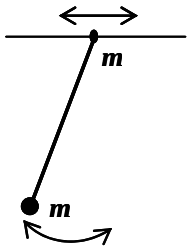
Pan Pearl River Delta Physics Olympiad 2007
2007 年泛珠三角及中華名校物理奧林匹克邀請賽
Part-1 (Total 7 Problems) 卷-1（共 7 題）
(9:30 am – 12:30 pm, 02-26-2007)

Q.1 (3 points) 題 1（3 分）

An airplane is initially rising up at speed v_0 at an angle θ to the horizon. Find the trajectory of the plane such that weightless condition can be achieved in the plane.

一架飛機以與水平面成θ角的初速度v_0上升。求飛機以什麼樣的軌跡飛行，能使飛機裏的物體處於失重狀態。

Q.2 (6 points) 題 2（6 分）

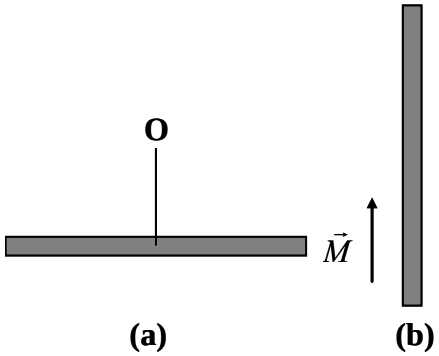


As shown, two identical weights are fixed on the two ends of a uniform rigid rod of length L. The upper weight is restricted to move on a smooth horizontal rail and the rod is free to swing along the rail. The masses of the weights and the rod are equal. Find the small angle vibration frequency of the system.

如圖所示，兩個質量爲 m 的重塊分別固定在一根長度爲 L 質量爲 m 的均勻杆兩端。上面的重塊可以沿光滑的水平軌道滑行，杆可沿軌道方向自由擺動。求整個系統的小角度振動頻率。

Q.3 (6 points) 題 3（6 分）

(a) A disc shaped medium block of radius R and thickness d (<< R) is uniformly magnetized with magnetization $\vec{M}$ perpendicular to the disc plane. Find the magnetic field at point-O on the central axis of the disk and at a distance h from the cavity center.



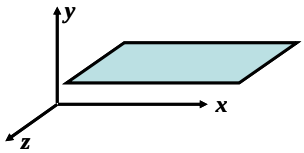
一半徑爲 R，厚度爲 d (<< R)的圓盤形均勻磁化介質，磁化強度爲 $\vec{M}$ 。盤的表面垂直於 $\vec{M}$ 。求圓盤中心軸上到圓盤中心距離爲 h 的點 O 的磁場。

(b) A long and thin cylindrical medium is uniformly magnetized with magnetization $\vec{M}$ along the cylinder long axis. Find the magnetic field inside and outside the medium.

一細長圓柱型介質沿柱軸方向均勻磁化，磁化強度爲 $\vec{M}$ 。求介質裏、外的磁場。

Q.4 (5 points) 題 4（5 分）

A large flat dielectric slab of thickness d and dielectric constant ε is moving along the x-direction at speed v. Its large surface plane is perpendicular to the y-axis. A magnetic field of strength B is applied along the z-direction. Find the surface bound charge density on the two large surfaces of the slab, and the electric field in the slab.



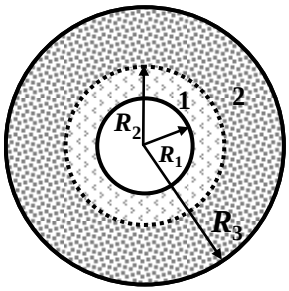
一個厚度爲 d，介電常數爲ε的大平板以速度 v 沿 X-方向運動。它的表面與 Y-軸垂直。Z-方向加有磁場 B。求平板兩表面上的束縛電荷密度，以及平板中的電場。

Q.5 (10 points) 題 5（10 分）

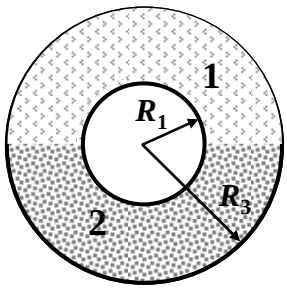
The space between two concentric conductor spherical shells of radii R_1 and R_3 is filled with two types of media. The dielectric constant and the conductivity of medium-1 and medium-2 are ε_1, σ_1 and ε_2, σ_2, respectively. The voltage difference between the two shells is V_0.

(a) In case-A, the media form two concentric shells with the conductor shells, and the radius of the boundary between the two media is R_2. Find the following: (i) total current from the inner shell to the outer shell; (ii) total free charge on the two conductor shells and on the boundary between the two media.

(b) In case-B, medium-1 fills the upper hemisphere and medium-2 fills the other half. Find the following: (i) total current from the inner shell to the outer shell; (ii) total free charge on the upper and lower halves of the two conductor shells.



Case-A



Case-B

如圖所示，兩個半徑分別爲R_1和R_3的同心導電球殼之間充滿了兩種介質。球殼之間電勢差爲V_0。介質 1 和 2 的介電常數和導電率分別爲 ε_1, σ_1 和 ε_2, σ_2。

(a) 兩種介質爲與導電球殼同心的球殼，其界面爲半徑爲R_2的球面。(i) 求兩導電球殼間的總電流；(ii) 求兩導電球殼以及兩介質之間界面上的電荷。

(b) 介質-1 填充上半部分，介質-2 填充下半部分。(i) 求兩導電球殼間的總電流；(ii) 求兩導電球殼上、下部分的電荷。

Q.6 (12 points) 題 6（12 分）

(a) Assume that atmosphere is made of diatom ideal gas in adiabatic equilibrium. Determine air pressure P, temperature T and density ρ as a function of altitude h, provided that their values at $h = 0$ are known. (Hint: Set up a differential equation for a thin layer of air at some altitude. $\int x^{\alpha} dx = \frac{1}{\alpha+1} x^{\alpha+1}$, where $\alpha \neq 1$ is a constant.) (6 points)

(a) 大氣可看成絕熱平衡下的雙原子理想氣體。求空氣壓強 P、溫度 T 和密度ρ作爲高度 h 的函數，假定它們在 $h = 0$ 處的值爲已知。（提示：對某高度的一薄層氣體建立微分方程。$\int x^{\alpha} dx = \frac{1}{\alpha+1} x^{\alpha+1}$ ，$\alpha \neq 1$ ）(6 分)

(b) When the partial pressure of water vapor in air exceeds the saturated water vapor pressure (P_s) at a given temperature, the water vapor will condense into droplets which fall down as rain. P_s = 55.35 mmHg at 40°C, and P_s = 6.50 mmHg at 5 °C. The air/vapor mixture can be considered as diatom ideal gas and the mass of a water molecule is approximately the same as an 'air' molecule. In the humid air at sea level at 40°C the water vapor partial pressure is 90 % of P_s. The density of air is ρ_0 = 1.18 kg m^{-3} at 20 °C and 1.0 atm. The humid air then rises adiabatically to an altitude where the temperature is 5 °C. Ignore air pressure change due to the reduction of water vapor.

(b1) How much rain can one cubic meter of the humid air at sea level generate? (5 points)
(b2) Use the results in (a), find the altitude where the temperature is 5 °C. (1 point)

(b) 當空氣中水蒸汽的分壓強超過該溫度下的飽和水蒸汽壓(P_s)時，水蒸汽將凝聚成滴導致下雨。已知 40°C時P_s = 55.35 mmHg，5 °C時 P_s = 6.50 mmHg。空氣/水蒸汽的混合物可當作是雙原子理想氣體，水分子的質量近似等於 '空氣' 分子的質量。40°C時海平面上的潮濕空氣中，水蒸汽分壓是P_s的 90 %。已知 20 °C時，1 個大氣壓下的空氣密度 ρ_0 = 1.18 kg m^{-3}。忽略由於水蒸汽的減少導致的氣壓改變。該潮濕空氣絕熱上升到某一高度，該處溫度爲 5 °C。

（b1）一立方米海平面上的潮濕空氣能夠產生多少雨？(5 分)
（b2）用（a）的結果，求溫度爲 5 °C 處的高度。(1 分)

Q7 (8 points) 題 7（8 分）

(i) Find the torque on an electric dipole $\vec{p}$ in a uniform electric field $\vec{E}$. (1 point)

(ii) A medium is uniformed polarized with polarization $\vec{P}$ by an electric field $\vec{E}$. Find the torque per volume on the medium exerted by the electric field. (1 point)

(iii) An electromagnetic wave $\vec{E} = E_0(\vec{x}_0 + \vec{y}_0)e^{i(kz-\omega t)}$ is propagating along the z-axis in an isotropic medium. In such medium the relation between the electric displacement $\vec{D}$ and $\vec{E}$ is given by $\vec{D} = \varepsilon_0\varepsilon\vec{E}$, so $\vec{D}$ and $\vec{E}$ are always pointing in the same direction. Find the torque per volume on the medium exerted by the electromagnetic wave. (1 point)

(iv) An electromagnetic wave $\vec{E} = E_0(\vec{x}_0 e^{ik_1 z} + \vec{y}_0 e^{ik_2 z})e^{-i\omega t}$ is propagating along the z-axis in an anisotropic medium. In such medium the electric displacement is $\vec{D} = \varepsilon_0 E_0(\varepsilon_x \vec{x}_0 e^{ik_1 z} + \varepsilon_y \vec{y}_0 e^{ik_2 z})e^{-i\omega t}$, so $\vec{D}$ is not parallel to $\vec{E}$. Note that $k_1 = \frac{\omega}{c}\sqrt{\varepsilon_x}$ and $k_2 = \frac{\omega}{c}\sqrt{\varepsilon_y}$, where c is the speed of light in vacuum. Find the time-averaged (over one period) torque per volume on the medium exerted by the electromagnetic wave. (3 points)

(v) Following (iv), find the time-averaged total torque on a section of cylindrical shaped medium of unit cross section area with its long axis along the z-direction from $z = 0$ to $z = d$, and the smallest value of d at which the total torque is maximum. (2 points)

(i) 求一個電偶極子 $\vec{p}$ 在電場 $\vec{E}$ 中受到的力矩。(1 分)

(ii) 某介質在電場 $\vec{E}$ 中均勻極化，極化強度爲 $\vec{P}$ 。求單位體積介質在該電場中受到的力矩。(1 分)

(iii) 在一各向同性的介質中，電磁波 $\vec{E} = E_0(\vec{x}_0 + \vec{y}_0)e^{i(kz-\omega t)}$ 沿z-軸傳播 。在該介質中電位移矢量 $\vec{D}$ 和電場 $\vec{E}$ 的關係滿足 $\vec{D} = \varepsilon_0\varepsilon\vec{E}$ ，因此 $\vec{D}$ 和 $\vec{E}$ 總是保持同一方向。求單位體積介質在該電磁波中受到的力矩。(1 分)

(iv) 在一各向異性的介質中，電磁波 $\vec{E} = E_0(\vec{x}_0 e^{ik_1 z} + \vec{y}_0 e^{ik_2 z})e^{-i\omega t}$ 沿z-軸傳播 。在該介質中電位移矢量爲 $\vec{D} = \varepsilon_0 E_0(\varepsilon_x \vec{x}_0 e^{ik_1 z} + \varepsilon_y \vec{y}_0 e^{ik_2 z})e^{-i\omega t}$ ，因此通常 $\vec{D}$ 和 $\vec{E}$ 不平行。這裏 $k_1 = \frac{\omega}{c}\sqrt{\varepsilon_x}$ ， $k_2 = \frac{\omega}{c}\sqrt{\varepsilon_y}$ ，c是真空中光速。求單位體積介質在該電磁波中受到的一個週期裏的平均力矩。 (3 分)

(v) 根據 (iv), 求長軸平行於 z-軸，單位橫截面積的圓柱形介質中 $z = 0$ 到 $z = d$ 部分所受的一個週期的平均力矩，以及使力矩最大所需的 d 的最小值。（2 分）

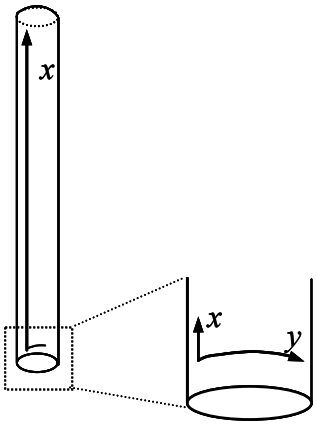
THE END 完

Pan Pearl River Delta Physics Olympiad 2007
2007 年泛珠三角及中華名校物理奧林匹克邀請賽
Part-2 (Total 3 Problems) 卷-2（共 3 題）
(2:30 pm – 5:30 pm, 02-26-2007)

Q1 Folded Space (6 points) 題 1 卷起的空間（6 分）

(a) Consider a one-dimensional standing electromagnetic wave in the form of $E(x) = A\sin(k_x x)$ along the x-direction confined within the space between $x = 0$ and $x = a$. The wave must vanish at these two end points. Find the allowed values of k_x. (1 point)

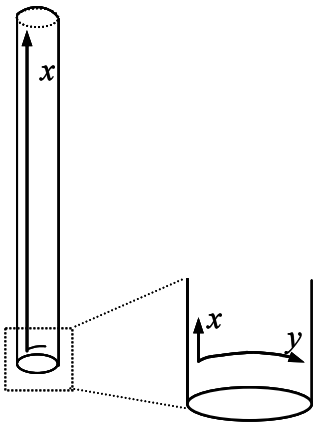
(b) The String Theory predicts that our space is more than three-dimension, and the additional hidden dimensions are folded up like the dimension y on the surface of a thin cylinder shown in the figure. Suppose the radius of the cylinder is b (<< a), and the electromagnetic wave on the surface now takes the form $E(x, y) = A\sin(k_x x)\cos(k_y y)$, where y is the coordinate of the folded space around the cylinder. Find the allowed values of k_y. (3 points)



(c) The photon energy is given by $W = \frac{hc}{2\pi}\sqrt{k_x^2 + k_y^2}$, and hc = 1239 (eV × nanometer), where eV stands for electron volt and 1 nanometer is 10^{-9} meters. The highest energy photons human can make so far is about 1.0×10^{12} eV. If this is sufficient to create a photon in the folded space, what should be the value of b? (2 points)

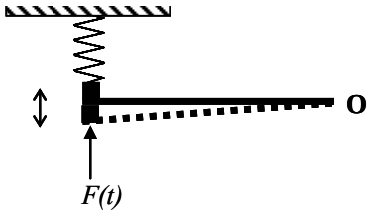
(a) 一維電磁駐波 $E(x) = A\sin(k_x x)$ 在 x-方向限制在$x = 0$ 和 $x = a$之間。 在兩個端點處駐波消失。求 k_x的可能值。(1 分)

(b) 弦理論認爲物理空間多於三維，多出的隱藏維空間象細圓柱的表面一樣卷了起來，如圖中y坐標所示。設圓柱的半徑爲b (<< a), 在圓柱面上電磁波的形式爲 $E(x, y) = A\sin(k_x x)\cos(k_y y)$ ，其中y 是繞圓柱的折疊空間的坐標。求 k_y的可能值。(3 分)



(c) 光子能量 $W = \frac{hc}{2\pi}\sqrt{k_x^2 + k_y^2}$, 其中 hc = 1239 (eV × nm)，eV 表示 1 電子伏特, 1 nm 等於 10^{-9} 米。目前人類能產生的最高能量的光子大約爲 1.0×10^{12} eV。如果該能量能夠產生一個折疊空間的光子，b的值滿足什麼條件？(2 分)

Q2 Atomic Force Microscope (AFM) in thermal noise (22 points)
題 2 熱噪聲下的原子力顯微鏡 (22 分)

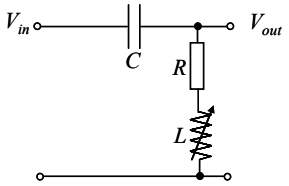


(i) An AFM is modeled as a uniform rigid rod of length l and mass m_1 with a point mass m_2 on one end (the tip), and the other end is fixed at point O around which the rod is free to rotate. A spring of force constant K is attached to the tip. Find the resonant frequency ω_0 of the AFM. (4 points)
原子力顯微鏡能夠簡化爲一個長度爲l，質量爲m_1的均勻硬杆，一端有一個質量爲 m_2 的質點 (針尖)，另一端固定在點 O ，杆可繞點 O自由轉動。一個彈性係數爲 K 的彈簧連著針尖。求原子力顯微鏡的共振頻率ω_0。(4 分)

(ii) Given an external driving force $F(t) = F_1 \cos(\omega_1 t)$, derive the differential equation for the small vertical displacement $x(t)$ of the tip from its equilibrium position, and solve it using a trial solution $x(t) = A_1 \cos(\omega_1 t + \Phi_1)$ where the amplitude A_1 and phase Φ_1 are to be determined. (4 points)
給定一個外驅動力 $F(t) = F_1 \cos(\omega_1 t)$, 推導針尖離平衡位置的小位移$x(t)$的微分方程，並用試探解 $x(t) = A_1 \cos(\omega_1 t + \Phi_1)$ 解它，其中振幅A_1 和位相Φ_1 待定。(4 分)

(iii) Given two driving forces $F(t) = F_1 \cos(\omega_1 t) + F_2 \cos(\omega_2 t)$, find $x(t)$. (4 points)
給定兩個外驅動力 $F(t) = F_1 \cos(\omega_1 t) + F_2 \cos(\omega_2 t)$, 求 $x(t)$ 。 (4 分)

(iv) The driving force comes from thermal noise, which can be described as a sum of many harmonic driving forces $F_{thermal}(t) = \sum_n F_n \cos(\omega_n t)$ in the entire frequency range. Find $x(t)$ under the thermal noise driving force. (2 points)
驅動力來自於熱噪聲，它能夠寫成覆蓋所有頻率的許多簡諧驅動力的和 $F_{thermal}(t) = \sum_n F_n \cos(\omega_n t)$ 。求熱驅動力下的 $x(t)$ 。(2 分)



(v) Consider the electronic band pass filter as shown. Given the input voltage $V_{in}(t) = V_0 e^{i\omega t}$, find the value of inductance L such that the **<u>denominator</u>** of the absolute value of the output voltage is minimum. (2 points)
考慮一個如圖所示的電子帶通濾波器。輸入電壓爲 $V_{in}(t) = V_0 e^{i\omega t}$ ，求使輸出電壓絕對值<u>分母最小</u>的電感L 的值。(2 分)

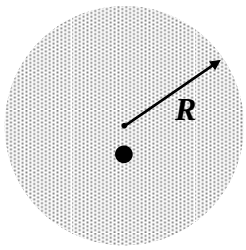
(vi) The AFM signal which is proportional to the solution $x(t)$ in (iv) is applied as the input signal to the filter. Assuming that only the signal with the frequency $\omega_n = \omega$, where ω makes the denominator of the output voltage amplitude minimum in (v), can pass through the filter, draw a sketch of the amplitude of the output voltage vs L if $F_n = 1$ for all n, and describe briefly how the AFM resonant frequency in (i) can be found experimentally. (6 points)
將正比於（iv）中 $x(t)$的原子力顯微鏡信號輸入到電子濾波器。假設僅有頻率 ω_n等於（v）中使輸出電壓絕對值分母最小的ω 的信號能通過該濾波器，假定對所有n，$F_n = 1$ ，試畫出輸出電壓的大小隨L 變化的簡圖， 並簡單描述實驗上如何找到（i）中所述原子力顯微鏡的共振頻率。(6 分)

Q3 The Lorentz-Lorenz Relation (22 points) 題 3 洛倫茲-洛倫茲關係 (22 分)

The dielectric constant $\varepsilon(\omega)$ of a dielectric medium is given by the so called Lorentz-Lorenz Relation $\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+n}\cdot\varepsilon_0=\frac{1}{3}K(\omega)$, where n is a number and K is a material-related constant that depends explicitly on the frequency ω of the applied electric field. You are to derive the relation through the steps below.

介質的介電常數 $\varepsilon(\omega)$ 滿足所謂的洛倫茲-洛倫茲關係 $\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+n}\cdot\varepsilon_0=\frac{1}{3}K(\omega)$，其中 K 與所加電場的頻率 ω 以及介質的物質常數有關，n 是一數字。下面逐步推出這一關係。

(i) An atom can be approximately treated as consisting of a uniform spherical electron density (electron cloud) of radius R with total charge $-Ze$ and positive charge nucleus Ze at the center, where e is the charge of a positron. The nucleus mass is much larger than an electron mass m_e. In a uniform external electric field E_0 the electron cloud is displaced slightly from the nucleus while maintaining its spherical shape. Find the displacement. (4 points)



一個原子可以近似看成由總電荷爲$-Ze$、均勻分佈成半徑爲R的球形電子雲，和處於中心的帶電Ze的原子核組成，其中e 是正電子的電荷。 核的質量遠大於電子質量 m_e。在均勻外電場E_0 中電子雲輕微偏離核，但保持球形。求偏離的位移。(4 分)

(ii) The atom is placed in an oscillating uniform electric field $E(t)=A\cos(\omega t)$. Find the induced dipole moment of the atom. (6 points)
將原子放在震盪的均勻外場 $E(t)=A\cos(\omega t)$ 中， 求原子的電偶極矩。(6 分)

(iii) In a medium the number of atoms per unit volume is N, find the polarization P of the medium. (2 points)
已知介質的單位體積原子數是 N，求介質的極化矢量 P 。(2 分)

(iv) Note that in (iii) the electric field is the external field E_{ext}. Consider a small sphere containing many atoms in the large medium. The total field E_{total} in the sphere consists of two contributions, namely that from the medium inside the sphere E_{self} and the external field E_{ext}. Given that the electric fields are uniform inside the sphere, find the relation between E_{ext} and E_{total}, and determine n and K in the Lorentz-Lorenz Relation. (7 points)
在 (iii) 中的電場是外加電場 E_{ext}。考慮大介質中的一個包含很多原子的小球。球中的總電場 E_{total} 來源於兩部分，一是球中的介質產生的電場 E_{self} ，二是外加電場E_{ext}。已知球中的電場是均勻的，求 E_{ext} 和 E_{total}的關係，並求洛倫茲-洛倫茲關係中的n 和K。 (7 分)

(v) Use the result in (iv), briefly explain the Mirage phenomenon. (3 points)
用(iv)的結果簡單解釋海市蜃樓現象。（3 分）

THE END 完