Hong Kong Physics Olympiad Lesson 2

2.1 **Forces and Motions**

2.2 **A Block on a Wedge**

2.2 **Friction and Motions**

2.1 Forces and Motions

(1) Newton's first law

Consider a body on which no net force acts. If the body is at rest, it will remain at rest. If the body is moving with a constant velocity, it will continue to do so.

Inertial frame of reference: A non-accelerating frame of reference in which Newton's first law is valid.

(2) Newton's second law

The rate of change of momentum of a body is proportional to the resultant force and occurs in the direction of force. i.e. $\sum \boldsymbol{F} \propto m\frac{d\boldsymbol{v}}{dt} = m\boldsymbol{a}$. Now, one Newton is defined as the force which gives a mass of 1.0 kilogram an acceleration of 1.0 meter per second per second. Hence if m = 1.0 kg and a = 1.0 ms^{-2}, then F = 1.0 N gives the proportional constant equals one.

$$\boxed{\sum \boldsymbol{F} = m\boldsymbol{a} = m\frac{d\boldsymbol{v}}{dt}}$$

$\Sigma\, \boldsymbol{F}$: vector sum of all forces

$$\begin{cases} \sum F_x = ma_x \\ \sum F_y = ma_y \\ \sum F_z = ma_z \end{cases}$$

Unit: International system of units, or metric system SI

$\boldsymbol{F}$: Newton (N)

m: Kilogram (kg)

$\boldsymbol{a}$: m/s^2

If $\Sigma\boldsymbol{F} = 0$ $\Rightarrow$ $\boldsymbol{a} = 0$ or $\boldsymbol{v}$ = const. $\Rightarrow$ **Newton's first law**

(3) **Newton's third law**:

Forces come in pairs, if a hammer exerts a force on a nail; the nail exerts an equal but oppositely directed force on the hammer.

i.e. $\boldsymbol{F}_{AB} = -\boldsymbol{F}_{BA}$

Action force = Reaction force

Note that they act on different bodies, so they will not cancel.

Remark:

A man stands on a platform scale. The reaction from the scale to the man shows the weight of the man.

Weight $\boldsymbol{W} = m\boldsymbol{g}$, Reaction $\boldsymbol{R} = -m\boldsymbol{g}$

Mass m: a scalar, measured in kg

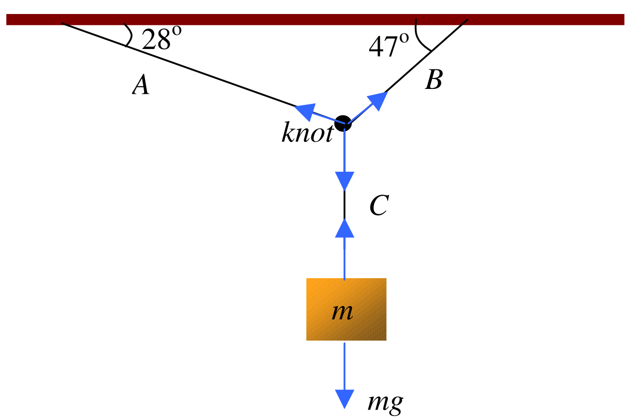
Weight $\boldsymbol{W}$: a vector, measured in N

Example

A block of mass $m = 15\ kg$ hanging from three cords, find the tension in three cords.

Answer:

Equilibrium condition at knot: $\Sigma \boldsymbol{F} = 0$ (two eqs.)



x-direction $F_{Bx} + F_{Ax} = 0$

or $F_B \cos 47 - F_A \cos 28 = 0$ (1)

y-direction $F_{Ay} + F_{By} + F_{Cy} = 0$

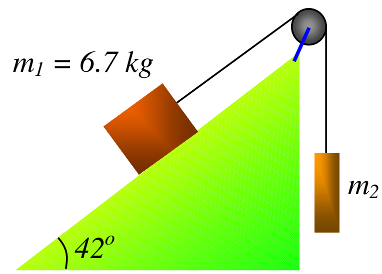
or $F_B \sin 47 + F_A \sin 28 - mg = 0$ (2)

From Eqs.(1) and (2), we obtain

$F_A = 104\ N$, $F_B = 135\ N$

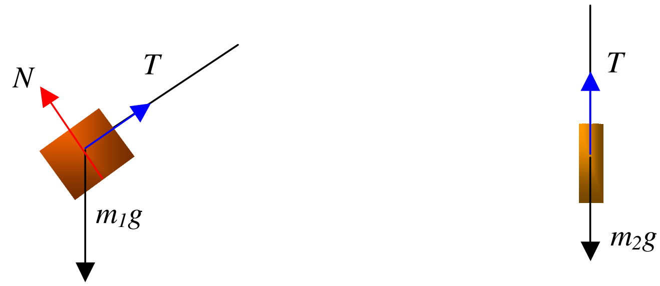
Example

Two blocks are connected by a string as shown in the figure. The smooth inclined surface makes an angle of 42° with the horizontal, and the block on the incline has a mass of $m_1 = 6.7$ kg. Find the mass of the hanging block m_2 that will cause the system to be in equilibrium.



Answer:

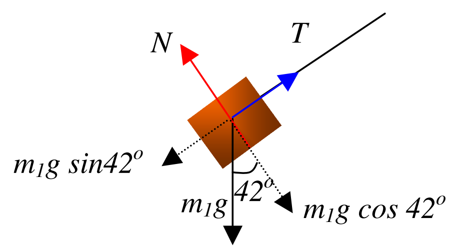
The free-body diagrams of the blocks m_1 and m_2 are shown as follows.



The down-plane component of the 6.7-kg mass is given by

$m_1 g \sin 42^{o} = (6.7\, kg)(9.8\, ms^{-2}) \sin 42^{o} = 43.9\, N$

For equilibrium, this force is balanced if the tension has the same magnitude. On the other hand, the hanging block is also in equilibrium, the weight of it, m_2g, must balance the tension force, hence we can write

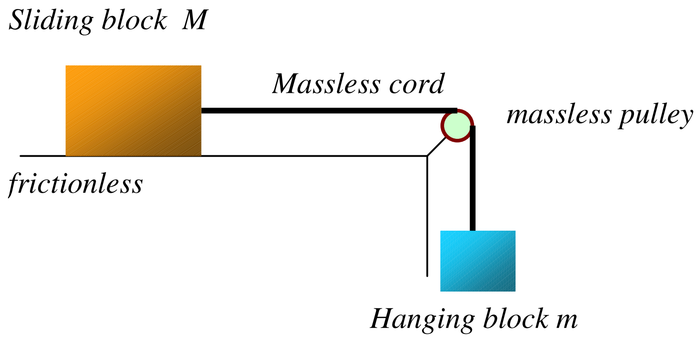


$$m_2 g = 43.9\,.$$

The mass of the hanging block is solved as (43.9 N)/g = 4.48 kg.

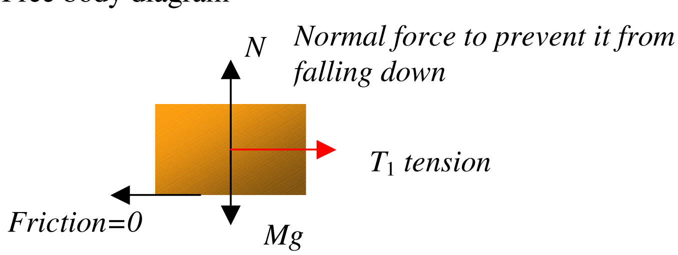
Example

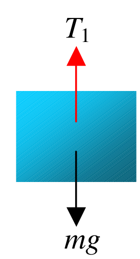
Find the acceleration and the tension of the system.



Answer:

(a) Free body diagram





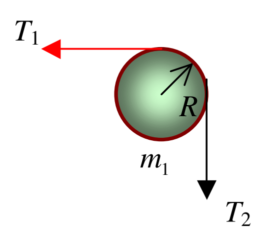
(b) About the pulley

If $m_1 \neq 0$ and the pulley is rotating, then $T_2 \neq T_1$.

In our problem, the pulley is massless and frictionless

i.e. $m_1 = 0$, hence $T_1 = T_2 = T$.

The string slides on the pulley.



(c) Cord length is fixed (no extension in the string)

$$\therefore a_1 = a_2 = a$$

(d) Using Newton's second law

$$\begin{cases} T = Ma & (1) \\ mg - T = ma & (2) \end{cases}$$

(1) + (2) $\quad mg = (M + m)a$

$$\therefore a = \frac{m}{M+m} g \qquad T = \frac{Mmg}{M+m}$$

Example

A man of mass 50 kg is now rides in an elevator. Given that the elevator have the following motions.

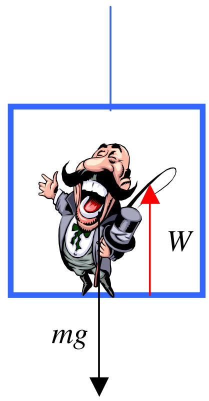
(i) moving upward with an acceleration 2ms^{-2},

(ii) moving upward with a deceleration 2ms^{-2},

(iii) moving downward with an acceleration 2ms^{-2},

(iv) moving downward with a deceleration 2ms^{-2}.

Calculate the man's weight.



Answer:

Assuming upward is positive and by Newton's second law, we have

$$W - mg = ma.$$

Hence, the weight W is represented by

$$W = mg + ma.$$

(i) For upward acceleration, a is positive, hence the weight is increased.

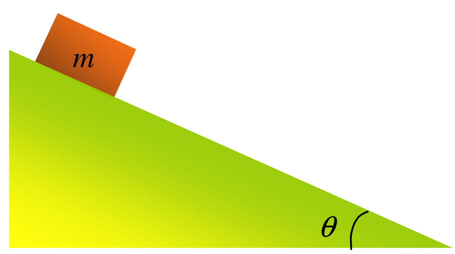
(ii) For upward deceleration, a is negative, hence the weight is decreased.

(iii) For downward acceleration, a is negative, hence the weight is decreased.

(iv) For downward deceleration, a is positive, hence the weight is increased.

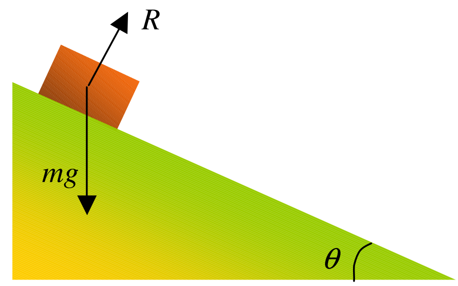
Example

Find the vertical and horizontal acceleration of a small block which slides on a frictionless inclined plane.



Answer:

The net forces that act on the block are the weight mg and the reaction R from the inclined plane.



Since there is no acceleration in the direction of R,

$$R = mg\cos\theta.$$

Hence, the horizontal force that acts on the block is

$$mg\cos\theta\sin\theta.$$

The horizontal acceleration of the block is

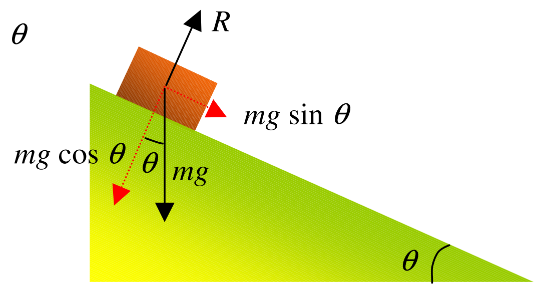
$$mg\cos\theta\sin\theta/m = g\cos\theta\sin\theta.$$

The vertical force that acts on the block is $mg - R\cos\theta$

i.e. $\quad mg - (mg\cos\theta)\cos\theta = mg\sin^2\theta.$

The vertical acceleration of the block is

$$mg\sin^2\theta/m = g\sin^2\theta.$$



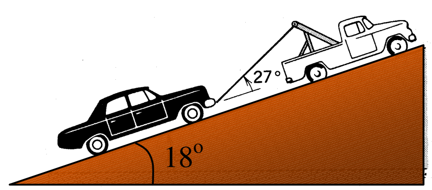
Example

A 1200-kg car is being towed up an 18° incline by means of a rope attached to the rear of a truck. The rope makes an angle of 27° with the incline. What is the greatest distance that the car can be towed in the first 7.5 s starting from rest if the rope has a breaking strength of 46 kN? Ignore all resistive forces on the car.

Answer:

The net force makes the car to accelerate

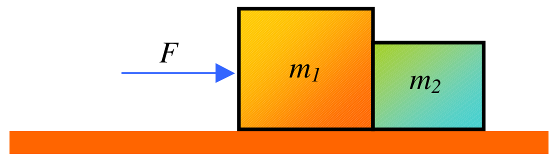
$$46000\cos 27° - 1200(9.8)\sin 18° = 1200a$$
$$a = 31.1\ \mathrm{m/s^2}$$



The distance traveled $= 0.5(31.1)(7.5^2) = 874.7$ m

Example

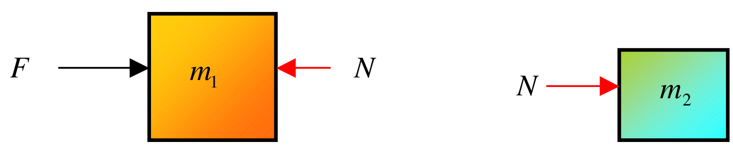
Two blocks are in contact on a frictionless table. A horizontal force is applied to one block, as shown in figure.



(a) If $m_1 = 2.3\,\text{kg}$, $m_2 = 1.2\,\text{kg}$, and $F = 3.2\,\text{N}$, find the force of contact between the two blocks.

(b) Show that if the same force F is applied to m_2, but not to m_1, the force of contact between the blocks is $2.1\,\text{N}$, which is not the same value derived in (a). Explain.

Answer:



a)

Consider mass m_1

$$F - N = m_1 a \qquad (1)$$

Consider mass m_2

$$N = m_2 a \qquad (2)$$

(1) + (2) gives

$$F = (m_1 + m_2)a$$

$$a = \frac{F}{m_1 + m_2}$$

$$\therefore N = \frac{m_2 F}{m_1 + m_2} \qquad (3)$$

for $m_1 = 2.3\,\text{kg}$, $m_2 = 1.2\,\text{kg}$, and $F = 3.2\,\text{N}$,

$$N = 1.10\,\text{N}$$

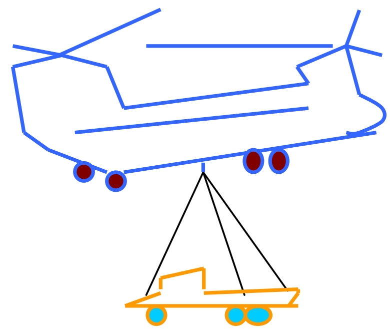
b) Interchange m_1 and m_2, one obtains

$$N = 2.1\,\text{N}$$

In order to preserve constant acceleration, greater force of the contact is needed to move the mass m_1.

Example

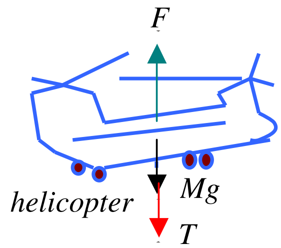
A 15000-kg helicopter is lifting a 4500-kg car with an upward acceleration of $1.4\,\mathrm{m/s^2}$. Calculate (a) the vertical force the air exerts on the helicopter blades and (b) the tension in the upper supporting cable.

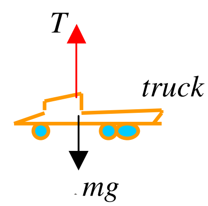


Answer:

a) $$F-(15000+4500)g=(15000+4500)\cdot 1.4$$

$$F=2.184\times 10^5\,\mathrm{N}$$



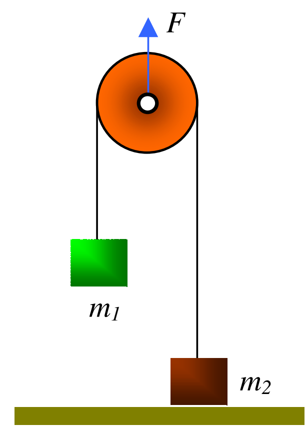


b) $$F-15000g-T=15000\times 1.4$$

$$T=5.04\times 10^4\,N$$

Example

Someone exerts a force F directly up on the axle of the pulley shown in figure. Consider the pulley and string to be massless and the bearing frictionless. Two objects, m_1 of mass 1.2 kg and m_2 of mass 1.9 kg, are attached as shown to the opposite ends of the string, which passes over the pulley. The object m_2 is in contact with the floor. (a) What is the largest value the force F may have so that m_2 will remain at rest on the floor? (b) What is the tension in the string if the upward force F is 110 N? (c) With the tension determined in part (b), what is the acceleration of m_1?



Answer:

a) When the tension of the string greater than the weight of m_2, m_2 leaves the floor. Therefore the largest value of force F for m_2 remain at rest on the floor is

$$\begin{aligned} F &= 2\times1.9\times9.8 \\ &= 37.24\ \text{N} \end{aligned}$$

b) Mass of the pulley = 0 kg and net force acts on the pulley = $110 - 2\,T$

$$\therefore 110 - 2T = 0$$
$$T = 55\ \text{N}$$

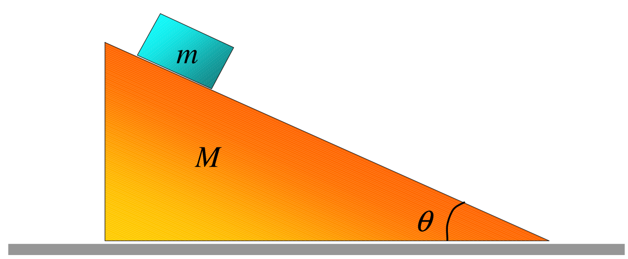
c) Net force acts on m_1 = $55 - 1.2 \times 9.8 = 43.24$ N

Acceleration of m_1 = $43.24 / 1.2 = 36.0\,\text{m}/s^2$

2.2 A Block on a Wedge

Example

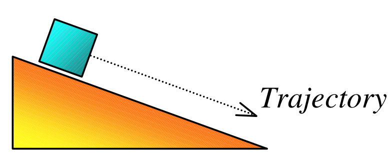
A wedge with mass M rests on a frictionless horizontal tabletop. A block with mass m is placed on the wedge. There is no friction between the block and the wedge. The system is released from rest. Calculate 1) the acceleration of the wedge, 2) the horizontal and vertical components of the acceleration of the block, check the limit when $M \to \infty$.



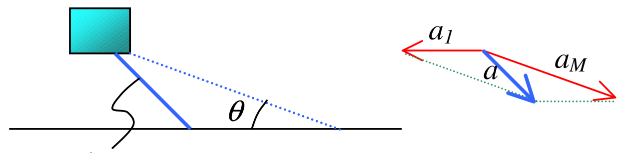
Answer:

Before doing this problem, we look at two remarks first.

- If M is not moving then we have the trajectory sketched in the right.



- If M is moving to the left, then the actual trajectory is sketched as below, where a_M is the acceleration of the block relative to the wedge of mass M.

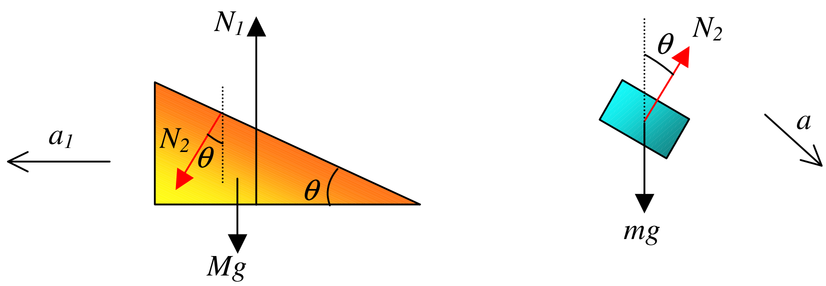


Now, we look at the force diagram of the wedge and the block.

For M: $\quad N_2 \sin\theta = Ma_1 \qquad (1)$

For m: $\quad mg - N_2\cos\theta = ma_M \sin\theta \qquad (2)$

$\quad N_2 \sin\theta = m(a_M \cos\theta - a_1) \qquad (3)$



This is a set of simultaneous equations with 3 unknowns. After solving, we obtain

$$\therefore \begin{cases} a_1 = \dfrac{mg\sin\theta\cos\theta}{M + m\sin^2\theta} \\ a_M = \dfrac{(M+m)g\sin\theta}{M + m\sin^2\theta} \end{cases}$$

where M is very large $M \to \infty$, we have

$$a_1 \to 0 \text{ and } a_M = g\sin\theta .$$

Since the horizontal and vertical components of a are given by

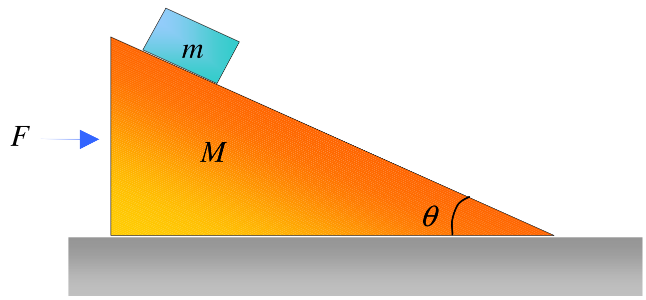
$$a_x = a_M \cos\theta - a_1 \text{ and } a_y = a_M \sin\theta ,$$

hence we have $\begin{cases} a_x = \dfrac{Mg\sin\theta\cos\theta}{M + m\sin^2\theta} \\ a_y = \dfrac{(M+m)g\sin^2\theta}{M + m\sin^2\theta} \end{cases}$

The reaction between the wedge and the block is given by the expression of a_1 and equation (1), i.e. $\quad N_2 = \dfrac{mMg\cos\theta}{M + m\sin^2\theta}$.

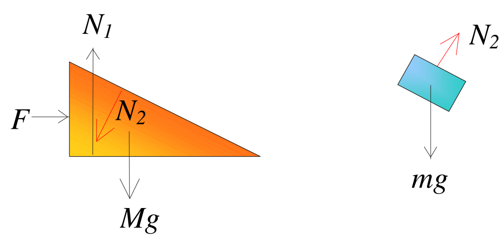
Example

A right triangular wedge of mass M and inclination angle θ, supporting a small block of mass m on its side, rests on a horizontal frictionless table, as shown in figure. (a) Assuming all surfaces are frictionless, what horizontal acceleration a must M have relative to the table to keep m stationary relative to the wedge? (b) What horizontal force F must be applied to the system to achieve this result? (c) Suppose no force is supplied to M, describe the resulting motion.



Answer:

a, b) In this case M and m are moving with the same acceleration a. Note that a is



along x-direction only.

$$\text{for } M \text{ and } m: \begin{cases} F - N_2 \sin\theta = Ma & (1) \\ mg - N_2 \cos\theta = 0 \Rightarrow N_2 = \dfrac{mg}{\cos\theta} & (2) \\ N_2 \sin\theta = ma & (3) \end{cases}$$

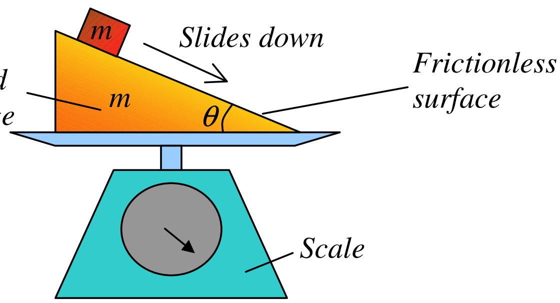
From (2) and (3), $a = g\tan\theta$

Substitute (3) into (1) obtains $F = (M+m)a = (M+m)g\tan\theta$

c) The block m will slide down along the wedge while the wedge will move backwards

Example

A block of mass *m* slides down on the frictionless inclined surface of a wedge which is fixed on a scale. If the mass of the wedge is also *m*, find the weight of the system recorded by the scale.



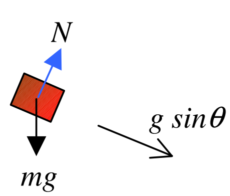
A. $mg + mg\cos\theta$ **B.** $mg + mg\cos\theta\sin\theta$

C. $mg + mg\cos^2\theta$ **D.** $mg + mg\sin^2\theta$

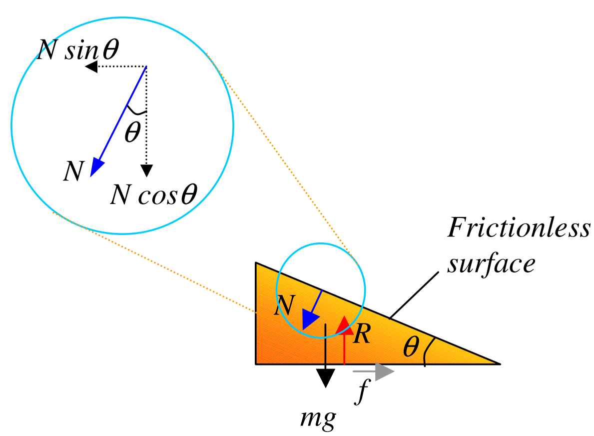
E. $2\,mg$

Answer:

Since the wedge is fixed on the scale, the block slides down the wedge with an acceleration $g\sin\theta$. On the other hand, we notice that there is no acceleration in the direction perpendicular to the inclined plane, hence the normal reaction, N given by the wedge to the block is $mg\cos\theta$. The same force acts on the wedge by the block. As the scale measures the reaction force R which is given by $mg + N\cos\theta$, that is $R = mg + mg\cos^2\theta$.



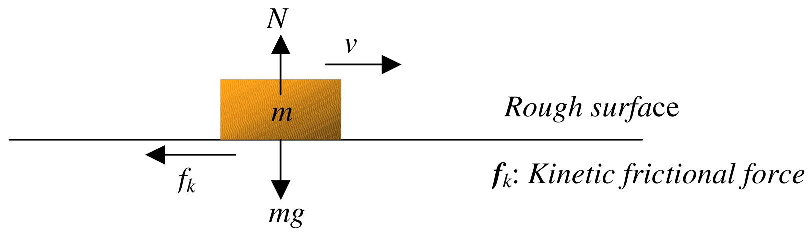
The block slides down along the inclined



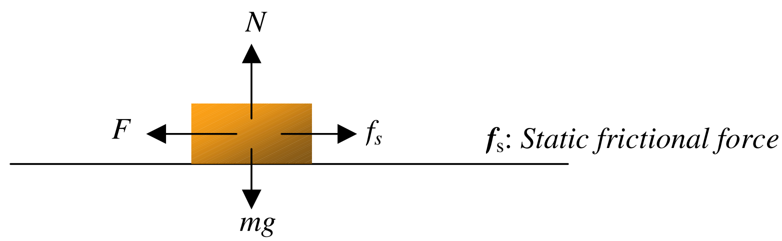
Fixed wedge

2.3 Friction and Motions

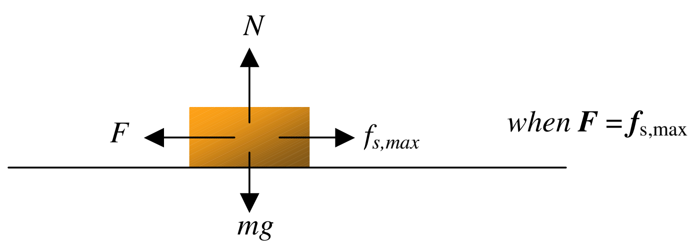
a)



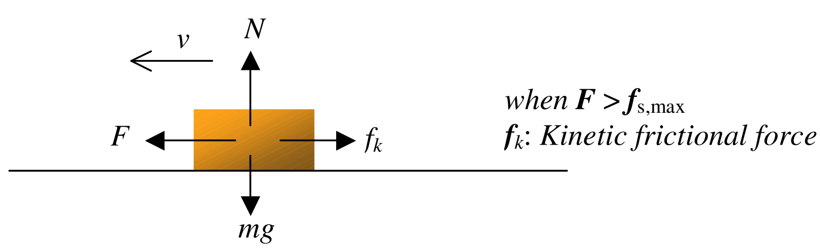
b)



c)



d)



- Laws of friction

1) Static $0 \leq f_s \leq f_{s,max}$ $(f_{s,max} = \mu_s N)$ μ_s : coefficient of static friction

  - It is independent of the area of contact between the surfaces.
  - It is parallel to the surface of contact, and in the direction that opposes relative motion.

2) Kinetic $f_k = \mu_k N$ (N = normal force) μ_k: coefficient of kinetic friction

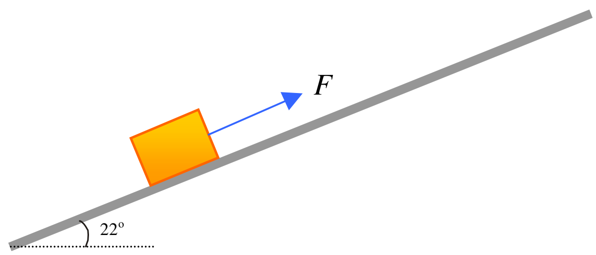
  - It is independent of the area of contact between the surfaces.
  - It is independent of the relative speed of the surfaces.

Remarks:

It should be noted that in most cases, $\mu_k < \mu_s$, and all these coefficients are positive number smaller than one.

Example

A 7.96 kg block rests on a plane inclined at 22° to the horizontal, as shown in figure. The coefficient of static friction is 0.25, while the coefficient of kinetic friction is 0.15. (a) What is the minimum force F, parallel to the plane, which can prevent the block from slipping down the plane? (b) What is the minimum force F that will start the block moving up the plane? (c) What force F is required to move the block up the plane at constant velocity?



Answer:

a) When F is at its minimum, f_s is directed up-hill and assumes its maximum value: $f_{s,\max} = \mu_s N = \mu_s mg\cos\theta$. Thus for the block $mg\sin\theta - F_{\min} - \mu_s mg\cos\theta = 0$, which gives

$$\begin{aligned} F_{\min} &= mg(\sin\theta - \mu_s\cos\theta) \\ &= (7.96\times 9.8)(\sin 22° - 0.25\times\cos 22°) \\ &= 11.14\ \text{N} \end{aligned}$$

b) To start moving the block up the plane, we must have $F - mg\sin\theta - \mu_s mg\cos\theta \geq 0$, which gives the minimum value of F required:

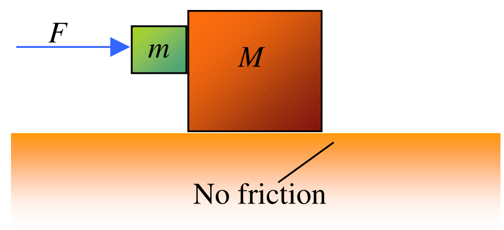
$$\begin{aligned} F_{\min} &= mg(\sin\theta + \mu_s\cos\theta) \\ &= (7.96\times 9.8)(\sin 22° + 0.25\times\cos 22°) \\ &= 47.3\ \text{N} \end{aligned}$$

c) In this case the block is already in motion, so we should replace μ_s with μ_k in calculation the frictional force. Thus $F - mg\sin\theta - \mu_k mg\cos\theta = ma = 0$, which gives the value of F:

$$F = mg(\sin\theta + \mu_k \cos\theta) = (7.96\times 9.8)(\sin 22° + 0.15\times\cos 22°)$$
$$= 40.1\,\text{N}$$

Example

The two blocks, m = 16 kg and M = 88 kg, shown in figure are free to move. The coefficient of static friction between the blocks is $\mu_s = 0.38$, but the surface beneath M is friction-less. What is the minimum horizontal force F required to hold m against M?



Answer:

The free-body diagrams for both m and M are shown to the right. When $\vec{F} = \vec{F}_{\min}$

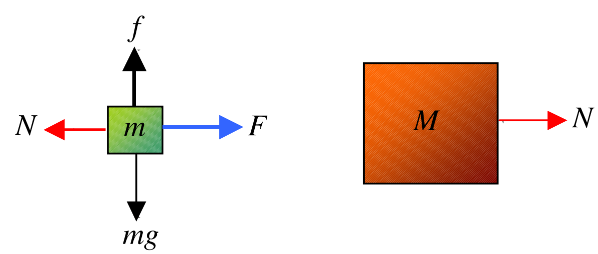
$$f_s = f_{s,\max} = \mu_s N\,,$$

where N is the normal force exerted by m on M. The equations of motion for blocks m and M are

$$\vec{F}_{\min} + \vec{f}_s + m\vec{g} + \vec{N} = m\vec{a}$$

and $$\vec{N} = M\vec{a}$$

respectively.



The vector equation above for block m in component form is

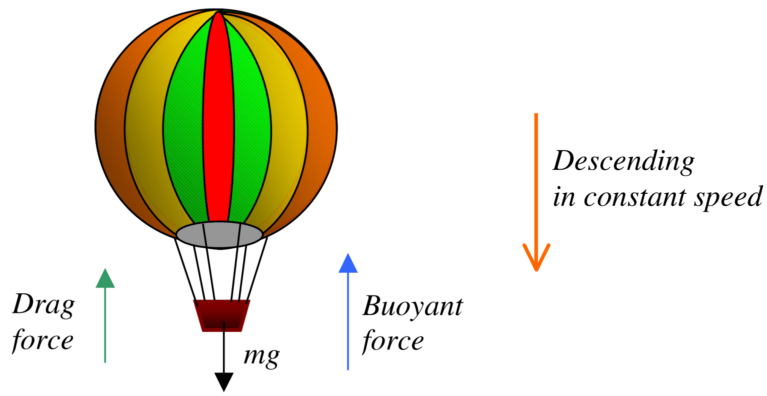
$$\begin{cases} F_{\min} - N = ma \\ f_s = \mu_s N = mg; \end{cases}$$

while the equation for M can be written as $N = Ma$. After solving, we obtain the minimum force $F_{\min}$

$$F_{\min} = \frac{mg}{\mu_s}\left(1 + \frac{m}{M}\right) = \frac{16\times 9.8}{0.38}\left(1 + \frac{16}{88}\right) = 4.9\times 10^2\,\text{N}\,.$$

Example

A balloon is descending through still air at a constant speed of 1.88 m/s. The total weight of the balloon, including payload, is 10.8 kN. A constant upward buoyant force of 10.3 kN is exerted on the balloon. The air also exerts a drag force given by $D = bv^2$, where v is the speed of the balloon and b is a constant. The crew drops 26.5 kg of ballast. What will be the eventual constant downward speed of the balloon?



Answer:

As the balloon is descending in constant speed, the upward and downward forces balance each other. The total weight of the balloon = buoyant force + drag force

$$10.8 \times 10^3 \text{ N} = 10.3 \times 10^3 \text{ N} + \text{drag force}$$

We obtain the drag force = 500 N.

Having the drag force and speed, we can find the constant *b*.

$$D = bv^2$$
$$500 = b(1.88)^2$$
$$b = 141.5\, kgm^{-1}$$

After dropping the ballast, the total weight of the balloon is given by

$$10.8 \times 10^3 \text{ N} - 26.5 \times 9.8 \text{ N}$$
$$= 1.05 \times 10^4 \text{ N}$$

To achieve another constant speed, the net force should be zero again.

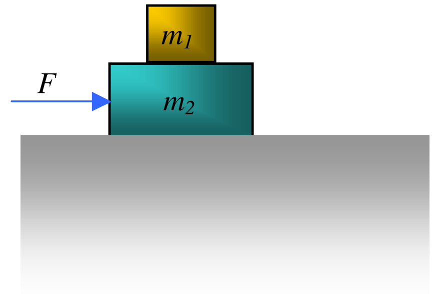
$$1.05 \times 10^4\ N = 10.3 \times 10^3\ N + 141.5 \times v^2$$
$$v = 1.30 \text{ m/s}$$

Example

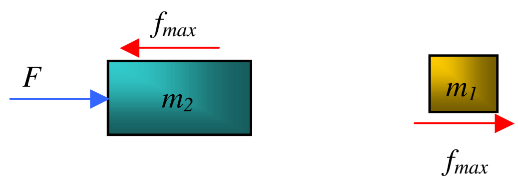
Two blocks, stacked one on the other, slide on a frictionless, horizontal surface. The surface between the two blocks is rough, however, with a coefficient of static friction equal to 0.47.

(a) If a horizontal force F is applied to the 5.0-kg bottom block (m_2), what is the maximum value F can have before the 2.0-kg top block (m_1) begins to slip?

(b) If the mass of the top block is increased, does the maximum value of F increase, decrease, or stay the same? Explain.



Answer:



The maximum frictional force that the 2.0-kg block can experience is f_{max}, where $f_{\max} = \mu m_1 g$, that is f_{max} = (0.47)(2.0 kg)(9.8 ms^{-2}) = 9.21 N.

The acceleration of the 2-kg block, $a = f_{max} / m_1$ = 4.61 ms^{-2}.

There will be no slipping when the two block move as one. That is the 5-kg block moves with the same acceleration 4.606 ms^{-2}.

According to Newton's second law, we have $F - f_{max} = m_2 a$,

hence $F = f_{max} + m_2 a$ = 9.21 N + (5.0 kg)(4.61 ms^{-2}) = 32.3 N.

We observe that the horizontal force $F = \mu m_1 g + \mu m_2 g = \mu g(m_1 + m_2)$. The horizontal force F increases with the mass of the top block.