Part-I 第一卷

Q1 題 1 **(8 points 8 分)** Solution 解:

For resistance *R*, only half of the area is conducting, as the other half is blocked by medium-2. Let the voltage between the plates be *V*, then the electric field

先求電阻 *R*。介質-2 不導電，所以只有一半的導電板導電。令兩板之間的電壓爲 *V*，則

電場爲

$E = V/d$, (1 point 1 分)

The current density is 電流密度爲 $J = \sigma_1 E$, (1 point 1 分)

The current is 電流爲 $I = \frac{1}{2} J a^2 = \frac{a^2}{2} \sigma_1 \frac{V}{d}$. (1 point 1 分)

So the resistance is 因此電阻爲 $R = \frac{V}{I} = \frac{2d}{\sigma_1 a^2}$. (1 point 1 分)

For capacitance, it can be treated as two capacitors in parallel. The capacitance on the right is

再求電容。總電容可當作是左右兩個電容幷聯。右邊的電容爲

$C_1 = \frac{\varepsilon_0 \varepsilon_1 a^2}{2d}$. (1 point 1 分)

For the left half, let the electric displacement be D_2 which is the same throughout the region.

在左半邊，令電位移爲 D_2，電位移處處相同。 (1 point 1 分)

The total free charge on the left half is

左半邊的總自由電荷爲

$Q = \frac{a^2}{2} D_2$. (1 point 1 分)

The total voltage between the two plates is

兩導電板之間的總電壓爲

$V = \frac{1}{2} E_1 d + \frac{1}{2} E_2 d = \frac{d}{2\varepsilon_0} D_2 (\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2})$. (1 point 1 分)

So 因此 $C_2 = \frac{\varepsilon_0 a^2}{d} \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$,

$C = C_1 + C_2 = \frac{\varepsilon_0 a^2}{d} \left(\frac{\varepsilon_1}{2} + \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \right)$. (1 point 1 分)

Q2 題 2 (10 points 10 分) solution 解:

(a) Let the mass of the blackhole be *M*, then

令黑洞的質量爲 *M*，則

$$\frac{GM}{r^2} = \frac{v^2}{r}$$, (2 points 2 分)

So 因此 $\sqrt{\frac{GM}{r}} = v$, (1 point 1 分)

$n = -\frac{1}{2}$. (1 point 1 分)

(b) Let the areal mass density be σ, then

令質量面密度爲σ，則

$$\frac{G\pi\sigma r^2}{r^2} = \frac{v^2}{r}$$, (2 points 2 分)

So 因此 $\sqrt{G\pi\sigma r} = v$, (1 point 1 分)

$n = \frac{1}{2}$. (1 point 1 分)

(c) Anything reasonable is fine. It DOES NOT have to be dark matter.

任何有一定理由的解釋都行，不一定非暗物質不可。(2 points 2 分)

Q3 題 3 (10 points 10 分) Solution 解:

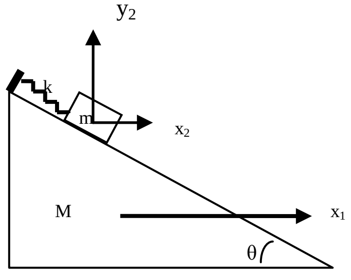
Method 1: Using conservation of energy

方法-1：利用能量守恒

Equations 方程:

$$Mx_1 = -mx_2, \quad y_2 = (x_2 - x_1)\tan\theta = (1 + \frac{m}{M})x_2 \tan\theta$$

(1 point 1 分)



All coordinates are in the rest frame. 所有坐標取自靜止參照系。

The total kinetic energy of the system is

總動能爲

$$T = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{2}m\dot{y}_2^2 \quad \text{(1 point 1 分)}$$

$$= \frac{1}{2}M\frac{m^2}{M^2}\dot{x}_2^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{2}m\tan^2\theta \bullet \dot{x}_2^2(1+\frac{m}{M})^2 \quad \text{(1 point 1 分)}$$

The total potential energy 總勢能爲:

$$V = \frac{1}{2}k[(x_2 - x_1)^2 + y_2^2] - mgy_2 \quad \text{(1 point 1 分)}$$

$$= \frac{1}{2}k[(1+\frac{m}{M})^2 x_2^2 + (1+\frac{m}{M})^2 x_2^2 \tan^2\theta] - mg(1+\frac{m}{M})x_2\tan\theta$$

$$= \frac{1}{2}kx_2^2(1+\frac{m}{M})^2 \bullet \frac{1}{\cos^2\theta} - mg(1+\frac{m}{M})x_2\tan\theta \quad \text{(1 point 1 分)}$$

Using 利用 $ax^2 - bx = a(x - \frac{b}{2a})^2 - \frac{b^2}{4a}$, we get 得

$$V = \frac{1}{2}k(1+\frac{m}{M})^2 \frac{1}{\cos^2\theta}(x_2 - \frac{mMg\sin 2\theta}{k(M+m)})^2 - \frac{(mg\sin\theta)^2}{2k} \quad \text{(1 point 1 分)}$$

Make a transfer of coordinate 取坐標變換

$$x \equiv x_2 - \frac{mMg\sin 2\theta}{k(M+m)}, \text{(1 point 1 分)}$$

We reach an expression for the total energy that is of the form of $T + V = ax^2 + b\dot{x}^2 + c$ which must be constant by energy conservation. Let $x = A\cos(\omega t + \phi)$, we get

$$T + V = aA^2\cos^2(\omega t + \phi) + bA^2\omega^2\sin^2(\omega t + \phi) + c = A^2(a - b\omega^2)\cos^2(\omega t + \phi) + bA^2\omega^2 + c.$$

For the total energy to be constant the $\cos^2(\omega t + \phi)$ term must be zero all the time, which leads to $\omega^2 = a/b$.

我們得到總能量的表達式爲 $T + V = ax^2 + b\dot{x}^2 + c$ 。因能量守恒總能量應爲常數。令 $x = A\cos(\omega t + \phi)$，得

$$T + V = aA^2\cos^2(\omega t + \phi) + bA^2\omega^2\sin^2(\omega t + \phi) + c = A^2(a - b\omega^2)\cos^2(\omega t + \phi) + bA^2\omega^2 + c$$

作爲常數，上式中 $\cos^2(\omega t + \phi)$ 項必須爲零。因此 $\omega^2 = a/b$ 。

Therefore, the oscillation frequency ω in this case is

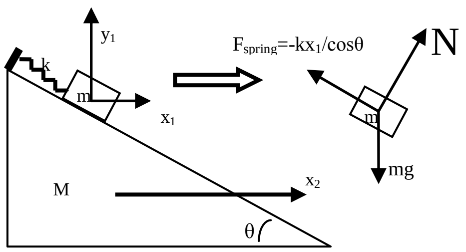
由上式得系統的頻率ω爲

$$\omega^2 = \frac{k(1+\frac{m}{M})^2/\cos^2\theta}{\frac{m}{M^2}+m+m(1+\frac{m}{M})\tan^2\theta} = \frac{k}{m}\frac{1+\frac{m}{M}}{\cos^2\theta+(1+\frac{m}{M})\sin^2\theta} = \frac{k}{m}\left(\frac{M+m}{M+m\sin^2\theta}\right).$$ (1 point 1 分)

For $\theta = 0$, we have 當$\theta = 0$，得$\omega^2 = k\left(\frac{1}{m}+\frac{1}{M}\right)$ (1 point 1 分),

For $\theta = 90°$ we have 當$\theta = 90°$，得$\omega^2 = \frac{k}{m}$. (1 point 1 分)

Method 2: Analytical Mechanics 方法-2：分析力學



Force figure 力圖 (2 points 2 分)

Equations 方程

$$m\ddot{x}_1 = -\frac{kx_1}{\cos\theta}\cos\theta - N\sin\theta + m\ddot{x}_2,$$

x_1 is in the frame on the slope. x_1是相對與斜面的橫坐標。 (1 point 1 分)

$$y_1 = x_1\tan\theta$$

$$m\ddot{x}_1\tan\theta = N\cos\theta - mg - kx_1\tan\theta$$ (1 point 1 分)

$$M\ddot{x}_2 = -(N\sin\theta + kx_1)$$ (1 point 1 分)

Process 解方程過程

Step 1: Eliminate $\ddot{x}_2$ 步驟-1：消去$\ddot{x}_2$

$$m\ddot{x}_1 = -kx_1 - N\sin\theta - \frac{m}{M}(N\sin\theta + kx_1)$$

$$m\ddot{x}_1\tan\theta = N\cos\theta - mg - kx_1\tan\theta$$

Step 2: Eliminate N　　步驟-2：消去 N

$$m\ddot{x}_1 + (1+\frac{m}{M})kx_1 = -N\sin\theta(1+\frac{m}{M})$$

$$m\ddot{x}_1\tan\theta + mg + kx_1\tan\theta = N\cos\theta$$

Which means 整理後得

$$m\cos\theta\ddot{x}_1 + \cos\theta(1+\frac{m}{M})kx_1 = -\sin\theta(1+\frac{m}{M})[m\tan\theta\ddot{x}_1 + mg + kx_1\tan\theta]$$ (2 points 2 分)

By assuming $x_1 = Ae^{i\omega t}$, the oscillation frequency ω is obtained

設解 $x_1 = Ae^{i\omega t}$ ，得頻率ω

$$\omega^2 = \frac{k(1+\frac{m}{M})}{m}\frac{\cos\theta + \frac{\sin^2\theta}{\cos\theta}}{\cos\theta + (1+\frac{m}{M})\frac{\sin^2\theta}{\cos\theta}} = \frac{k}{m}\left(\frac{M+m}{M+m\sin^2\theta}\right).$$ (1 point 1 分)

For $\theta = 0$, we have 當 $\theta = 0$ ，得 $\omega^2 = k\left(\frac{1}{m}+\frac{1}{M}\right)$ (1 point 1 分),

For $\theta = 90°$ we have 當 $\theta = 90°$，得 $\omega^2 = \frac{k}{m}$. (1 point 1 分)

Q4 題 4 (10 points 10 分)　　Solution 解:

(a) Let the magnetic field be $\vec{B}(z,t) = \vec{B}_0 e^{i(\tilde{k}z-\omega t)}$. The k-vector of the wave is $\vec{k} = \tilde{k}\vec{z}_0$.

Using the equation $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$,

令磁場的表達式爲 $\vec{B}(z,t) = \vec{B}_0 e^{i(\tilde{k}z-\omega t)}$ 。k-矢量爲 $\vec{k} = \tilde{k}\vec{z}_0$ 。利用方程 $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

we get 得 $\vec{B}(z,t) = \frac{\vec{k}\times\vec{E}(z,t)}{\omega}$, (1 point 1 分)

$$= \frac{\tilde{k}}{\omega}E_0(\vec{z}_0\times\vec{x}_0)e^{i(\tilde{k}z-\omega t)} = \frac{\tilde{k}}{\omega}E_0\vec{y}_0 e^{i(\tilde{k}z-\omega t)}$$

So 因此 $\vec{B}_0 = \frac{\tilde{k}}{\omega} E_0 \vec{y}_0$. (1 point 1 分)

(b) Note that 利用 $\tilde{k} = \frac{\omega}{c}\sqrt{\varepsilon} + i\frac{\sigma}{2}\sqrt{\frac{\mu_0}{\varepsilon\varepsilon_0}} \equiv k_R + ik_I$.

So 得 $<\vec{S}> = \frac{1}{\mu_0} <(\vec{E}\times\vec{B})> = \frac{1}{2\mu_0}\mathrm{Re}(\vec{E}\times\vec{B}^*)$ (1 point 1 分)

$= \frac{E_0^2}{2\mu_0\omega}\vec{z}_0 \,\mathrm{Re}(k_R - ik_I)e^{-2k_I z} = \frac{1}{2\mu_0}\frac{k_R}{\omega}E_0^2\vec{z}_0 e^{-2k_I z} = \frac{1}{2\mu_0 c}E_0^2\vec{z}_0 e^{-2k_I z}$ (1 point 1 分)

(c) $q = -\frac{d<S>}{dz} = \frac{k_I k_R}{\mu_0\omega}E_0^2 e^{-2k_I z} = \frac{\sigma}{2}\frac{\frac{\omega}{c}\sqrt{\varepsilon}\sqrt{\frac{\mu_0}{\varepsilon\varepsilon_0}}}{\mu_0\omega}E_0^2 e^{-2k_I z} = \frac{\sigma}{2}E_0^2 e^{-2k_I z}$. (2 points 2 分)

(d) Joule Heat 焦爾熱 = $<\vec{J}\cdot\vec{E}> = \frac{1}{2}\mathrm{Re}(\vec{J}\cdot\vec{E}^*) = \frac{\sigma}{2}\mathrm{Re}(\vec{E}\cdot\vec{E}^*) = \frac{\sigma}{2}E_0^2 e^{-2k_I z}$ (2 points 2 分)

(e) The energy loss of EM wave is equal to the Joule Heat.

電磁波能量的損失等于焦爾熱。(2 points 2 分)

Q5 (12 points) 題 5（12 分） Solution 解：

(a) First law 熱力學第一定律: $dU = -PdV + \delta Q$.

The equation of adiabatic processes is 絕熱過程的方程式爲

$\delta Q = dU + PdV = d(3PV) + PdV = 4PdV + 3VdP = 0$ (1 point 1 分)

$\Rightarrow PV^{4/3} = \text{Constant}$

$\Rightarrow PV^{4/3} =$ 常數 (1 point 1 分)

(b) In the Carnot cycle 在卡諾循環過程中:

$$(P_1,V_1) \xrightarrow{\text{Isothermal}} (P_1,V_2) \xrightarrow{\text{Adiabatic}} (P_2,V_3) \xrightarrow{\text{Isothermal}} (P_2,V_4) \xrightarrow{\text{Adiabatic}} (P_1,V_1).$$

Isothermal = 等溫；Adiabatic = 絕熱

The heat supplied to the gas during the first isothermal process is

第一個等溫過程中吸收的熱量為

$$Q_1 = 3P_1V_2 - 3P_1V_1 + \int_{V_1}^{V_2} P_1 dV = 4P_1(V_2 - V_1)\,.$$ (1 point 1 分)

(c) Similarly, the heat supplied to the gas during the second isothermal process is

第二個等溫過程中吸收的熱量為

$$Q_2 = 4P_2(V_4 - V_3)\,.$$ (1 point 1 分)

(d) From (a) we have 由(a)得: $P_1V_2^{4/3} = P_2V_3^{4/3}$, $P_2V_4^{4/3} = P_1V_1^{4/3}$ (1 point 1 分)

By definition 由定義

$$\frac{T_1}{T_2} = -\frac{Q_1}{Q_2} = -\frac{P_1(V_2 - V_1)}{P_2(V_4 - V_3)} = -\frac{P_1^{1/4}P_2^{3/4}(V_3 - V_4)}{P_2(V_4 - V_3)} = \frac{P_1^{1/4}}{P_2^{1/4}}\,.$$ (2 points 2 分)

Therefore, one may define the absolute temperature by $T = AP^{1/4}$, where A is an arbitrary constant. Since $T = 1$ when $P = 1$, $T = P^{1/4}$.

因此我們可以定義溫度為 $T = AP^{1/4}$，其中 A 為任意常數。由 $P = 1$ 時 $T = 1$，得 $T = P^{1/4}$。

(1 point 1 分)

(e) The internal energy is then 內能為 $U = 3T^4V$. (1 point 1 分)

Hence the heat capacity is 因此熱容量為 $C_V = \left(\frac{\partial U}{\partial T}\right)_V = 12T^3V$. (1 point 1 分)

(f) The entropy is 熵為 $S = \int_0^T C_V \frac{dT}{T} = 12V\int_0^T T^2 dT = 4T^3V = 4P^{3/4}V$. (2 points 2 分)

Part-II 第二卷

Q1 (16 points) 題 1（16 分） Solution 解：

(a) Consider a thin layer of gas of unit area and thickness *dr*. The pressure at *r* should be larger than the pressure at *r* + *dr* in order to balance the gravity $\frac{GMmn}{r^2}$. So we have $\frac{dP}{dr} = -\frac{GM_s mn}{r^2}$, where *n* is the molecular number density of the gas.

考慮離太陽 *r* 處一厚度爲 *dr* 的單位面積氣體。在 *r* 處的氣壓應比在 *r* + *dr* 處的大一點，從而平衡太陽的引力 $\frac{GMmn}{r^2}dr$ 。因此有 $\frac{dP}{dr} = -\frac{GM_s mn}{r^2}$ ，其中 *n* 爲氣體的分子數密度。(1 point 1 分)

We also have the ideal gas law $P = nkT_0$. Replace *P* with *n* we get $\frac{dn}{n} = -\frac{GM_s m}{kT_0}\frac{dr}{r^2}$.

另有理想氣體方程 $P = nkT_0$。將 *P* 用 *n* 代入，得 $\frac{dn}{n} = -\frac{GM_s m}{kT_0}\frac{dr}{r^2}$ 。(1 point 1 分)

Finally, $\rho = \rho_0 e^{\alpha/r}$, where $\alpha = \frac{GM_s m}{kT_0}$.

最後得 $\rho = \rho_0 e^{\alpha/r}$ ，其中 $\alpha = \frac{GM_s m}{kT_0}$ 。(1 point 1 分)

(b) When $r -> \infty$, $\rho -> \rho_0$ instead of zero. That means the gas ball is infinitely large, which is unphysical.

當 $r -> \infty$ ， $\rho -> \rho_0$ 而不是零。這意味著氣體球是無限大的，不符合實際情況。(2 points 2 分)

(c) The amount of energy per second through any concentric sphere shells should be constant. That is, $J_0 = 4\pi r^2 I$.

每秒鐘穿過任意一個同心圓殼的能量應爲常數。所以 $J_0 = 4\pi r^2 I$ 。(1 point 1 分)

Then 從而得 $\frac{J_0}{4\pi r^2} = I$. (2 points 2 分)

(d) $I(r) = -\sigma\frac{dT}{dr}$, so 因此 $\frac{dT}{dr} = -\frac{J_0}{4\pi\sigma r^2}$, (1 point 1 分)

Then 由此得 $T = \frac{J_0}{4\pi\sigma r}$.

The integral constant should be zero, as T should be zero at large distance.

因爲在無窮遠處溫度須爲零，所以由積分産生的常數必須爲零。(2 points 2 分)

(e) Again 重力和壓力平衡, $\frac{dP}{dr} = -\frac{GM_s mn}{r^2}$.

But now $P = nkT(r) = \frac{J_0}{4\pi\sigma r} kn$. Replace n by P in the first equation, we get $\frac{dP}{dr} = -\frac{4\pi GM_s m\sigma P}{kJ_0 r}$,

現在 $P = nkT(r) = \frac{J_0}{4\pi\sigma r} kn$ 。將 P 代入 n，得 $\frac{dP}{dr} = -\frac{4\pi GM_s m\sigma P}{kJ_0 r}$ (1 point 1 分)

which leads to $P = P_0\left(\frac{r}{r_0}\right)^{-\beta}$, where $\beta = \frac{4\pi GM_s m\sigma}{kJ_0}$.

從而得 $P = P_0\left(\frac{r}{r_0}\right)^{-\beta}$ ，其中 $\beta = \frac{4\pi GM_s m\sigma}{kJ_0}$ 。(1 point 1 分)

$\rho = \frac{4\pi\sigma mrP_0}{kJ_0}\left(\frac{r}{r_0}\right)^{-\beta}$. (1 point 1 分)

This time P and ρ go to zero at large r. 現在的 P 和ρ在無窮遠處爲零。

(f) From the surface temperatures of the planets we know today we estimate that r_0 is about the radius of the orbit of Mars.

由現在各行星的表面溫度我們可以推測大概和火星的軌道相近。(2 points 2 分)

Q2 (16 points) 題 2（16 分） Solution 解：

(a) The number of electrons crossing the junction per second is I/e.

每秒鐘通過界面的電子數爲 I/e。(1 point 1 分)

On average, there are $(\alpha - 0.5)I/e$ electrons flip their spins.

平均有$(\alpha - 0.5)I/e$的電子的自旋反轉。(1 point 1 分)

The net angular momentum change per second is then $(\alpha - 0.5)\frac{Ih}{4\pi e}\times 2$.

每秒鐘角動量的淨變化爲 $(\alpha-0.5)\dfrac{Ih}{4\pi e}\times 2$ 。(1 point 1 分)

This is equal to the torque so $\tau=(\alpha-0.5)\dfrac{h}{2\pi e}I$.

角動量的淨變化等于力矩 $\tau=(\alpha-0.5)\dfrac{h}{2\pi e}I$ 。(1 point 1 分)

(b) The equation of motion is $J\dfrac{d^2\theta}{dt^2}=-\eta\dfrac{d\theta}{dt}-\kappa\theta$.

導綫扭擺的運動方程爲 $J\dfrac{d^2\theta}{dt^2}=-\eta\dfrac{d\theta}{dt}-\kappa\theta$ 。(1 point 1 分)

Let $\theta(t)=\theta_0 e^{i\tilde{\omega}t}$, where $\tilde{\omega}$ is complex,

令 $\theta(t)=\theta_0 e^{i\tilde{\omega}t}$ ，其中 $\tilde{\omega}$ 是複數，we get 得(1 point 1 分)

$\tilde{\omega}^2-i\gamma\tilde{\omega}-\omega_0^2=0$, where 其中 $\omega_0^2\equiv\kappa/J$, $\gamma\equiv\eta/J$. (1 point 1 分)

Let $\tilde{\omega}=\omega_R+i\omega_I$ and solving the equation, we get $\theta=\theta_0 e^{-\omega_I t}e^{i\omega_R t}$,

令 $\tilde{\omega}=\omega_R+i\omega_I$ ，并解上述方程，得 $\theta=\theta_0 e^{-\omega_I t}e^{i\omega_R t}$ ，(1 point 1 分)

Where 其中 $\omega_I=\gamma/2$, (1 point 1 分) ， $\omega_R=\sqrt{\omega_0^2+\gamma^2/4}$. (1 point 1 分)

(c) This is a forced oscillation with the force given by $\tau(t)=(\alpha-0.5)\dfrac{h}{2e}I_0 e^{i\omega t}=\tau_0 e^{i\omega t}$.

這是個受迫振動問題，驅使力矩爲 $\tau(t)=(\alpha-0.5)\dfrac{h}{2e}I_0 e^{i\omega t}=\tau_0 e^{i\omega t}$ 。(1 point 1 分)

The equation of motion is 運動方程爲 $J\dfrac{d^2\theta}{dt^2}=-\eta\dfrac{d\theta}{dt}-\kappa\theta+\tau(t)$. (1 point 1 分)

Let 令 $\theta(t)=\theta_0 e^{i\omega t}$, (1 point 1 分)

we get the oscillation amplitude $\theta_0=\dfrac{\tau_0/J}{\omega_0^2-\omega^2+i\gamma\omega}$.

得振動幅度爲 $\theta_0=\dfrac{\tau_0/J}{\omega_0^2-\omega^2+i\gamma\omega}$ 。(1 point 1 分)

The speed of the side wing is 邊翼的速度爲 $v(t)=i\omega d\theta_0 e^{i\omega t}$, (1 point 1 分)

and the electromotive potential is 電動勢爲 $\xi(t)=BLv(t)=i\omega dBL\theta_0 e^{i\omega t}$ 。(1 point 1 分)

Q3 (18 points)　題 3（18 分）　Solution 解：

(a) The magnetic dipole experiences a torque $\vec{\tau} = \vec{m} \times \vec{B}_0$ which is always perpendicular to the $\vec{S} \sim \vec{z}_0$ plane. The torque will turn the direction of $\vec{S}$ so $\vec{S}$ rotates around $\vec{B}_0$ at constant angular speed.

磁偶極子受到的力矩爲$\vec{\tau} = \vec{m} \times \vec{B}_0$，其方向始終與$\vec{S} \sim \vec{z}_0$平面垂直。力矩改變$\vec{S}$的方向，因此$\vec{S}$繞著$\vec{B}_0$以勻角速度旋轉。(1 point 1 分)

Let the angle between $\vec{S}$ and $\vec{z}_0$ be θ. The torque is $mB_0 \sin\theta = \mu S B_0 \sin\theta$, while the change of angular momentum over time δt is $\delta S = S \sin\theta \delta\phi$.

令$\vec{S}$與$\vec{z}_0$之間的夾角爲θ。則力矩的大小爲$mB_0 \sin\theta = \mu S B_0 \sin\theta$，而角動量的變化爲$\delta S = S \sin\theta \delta\phi$。 (1 point 1 分)

Since 既然$S \sin\theta \delta\phi = \delta S = \mu S B_0 \sin\theta \delta t$ (1 point 1 分)

We have 我們得$\omega_0 = \dfrac{\delta\phi}{\delta t} = \mu B_0$. (1 point 1 分)

(b) In the reference frame rotating at angular velocity $\omega_0 \vec{z}_0 = -\mu B_0 \vec{z}_0$, the spin appears stationary.

在以角速度$\omega_0 \vec{z}_0 = -\mu B_0 \vec{z}_0$旋轉的參照系裏，自旋是不動的。(2 points 2 分)

(c) The effective B-field is 有效磁場爲$\vec{B}_\omega = -\dfrac{\vec{\omega}}{\mu}$. (2 points 2 分)

(d) In the rotating frame of $-\omega_1 \vec{z}_0$, $\vec{B}_1$ also appears static.

在以角速度$-\omega_1 \vec{z}_0$旋轉的參照系裏，$\vec{B}_1$是不動的。(1 point 1 分)

Let it be along the X' axis in the rotating frame, the total B-field is

$$\vec{B} = (B_0 - \frac{\omega_1}{\mu})\vec{z}_0^{'} + B_1 \vec{x}_0^{'}$$

令$\vec{B}_1$在旋轉參照系裏沿 X' 方向，則總磁場爲$\vec{B} = (B_0 - \dfrac{\omega_1}{\mu})\vec{z}_0^{'} + B_1 \vec{x}_0^{'}$。 (2 points 2 分)

(e) In this case, only $B_1\vec{x}_0^{'}$ remains. The spin will rotate around the $\vec{x}_0^{'}$ axis at angular speed $\omega_1 = \mu B_1$,

這時的磁場只剩下 $B_1\vec{x}_0^{'}$。自旋繞其以角速度 $\omega_1 = \mu B_1$ 旋轉， (2 points 2 分)

and the time to flip the spin is 倒轉自旋所需的時間爲 $t = \frac{1}{2}\frac{2\pi}{\omega_1} = \frac{\pi}{\mu B_1}$. (2 points 2 分)

(f) In this case the spin will rotate around the total B-field given by (c) at angular speed $\omega = \mu\sqrt{\left(B_0 - \frac{\omega}{\mu}\right)^2 + B_1^2}$.

這時的磁場由(c)給出。自旋的角速度爲 $\omega = \mu\sqrt{\left(B_0 - \frac{\omega}{\mu}\right)^2 + B_1^2}$ (3 points 3 分)

~~~~~~~~~ End 完 ~~~~~~~~
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