Pan Pearl River Delta Physics Olympiad 2008
2008 年泛珠三角及中華名校物理奧林匹克邀請賽
Part-1 (Total 6 Problems) 卷-1（共 6 題）
(9:00 am – 12:00 pm, 02-14-2008)

Q.1 (4 points) 題 1（4 分） A nucleus-A with mass m_A and initial velocity v_0 along the *x*-axis collides with a nucleus-B with mass m_B at rest. Some kinetic energy E is absorbed by nucleus-B and converted into nuclear energy during the collision. Since $E << m_B c^2$ where c is the speed of light in vacuum, the change of mass of nucleus-B can be neglected. After the collision nucleus-A moves at an angle θ = 90º to the *x*-axis. Find the speed of nucleus-A and the velocity of nucleus-B.

一質量爲 m_A的原子核-A 以沿 X-軸方向的初速度 v_0與一質量爲 m_B的靜止原子核-B 相撞。碰撞中一部分機械能 E 被原子核-B 吸收而轉變爲核能。由於 $E << m_B c^2$ ，其中 c 是真空光速，所以可忽略原子核-B 質量的變化。碰撞後原子核-A 沿與 X-軸成θ = 90º 的方向飛出。求原子核-A 的速率，以及原子核-B 的速度。

Q.2 (6 points) 題 2（6 分）

Two large parallel conductor plates are held at voltage difference V at distance d apart. A large dielectric slab of thickness $d/3$ and dielectric constant ε is placed midway in the gap between the plates and is moving parallel to the plates at speed v (<< speed of light).

(a) In the reference frame where the plates are stationary, find the magnetic field in the middle of the upper air gap, in the middle of the slab, and in the middle of the lower air gap.

(b) In the reference frame where the dielectric slab is stationary, repeat (a).

兩大導電板間距 d，之間的電壓差 V。一厚 $d/3$ 的大介質板，介電常數ε，在兩板中間以速度 v（<<光速）沿與板平行的方向運動。

(a) 在導電板靜止的參照系，求在上、下空隙中間位置以及介質板中間位置的磁場。

(b) 在介質板靜止的參照系，求在上、下空隙中間位置以及介質板中間位置的磁場。

Q.3 (7 points) 題 3（7 分）

An electron has intrinsic angular momentum I called spin, and a permanent magnetic dipole moment $\vec{M} = -\frac{ge}{2m}\vec{I}$ associated with the spin, where e is the positive electron charge, m is the electron mass, and g is a number called g-factor. An electron with its spin aligned along its initial velocity in the *x*-direction enters a region of uniform magnetic field in the *z*-direction. Show that if g is exactly 2, then the spin is always in the same direction as the velocity of the electron. (The real $g = 2.00232...$. The deviation from 2 can be calculated precisely by quantum electrodynamics.)

電子具有固有的角動量 I，稱爲自旋，和與自旋相關的磁偶極矩$\vec{M} = -\frac{ge}{2m}\vec{I}$ 。其中 e 爲正電子電荷，m 爲電子質量，g 爲 g -因子。一電子初始速度和自旋都沿 X-方向，進入一均勻沿 Z-方向的磁場。證明若 g 剛好等於 2，則電子之後的速度和自旋始終在同一方向。（實際的 $g = 2.00232...$ 。和 2 的差可用量子電動力學準確計算。）

Q.4 (12 points) 題 4（12 分）

Near the Earth's surface, the atmosphere could be considered as ideal gas at constant temperature T = 300 K and in a uniform gravitational field g = 9.8 m/s^2. The mass per mole of 'air' molecule is m = 0.029 kg. The gas constant is R = 8.31 J/(K*mol).

(a) For stationary atmosphere, set up a differential equation that relates the pressure $p(h)$ at height h with the gravitational field g, the atmosphere mole number density $\rho(h)$, and mass per mole m. **(1 point)**

(b) Using the ideal gas law and the result in (a), set up a differential equation for the pressure $p(h)$ as a function of height h. **(1 point)**

(c) Assuming that p_0 is the pressure at $h=0$, solve the differential equation in (b), and find the height where the pressure is $\frac{p_0}{2}$. (Hint: $\int \frac{dx}{x} = \ln|x| + \text{Constant}$) **(4 points)**

(d) In the case of wind with constant velocity v blowing in the atmosphere at all height, the differential equation is approximately $\frac{dp}{dh} + \frac{mv^2}{2}\frac{d\rho}{dh} = -m\rho g$. Find the pressure $p(h)$ at height h. **(3 points)**

(e) In (d) the effect of Earth spinning is ignored. In general, any object moving at velocity $\vec{v}$ on Earth will experience an additional 'inertia force field' $\vec{f}_{\text{int}} = 2\vec{\Omega}\times\vec{v}$, where $\vec{\Omega}$ is the angular velocity of the spinning Earth, in addition to the gravitational field g. Verify that the inertia force field can indeed be ignored even in the case of typhoons where the wind speed is up to 500 km/hr. **(1 point)**

(f) Using the wind speed in (e), find the height where the pressure is $\frac{p_0}{2}$. **(2 points)**

在地面附近, 大氣層可被當作溫度爲 T = 300 K 的等溫理想氣體，並處於均勻引力場 g = 9.8 m/s^2 之內。 每 mole 的大氣質量爲 m = 0.029 kg。 理想氣體常數 R = 8.31 J/(K*mol)。

(a) 對於靜止的大氣層，建立微分方程，把在高度 h 處的壓強 $p(h)$、重力加速度 g、 大氣分子 mole 密度 $\rho(h)$ 、和每 mole 的大氣質量 m 這些物理量聯繫起來。 **(1 分)**

(b) 利用理想氣體方程和(a)的結果, 建立 $p(h)$ 爲 h 的函數的微分方程。 **(1 分)**

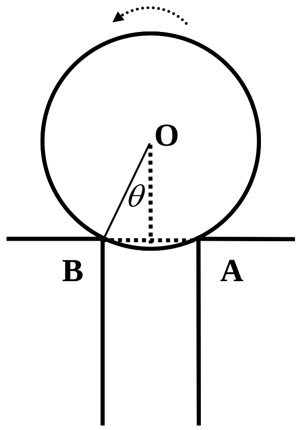
(c) 設在 $h=0$ 處壓強爲 p_0 ，解(b)的微分方程, 並找出壓強爲 $\frac{p_0}{2}$ 處的高度。 (提示: $\int \frac{dx}{x} = \ln|x| + \text{Constant}$) **(4 分)**

(d) 有風時，設大氣在所有高度的速度均爲 v , 則經過簡化的微分方程爲 $\frac{dp}{dh} + \frac{mv^2}{2}\frac{d\rho}{dh} = -m\rho g$ 。求在高度 h 處的壓強 $p(h)$. **(3 分)**

(e) 在(d)中沒有考慮地球的自轉。一般說來，任何以速度 $\vec{v}$ 在地球表面運動的物體，除了重力 g 外，均受到 '慣性力場' $\vec{f}_{\text{int}} = 2\vec{\Omega}\times\vec{v}$ 的作用，其中 $\vec{\Omega}$ 爲地球自轉的角速度。 驗證甚至在刮 500 km/hr 的颱風時地球自轉的效應仍可忽略。 **(1 分)**

(f) 用(e)的風速， 求壓強爲 $\frac{p_0}{2}$ 處的高度。**(2 分)**

Q.5 (11 points) 題 5（11 分）



A uniform solid sphere with mass M, radius R, and moment of inertia $I = \frac{2}{5}MR^2$ around its center is initially rolling without slipping on a horizontal surface. The speed of its center is v. It then encounters a ditch of width d such that $\sin\theta = \frac{d}{2R}$, as shown in the figure. For convenience you may use θ and R to replace d in the following calculations. The initial speed v is smaller than a value v_{max} such that when the sphere arrives at the near edge of the ditch at point-A, it falls off while keeping in touch with point-A without slipping, until it hits the other edge at point-B.

(a) Find the angular speed of the sphere right before it hits point-B. **(3 points)**

(b) Find the maximum initial speed v_{max} that the sphere can keep in touch with point-A without slipping before it hits point-B. **(2 points)**

(c) Assuming no slipping when the sphere hits point-B, find the minimum initial speed v_{min} such that the sphere can get over the ditch. **(4 points)**

(d) To satisfy both the conditions in (b) and (c), the angle θ must satisfy $f(\theta) > 0$. Determine $f(\theta)$. **(2 points)**

一均勻剛球質量 M，半徑 R，繞球心的轉動慣量 $I = \frac{2}{5}MR^2$ ，在平面上作純滾動，球心速度 v 。之後遇到一溝，溝寬 d ，如圖所示 $\sin\theta = \frac{d}{2R}$ 。爲方便起見，在以下的解答你可用 θ 和 R 來代替 d 。 球的初速度 v 比 某 v_{max} 小，使得當球到達溝的一邊 A 點處時, 一直與 A 點保持無滑動的接觸，直到球碰到溝另一邊的 B 點。

(a) 求碰到 B 點前的瞬間球的角速度。 **(3 分)**

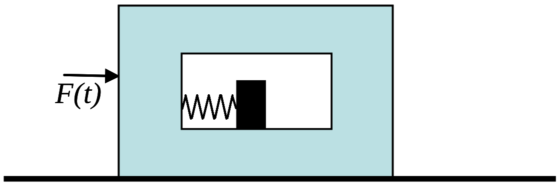
(b) 求球碰到溝另一邊的 B 點前一直與 A 點保持無滑動接觸的最大初速度 v_{max}。 **(2 分)**

(c) 設球碰到 B 點後無滑動，求能使球滾過溝的最小初速度 v_{min} 。**(4 分)**

(d) 要使(b) 和 (c)的條件都成立，角度 θ 必須滿足 $f(\theta) > 0$ 。 求 $f(\theta)$ 。**(2 分)**

Q.6 (10 points) 題 6（10 分）

Consider a big block of mass M_1 placed on a smooth horizontal surface with a hollow rectangular cave carved out in the interior, as shown in the figure. Inside the cave are a spring of force constant K with one end attached to the wall, and a smaller block of mass M_2 attached to the other end of the spring which can move on the smooth horizontal cave surface. The natural length of the spring is about half the length of the cave. When the small block is moving relative to the big block it never hits the wall of the cave. Both blocks are confined to motion along the x – axis. A periodic external force $F(t) = F_0 \cos(\omega t)$ is applied to the big block which can push and pull the block. Let the position of the big block be $X_1(t)$, and since the interior of the big block is hidden from the external observers, the effective mass of the system is then $M_{eff} = \dfrac{F(t)}{\ddot{X}_1(t)}$, where $\ddot{X}_1(t) \equiv \dfrac{d^2 X_1}{dt^2}$ is the second derivative of $X_1(t)$ to time.



(a) Assume that the two blocks are in simple harmonic motions at the same frequency as that of the driving force $F(t)$, find the effective mass of the system. **(7 points)**

(b) Find the range of frequencies in which the effective mass is negative. **(3 points)**

如圖，一質量 M_1 的大物塊放在光滑平面上，大塊內有一長方空腔，空腔內有一力常數 K 的彈簧，一端固定在空腔壁上，另一端系有一質量 M_2 的小物塊，放在光滑空腔底平面上。彈簧的自然長度約爲空腔長度的一半，小物塊運動時不會碰到兩邊的腔壁。所有運動都是沿 x – 軸的一維運動。一週期性外力 $F(t) = F_0 \cos(\omega t)$ 加在大物塊上，將它推前、拉後。設大物塊的位置爲 $X_1(t)$, 因大物塊的內部是看不見的，所以系統的有效質量爲 $M_{eff} = \dfrac{F(t)}{\ddot{X}_1(t)}$, 其中 $\ddot{X}_1(t)$ 爲 $X_1(t)$對時間的二次導數， $\ddot{X}_1(t) \equiv \dfrac{d^2 X_1}{dt^2}$ 。

(a) 假設兩物塊均以與外力 $F(t)$ 相同之頻率作簡諧運動，求系統的有效質量。 **(7 分)**

(b) 求有效質量爲負數的頻率範圍。**(3 分)**

THE END 完

Pan Pearl River Delta Physics Olympiad 2008
2008 年泛珠三角及中華名校物理奥林匹克邀請賽
Part-2 (Total 3 Problems) 卷-2（共 3 題）
(2:30 pm – 5:30 pm, 02-14-2008)

Q1 Point charge in a magnetic field (16 points) 題 1 磁場中的帶電粒子（16 分）

A point charge $-q$ ($q > 0$) with mass m moves without friction inside a region of magnetic field given by $\mathbf{B} = B_0 \dfrac{a}{r}\hat{\mathbf{z}}$ (a, $B_0 > 0$), where $r = \sqrt{x^2 + y^2}$ is the distance from the z-axis. At $t = 0$, the initial position and velocity of the charge is ($x = a$, $y = 0$, $z = 0$) and ($v_x = 0$, $v_y = v_0$, $v_z = 0$), where $v_0 > 0$.

(a) Show that the particle always stays on the xy plane. **(1 point)**

(b) Find the initial speed at which the charge should be launched so that it can perform circular motion around the origin. **(2 point)**

(c) To obtain the motion for arbitrary v_0, set up a differential equation, in the form of $\dfrac{dL}{dr} =$ ***constant***, for angular momentum L of the charge with respect to the z-axis, and solve the differential equation. If you cannot determine the ***constant***, just assume it is known and solve (d) and (e).
(Hint: $\vec{A}\times\left(\vec{C}\times\vec{B}\right)=\left(\vec{A}\cdot\vec{B}\right)\vec{C}-\left(\vec{A}\cdot\vec{C}\right)\vec{B}$) **(5 points)**

(d) From the result of (c), find the distance of the charge from the origin when it is moving in the tangential direction ($\vec{v}\perp\vec{r}$). Find the minimum v_0 above which the charge can never move in the tangential direction after it is initially launched. **(5 points)**

(e) Find the distance from the origin when the charge is moving in the radial direction ($\vec{v}$ parallel to $\vec{r}$). Find the minimum v_0 above which the charge can never move in the radial direction after it is initially launched. **(3 points)**

一質量 m 的點電荷 $-q$ ($q > 0$) 無阻力地在磁場 $\mathbf{B} = B_0 \dfrac{a}{r}\hat{\mathbf{z}}$ (a, $B_0 > 0$)中運動，其中 $r \equiv \sqrt{x^2 + y^2}$ 爲電荷離 z-軸的距離。

在 $t = 0$ 時, 電荷的位置和速度分別爲($x = a$, $y = 0$, $z = 0$) 和 ($v_x = 0$, $v_y = v_0$, $v_z = 0$), 其中 $v_0 > 0$。

(a) 證明粒子的運動始終在 xy 平面上。 **(1 分)**

(b) 若粒子以原點作圓周運動，求初速度 。 **(2 分)**

(c) 對一般初速度情況下求解。建立粒子繞 z-軸的角動量所滿足的微分方程，其形式應爲 $\dfrac{dL}{dr} =$ ***常數***，並解之。若無法確定***常數***，你可假設***常數***已知，並用來解答(d)和(e)。(提示： $\vec{A}\times\left(\vec{C}\times\vec{B}\right)=\left(\vec{A}\cdot\vec{B}\right)\vec{C}-\left(\vec{A}\cdot\vec{C}\right)\vec{B}$) **(5 分)**

(d) 從(c)的結果, 求當 $\vec{v}\perp\vec{r}$ 時粒子離原點的距離。若粒子在 $t = 0$ 後永遠不再有 $\vec{v}\perp\vec{r}$ 的情形出現，求 v_0 的最小值。 **(5 分)**

(e) 求當 $\vec{v}$ 平行於 $\vec{r}$ 時粒子離原點的距離。若粒子在 $t = 0$ 後永遠不再有 $\vec{v}$ 平行於 $\vec{r}$ 的情形出現，求 v_0 的最小值。 **(3 分)**

Q2 Electric Fan (16 points) 題 2 電扇（16 分）

Consider an electric fan with moment of inertia I around its central axis initially rotating about the axis at constant speed ω_0 driven by a motor. The fan will slow down once the motor is turned off and finally stops due to the friction forces from two sources. One is the fixed torque τ due to the friction between its rotating central axis and its holder. The other is the air resistance on the fan blades which is proportional to the instantaneous rotating speed $\omega(t)$, so the air resistance torque is $\gamma\omega(t)$, where γ is a constant.

(a) Theory

(a.1) Write down the differential equation for the instantaneous rotating speed $\omega(t)$ of the fan. **(2 points)**

(a.2) Given the initial speed ω_0, the time it takes for the fan to stop (t_s) can be expressed as $t_s = A \cdot \ln(1 + B\omega_0)$. Determine the constant A and B in terms of I, τ, and γ. **(4 points)**

(b) Design of experiment

Suppose the initial speed of the fan can be so slow that $\ln(1 + B\omega_0) \approx B\omega_0$, or can be so fast that $B\omega_0 >> 1$. The initial speed of the fan can be set and read from the meter on the fan controller. You are also given the following items: a ruler, a stop watch, and several pairs of small known mass blocks (about 1/10 the mass of the blades) that can be firmly attached to the blades of the fan, but their air resistance can be neglected. Design an experiment to determine τ, γ, and I. You should state clearly what data are to be collected and processed, what plot(s) should be drawn, and how to extract the parameters from the plot(s) to reach the final answers. **(10 points)**

一電扇繞其中心軸的轉動慣量 I ，初始時在馬達驅動下以角速度ω_0 繞中心軸轉動。馬達關了後，由於兩方面的摩擦阻力電扇的轉動會慢下來，直到停止。阻力之一爲中心軸與外套之間的固定摩擦阻力力矩τ。另一個是電扇葉片的空氣阻力力矩，與瞬時角速度 $\omega(t)$成正比，因此可表達爲$\gamma\omega(t)$, 其中 γ 爲常數。

(a) 理論

(a.1) 寫出電扇瞬時角速度的微分方程 **(2 分)**

(a.2) 給定初始角速度 ω_0, 從停止驅動到電扇停下的時間 t_s 可表達成 $t_s = A \cdot \ln(1 + B\omega_0)$ 。用 I、 τ、γ 來表達係數 A、B 。**(4 分)**

(b) 實驗設計

設電扇的初始角速度很慢，使 $\ln(1 + B\omega_0) \approx B\omega_0$ 得以成立；或很快，使 $B\omega_0 >> 1$ 得以成立。電扇的初始角速度可由馬達的控制器設置並讀出。另有標尺一把，計時器一個, 和幾對已知質量（約爲電扇葉質量的 1/10 ）、可粘在電扇葉上的小重塊。小重塊的空氣阻力可忽略。設計實驗步驟以確定 τ、γ、和 I 。 你必須清楚表述需要測量哪些實驗數據，如何作數據處理，如何作圖，從圖上得到哪些參數，最後得到結果。**(10 分)**

Q3 Negative Resistance Instability & Zero Resistance State (18 points)
題 3 負電阻不穩定性和零電阻狀態 (18 分)

Two years ago it was discovered that the resistance of a semiconductor device containing two dimensional electron gas becomes negative when it is in a constant magnetic field and under strong microwave radiation. Because a system including the device and other circuit elements such as normal resistors, capacitors, inductors, and voltage sources will be unstable (violating the second law of thermodynamics) if the *total* resistance of the system R_{sys} is negative, the system reorganizes itself into a new state with zero total resistance. The resistance of the device can be expressed as a current and charge dependent resistance

$$R_{NR} \equiv R(i,q) = R_0\left(\left(\frac{i}{i_o}\right)^2 + \left(\frac{q}{q_o}\right)^2 - 1\right),$$

where $R_0 > 0$ so that the resistance is negative when the current passing through the system i or charge storing in the system q is small, and becomes positive when there is large enough current or charge building up in the system. Here R_0, i_o, q_o are constants that depend only on the internal structure of the device, the strength and frequency of microwave radiation, and the strength of the magnetic field. When R_{NR} is connected to other normal circuit elements like capacitor, normal resistor, inductor, and voltage source, the usual Kirchhoff's circuit laws still apply, and we seek steady-state solutions only.

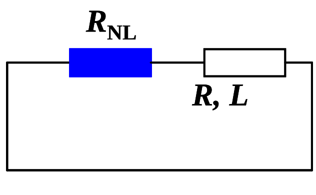
兩年前人們發現在微波照射下，在直流磁場中的由二維電子氣構成的半導體器件的電阻可成負值。由於含有這種器件和其它常規電阻、電感、電容、電源的系統的總電阻 R_{sys} 不能爲負，否則會違反熱力學第二定律，所以系統會自動調節，以達到一新的零電阻狀態。器件的電阻與電流、電荷有關的表達式爲

$$R_{NR} \equiv R(i,q) = R_0\left(\left(\frac{i}{i_o}\right)^2 + \left(\frac{q}{q_o}\right)^2 - 1\right),$$

其中 $R_0 > 0$。因此當系統的電流 i 或電荷 q 太小時，其電阻爲負。而當系統的電流或電荷夠大時，其電阻變爲正值。R_0, i_o, q_o 爲常數，只與器件的本身結構，以及磁場、微波頻率和照射強度有關。設對於 R_{NR} 和其它常規電阻、電感、電容、電源形成的電路，常規 Kirchhoff 電路原理仍然適用，並只考慮系統已到達穩定狀態下的解。）

a) We start with simple systems. 我們從簡單的系統開始。

(i) When R_{NR} is in series with a normal resistor with resistance *R*, find the DC current and the voltage drop across *R* and R_{NR}. (Consider both cases when $R > R_0$ and $R < R_0$.) **(2 points)**

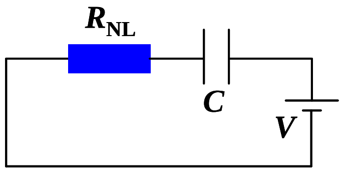


R_{NR} 和一常規電阻 *R* 串連，求電路的直流電流和 *R*、R_{NR} 上的電壓。(須考慮 $R > R_0$ 和 $R < R_0$ 這兩種情況。) **(2 分)**

(ii) When R_{NR} is in series with a normal inductor with inductance *L*, find the DC current and the voltage drop across *L* and R_{NR}. **(1 point)**

R_{NR} 和一常規電感 *L* 串連，求電路的直流電流和 *L*、R_{NR} 上的電壓。**(1 分)**

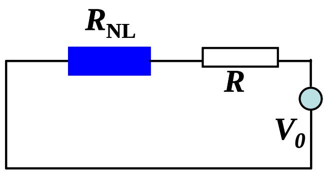
(iii) When R_{NR} is in series with a normal capacitance *C* and a DC voltage source *V*, find the minimum voltage *V* needed to make $R_{sys} = R_{NR} = 0$. Also find the DC current and the voltage drop across *C* and R_{NR}. (Hint: The charge *q* on the capacitor is the charge stored in the system that determines the value of R_{NR}.) **(2 points)**



R_{NR} 和一常規電容 C、直流電壓源 V 串連。求使 $R_{sys}=R_{NR}=0$ 所需的最小電壓，並求這時的直流電流和 C 、R_{NR} 上的電壓。（提示：電容上的電量 q 就是決定 R_{NR} 的儲存於系統的電荷量）**(2 分)**

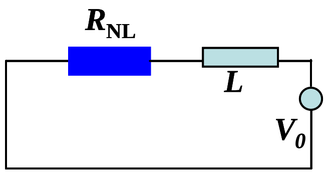
b) Now a small AC or DC voltage source V_0 is added in the above circuits. In the following calculations of the contribution of V_0 to the additional current and/or charge on top of the original ones, keep only their first order. In the cases when the small voltage source is AC, you should set up differential equations first.
現在以上的電路中加一直流或交流的小電源 V_0。在以下的計算由 V_0 所帶來的額外電流或電荷的過程中，只需保留它們的一次項。當小電源爲交流時，先建立微分方程。

(i) When R_{NR} is in series with a normal resistor with resistance R and a small DC voltage source V_0, find the DC current. You should consider both the cases $R > R_0$ and $R < R_0$, separately. **(3 points)**



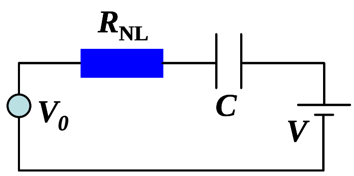
當 R_{NR} 和一常規電阻 R、小直流電壓源 V_0 串連，求直流電流。（分別考慮 $R > R_0$，和 $R < R_0$ 這二種情況。）**(3 分)**

(ii) When R_{NR} is in series with a normal inductor with inductance L and a small AC voltage source with amplitude V_0 and frequency ω, find the current through L and R_{NR}. **(3 points)**

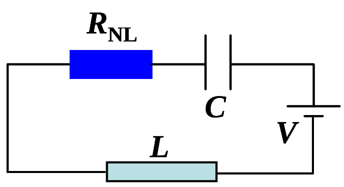


當 R_{NR} 和一常規電感 L、頻率爲 ω 幅度爲 V_0 的小交流電壓源串連，求電流。 **(3 分)**

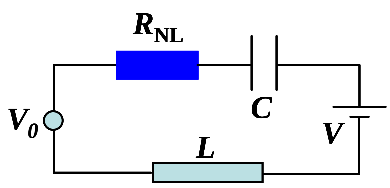
(iii) A small AC voltage source with amplitude V_0 and frequency ω is added to the circuit in part-a(iii), find the charge on the capacitor. **(2 points)**
a(iii)部分的電路里加上頻率爲ω幅度爲 V_0 的小交流電壓源，求電容上電荷。 **(2 分)**



(iv) Consider an LCR circuit with R_{NR} as the resistor in the presence of DC voltage source V, find the current in the circuit. **(2 points)**
在 LCR 電路中將常規電阻用 R_{NR} 代替，並加上直流電壓源 V，求直流電流。**(2 分)**



(v) A small AC voltage source with amplitude V_0 and frequency ω is added to the circuit in (iv). Find the current in the circuit. **(3 points)**
在以上(iv)的電路中加上頻率爲ω幅度爲 V_0 的小交流電壓源，求交流電流。**(3 分)**



THE END 完