cos\theta} = 0$$

$$gR\sin^2\theta = 2v^2\cos\theta$$

$$gR\sin^2\theta = 4\left[gH - gR(1+\cos\theta)\right]\cos\theta$$

$$R\left(1-\cos^2\theta\right) = 4\left[H - R(1+\cos\theta)\right]\cos\theta$$

$$R\left\{1-\left[\frac{2}{3}\left(\frac{H}{R}-1\right)\right]^2\right\} = 4\left[H - R\left(1+\frac{2}{3}\left(\frac{H}{R}-1\right)\right)\right]\frac{2}{3}\left(\frac{H}{R}-1\right)$$

$$12(H-R)^2 = 9R^2$$

$$H - R = \pm\frac{\sqrt{3}}{2}R$$

$$H = \left(1\pm\frac{\sqrt{3}}{2}\right)R$$

The solution $H = \left(1-\frac{\sqrt{3}}{2}\right)R < R$ is rejected because in this case $90^\circ < \theta < 270^\circ$ and the object will never leave the track.

Hence the solution is $H = \left(1+\frac{\sqrt{3}}{2}\right)R < \frac{5}{2}R$.