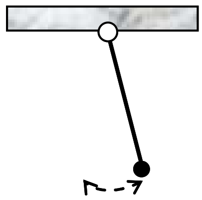
Pan Pearl River Delta Physics Olympiad 2011
2011 年泛珠三角及中华名校物理奥林匹克邀请赛
Part-1 (Total 6 Problems)　卷-1（共 6 题）
(9:00 am – 12:00 pm, 02-10-2011)

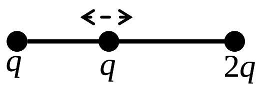
Math hints 数学提示:　　$(1+x)^{\alpha} \approx 1+\alpha x$ for $x << 1$, $\int x^n dx = \frac{1}{n+1}x^{n+1}, n \neq -1$.

Q1 (8 points) 题 1（8 分）

(a) A compound pendulum consists of a uniform rigid rod of mass *m* and length *L* and a small ball of mass *m* attached to the end of the rod, as shown in the figure. The upper end of the rod is attached to the joint on the ceiling about which the rod can rotate freely. Find the simple harmonic oscillation frequency of the pendulum. (4 points)
如图，一均匀硬杆，质量为 *m*，长度为 *L*，上端以光滑铰链系在天花板上，下端有一质量也为 *m* 的小球。求系统的简谐振动频率。（4 分）



(b) Two point charges *q* and 2*q* are fixed on a smooth horizontal rail separated by a distance *L*. A small cart carrying charge *q* and with mass *m* can move freely on the rail, as shown in the figure. Find the simple harmonic oscillation frequency of the cart. (4 points)
如图，两个点电荷 *q* 和 2*q* 分别固定在长为 *L* 的水平光滑导轨的两端。一质量为 *m* 的小车，带电 *q*，可在导轨上自由滑行。求小车的简谐振动频率。（4 分）

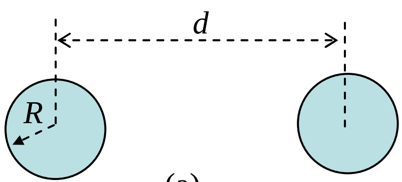


Q2 (9 points) 题 2（9 分）

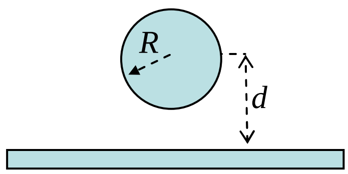
The mass of an electron at rest, in the unit of energy using the mass-energy equivalence relation, is 0.511×10^6 eV, or 0.511 MeV, and eV is electron-Volt. Find the momentum of an electron, in the unit of MeV/*c* (*c* is the speed of light in vacuum) and keep only two digits, when its kinetic energy is (i) 1.0×10^{-6} MeV, (ii) 1.0 MeV, and (iii) 1.0×10^6 MeV.
一静止电子的质量，若以质量-能量的等价关系而用能量为单位，为 0.511×10^6 eV, 或是 0.511 MeV。eV 是电子伏特的缩写。求电子有以下动能时的动量。（以 MeV/*c* 为单位， *c* 是真空中的光速。保留 2 位有效数字。）(i) 1.0×10^{-6} MeV, (ii) 1.0 MeV, and (iii) 1.0×10^6 MeV。

Q3 (8 points) 题 3（8 分）

(a) Find the capacitance per unit length between a pair of parallel cylindrical conductors with radius *R* and separated by a distance *d* (>> *R*), as shown in Fig. a. (b) A cylindrical conductor with radius *R* is placed at a distance *d* (>> *R*) above a grounded conductor plate, as shown in Fig. b. Find the capacitance per unit length between the two conductors.



(a)



(b)

(a) 如图a，两平行圆柱形导体半径为*R*，间距为*d* (>> *R*)。求单位长度两导体之间的电容。 (b) 如图b，一圆柱形导体半径为*R*，离一接地导体板距离为*d* (>> *R*)。求单位长度两导体之间的电容。

Q4 (10 points) 题 4（10 分）

This problem demonstrates why the energy density of gravitational field is negative.
本题演示为何重力场的能量密度是负的。

(a) First, consider two large parallel flat sheets separated by a distance D, one carrying uniform electric charge area density σ, and the other carrying –σ.
首先考虑两块平行大薄板，板之间的间距为 D，一板带均匀面电荷σ，另一板带均匀面电荷–σ。

(i) Find the electric force from one sheet exerted on a unit area of the other sheet. (1 point)
求一板对另一板单位面积的作用力。（1 分）

(ii) Find the electric field E near the sheets. (1 point)
求板附近的电场 E。（1 分）

(iii) Assume the total area of a plate is A. Let the separation distance D decrease by a small amount δD. Find the work done by the electric force. (1 point)
设板的面积为 A。令板的间距 D 减少一小量 δD，求电场作的功。（1 分）

(iv) Using the answer in (iii), find the energy density of the electric field and express the energy density in terms of E. (2 point)
利用(iii)的答案， 求电场的能量密度，并以电场 E 来表达。（2 分）

(b) Now, consider two neutral parallel flat sheets of uniform area mass density σ separated by a distance D.
现在考虑两块中性平行大薄板，板之间的间距为 D，质量面密度为σ。

(i) Find the gravitational force from one sheet exerted on a unit area of the other sheet. (1 point)
求一板对另一板单位面积的引力。（1 分）

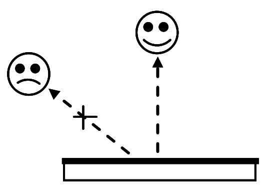
(ii) Find the gravitational field g near the sheets. (1 point)
求板附近的引力场 g。（1 分）

(iii) Assume the total area of a plate is A. Let the separation distance D decrease by a small amount δD. Find the work done by the gravitational force. (1 point)
设板的面积为 A。令板的间距 D 减少一小量 δD，求引力场作的功。（1 分）

(iv) Using the answer in (iii), find the energy density of the gravitational field and express the energy density in terms of g. (2 points)
利用(iii)的答案， 求引力场的能量密度，并以引力场 g 来表达。（2 分）

Q5 (5 points) 题 5（5 分）

A ‘magic’ transparent sheet can be placed onto the screen of a notebook computer to prevent peeping at large angles. Give a brief explanation of a possible working principle of such sheet.
将一‘神奇’透明膜贴在手提电脑的屏幕上，就可防止旁边的人偷看。试给出一个‘神奇’透明膜的工作原理。



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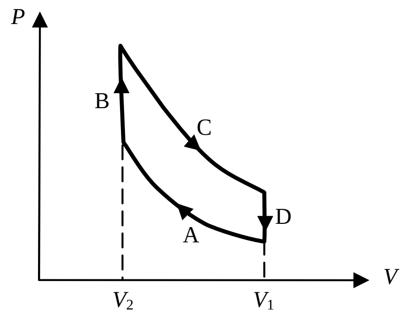
Q6 (10 points) 题 6（10 分）

The operation of an internal combustion engine in automobiles can be modeled by the following four-stroke cycle in the pressure-volume (P-V) diagram as shown in the figure:

A) Adiabatic compression from V_1 to V_2.
B) Heating raising the temperature from T_1 to T_2 at constant volume.
C) Adiabatic expansion from V_2 to V_1, producing mechanical work.
D) Cooling at constant volume lowering the temperature from T_3 to T_4.

Let n be the number of moles of gas operated in a cycle, C_V and C_P be the molar specific heat of the gas at constant volume and constant pressure, respectively. Also let $\gamma = C_P/C_V$.

(a) Express the temperature ratios T_2/T_3 and T_1/T_4 in terms of the volume ratio V_1/V_2. (2 points)
(b) Let Q be the heat absorbed by the gas in the heating stroke, and W_1 and W_3 be the work done by the gas in the compression and expansion strokes respectively. Calculate the ratios W_1/Q and W_3/Q. (6 points)
(c) Find the efficiency of the engine in terms of V_1, V_2, and γ. (2 points)



汽车内燃机的运转过程可用以上的四冲程过程在压强-体积(P-V)图上表述:

A) 从 V_1 到 V_2 的绝热压缩过程。
B) 等体积燃烧过程将气体温度从 T_1 提升到 T_2 。
C) 从 V_2 到 V_1 的绝热膨胀过程，并作功。
D) 等体积冷却过程将气体温度从 T_3 降到 T_4。

设 n 为气体的摩尔量，C_V 和 C_P 分别为单位摩尔的等体积和等压热容量， $\gamma = C_P/C_V$。

(a) 将温度比 T_2/T_3 和 T_1/T_4 以体积比 V_1/V_2 来表达。（2 分）
(b) 设 Q 为等体积燃烧过程中气体吸的热量，W_1 和 W_3 分别为气体在压缩和膨胀过程中作的功，求功热比 W_1/Q 、W_3/Q。（6 分）
(c) 求内燃机的效率，并以 V_1、V_2 、γ 来表达。（2 分）

《THE END 完》

Pan Pearl River Delta Physics Olympiad 2011
2011 年泛珠三角及中华名校物理奥林匹克邀请赛
Part-2 (Total 3 Problems) 卷-2（共 3 题）
(2:30 pm – 5:30 pm, 02-10-2011)

Math hints 数学提示: $\int \cos(x)dx = \sin(x)$; $(1+x)^{\alpha} \approx 1+\alpha x$ for $x << 1$.

Q1 Neutrino Oscillation (15 points) 题 1 中微子振荡（15 分）

(a) Neutrinos are very light particles with masses below 10 eV. Estimate the speed of a neutrino with total energy of 10^8 eV in the unit of the speed of light in vacuum *c*. (3 points)
中微子是具有很小质量（10 电子伏以下）的粒子。估算一个总能量为 10^8 电子伏的中微子的速度（以真空中光速 *c* 为单位）。（3 分）

(b) Neutrinos have one fascinating property that no other particles have, namely they keep changing continuously from one type to another and changing back as time goes by. For example, an electron neutrino (state-1) will change to a muon neutrino (state-2) and change back over a time period of *T*. Its mass will change from m_1 in state-1 to m_2 in state-2. This phenomenon is called ***neutrino oscillation***. Quantum mechanics says that the period *T* is determined by $T = \dfrac{h}{|E_1 - E_2|}$, where E_1 and E_2 are the total energy of the neutrino in state-1 and state-2, respectively, $|E_2 - E_1|$ is the absolute value of the energy difference, and *h* is the Planck constant. For a neutrino of **fixed momentum** *p*, find *T* in terms of m_1, m_2, *h*, *p*, and *c*. (7 points)
中微子有个很有趣的独特性质。随着时间的流逝，它可以逐渐从一种中微子变成另一种中微子，再逐渐变回原来的中微子，周而复始，永不停止。比如，电子中微子（状态-1）可以逐渐变成渺子中微子（状态-2），再逐渐变回成电子中微子。这一现象称为中微子振荡。一个循环所需的时间，即振荡周期，为 *T*。根据量子力学，振荡周期为 $T = \dfrac{h}{|E_1 - E_2|}$，其中 E_1 和 E_2 分别为中微子在状态-1 和状态-2 的总能量，$|E_2 - E_1|$ 是两者之差的绝对值，*h* 为普郎克常数。**若中微子的动量 *p* 为恒定值**，m_1、m_2、*h*、*p*、*c* 为已知，求振荡周期 *T*。（7 分）

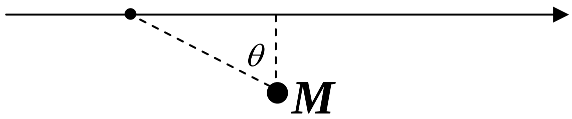
(c) Suppose electron neutrinos with energy of 10^8 eV change to muon neutrinos after traveling through the Earth, (You need to recall roughly the order of magnitude of the Earth diameter.) estimate the minimum value $|m_1^2 - m_2^2|$ in the unit of $(\text{eV})^2$. You may find the constant $hc = 1.24\times10^{-6}\ eV\cdot m$ useful. (5 points)
若能量为 10^8 电子伏的电子中微子在穿过地球的过程中变成了渺子中微子，估求 $|m_1^2 - m_2^2|$ 的最小值（以$(\text{电子伏})^2$为单位）。你需要自己给出地球直径的近似值，并可能会用到 $hc = 1.24\times10^{-6}\ eV\cdot m$。（5 分）

Q2 Bending Light with Gravity (15 points) 题 2 引力场里弯曲的光线（15 分）

(a) The path of light bent by gravitational field is an effect of General Relativity. However, one can analysis such phenomenon and obtain an approximate answer using ***Newtonian mechanics*** by assuming that a light beam is made of photons traveling at speed *c*, with the effective mass of each photon being equal to $m = E/c^2$, where *E* is the energy of the photon. Let us first consider the case of a classical particle flying by a fixed point mass *M* at high initial speed *v*. The trajectory of the particle will be bent due to the gravity of the point mass. Because of the high speed of the particle, the bending angle is very small and can be approximately calculated in the following way.

根据广义相对论，光线在引力场里会弯曲。若把光束看作是以光速 c 运动的具有能量 E 的光子，并赋予光子有效质量 $m = E / c^2$，则此现象也可以用***牛顿力学***来解释，并得到近似的答案。首先，我们考虑一初始速度 v 很高的经典粒子，飞过一质量为 M 的固定质点。由于引力作用，粒子的轨迹会弯曲。但由于粒子的初速度很高，弯曲的角度很小，因此可用以下的近似方法求得。

(i) Assume that the trajectory of the particle remains straight, as if the point mass was not there. The distance from the trajectory to the fixed point mass is b, as shown in the figure. Calculate the total impulse the particle receives from the point mass as it travels from the far left to the far right in terms of M, b, G the universal gravity constant, and m the mass of the particle. You may find it easier to use the angle θ as the integration variable. (4 points)



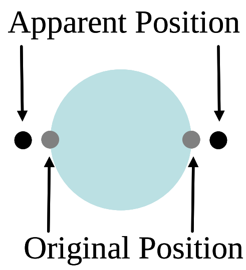
如图所示，假设粒子的轨迹一直为直线，不受引力的影响，轨迹离固定质点的距离为 b，求粒子从左到右飞过固定质点的过程中固定质点对粒子的冲量。在冲量的表达式里可能要用到引力常数 G 和粒子质量 m。用角度 θ 作变量可能简化积分运算。（4 分）

(ii) Determine the bending angle of the particle. (1 point)
求粒子轨迹的弯曲角度。（1 分）

(b) The angular distance of two stars in the sky happens to be the same as the sun's angular diameter.
天空中有两颗星的角距离刚好和太阳的角直径一样。

(i) The sun-earth distance is 1.5×10^{11} m, and the sun's diameter is 1.4×10^{9} m. Calculate the sun's angular diameter in the sky. (1 point)
已知日-地距离为 1.5×10^{11} 米，太阳的直径为 1.4×10^{9} 米。求太阳的角直径。（1 分）

(ii) When the sun happens to be right between the two stars in the sky, the stars appear to be 'pushed outwards' by the sun, as shown in the figure. Draw a diagram to briefly explain the phenomenon. (2 points)
当太阳刚好位于这两颗星的中间时，两颗星好象从它们原来的位置(Original Position) 被推到现在观察到的视觉位置(Apparent Position)。作简图解释此现象。（2 分）



(iii) Calculate the apparent angular distance between the two stars in (ii). The mass of the sun is 2.0×10^{30} kg. The speed of light in vacuum is $c = 3.0 \times 10^8 m/s$. The universal gravity constant is $G = 6.67 \times 10^{-11} N \cdot m^2 / kg^2$. (5 points)
求(ii)中观察到的两星之间的视觉角距离。已知太阳质量为 2.0×10^{30} kg，真空光速为 $c = 3.0 \times 10^8 m/s$，引力常数为 $G = 6.67 \times 10^{-11} N \cdot m^2 / kg^2$。（5 分）

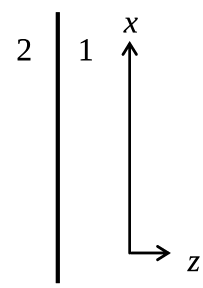
(iv) With the present technology, why the phenomenon in (ii) can only be observed during a total solar eclipse? (1 point)
为何上述现象用现有的技术只有在日全食时才能观察到？（1 分）

(v) If the moon's orbit size were doubled, could we still observe total solar eclipse? (1 point)
若月球的轨道半径为现在的两倍，还会有日全食现象吗？（1 分）

Q3　Magnetic monopoles (20 points)　题 3　磁单极子（20 分）

The uniqueness theorem of the theory of electric and magnetic fields ensures that virtual (image) charges could be placed ***outside*** the closed space in which the fields are to be determined to mimic the original boundary conditions. The fields ***in*** the closed space are then the same as the ones generated by the real charges inside the space and the virtual charges ***outside*** the space. For an interface without free charge and free electric current, the boundary conditions for electric field $\vec{E}$, electric displacement $\vec{D}$, magnetic field $\vec{B}$, and auxiliary field $\vec{H}$ are $\vec{E}_1^{//} = \vec{E}_2^{//}$, $\vec{D}_1^{\perp} = \vec{D}_2^{\perp}$, $\vec{B}_1^{\perp} = \vec{B}_2^{\perp}$, $\vec{H}_1^{//} = \vec{H}_2^{//}$. Here '⊥'means the components perpendicular to the interface, and '//' means the components parallel to the interface.

利用电、磁场理论的唯一性定理，我们可在一闭合空间外用虚拟电荷分布来产生和原来相同的边界条件，并用这些虚拟电荷分布和空间**内**原来的电荷分布求出闭合空间内的电、磁场。在一个无自由电荷、自由电流的界面，电场 $\vec{E}$、电位移矢量 $\vec{D}$、磁场 $\vec{B}$、磁辅助场 $\vec{H}$ 的边界条件分别为 $\vec{E}_1^{//} = \vec{E}_2^{//}$, $\vec{D}_1^{\perp} = \vec{D}_2^{\perp}$, $\vec{B}_1^{\perp} = \vec{B}_2^{\perp}$, $\vec{H}_1^{//} = \vec{H}_2^{//}$。这里'⊥'、'//'分别代表垂直、平行于界面的分量。



(a)　Consider the case of a point charge q inside medium-1 at a distance d from the interface between two ordinary dielectric media with dielectric constants ε_1 and ε_2, respectively. The Z-axis is perpendicular to the interface, the point charge q is on the Z-axis, and the interface is at z = 0. Two image charges q_1 and q_2 are used to solve this problem. In medium-2, the electric field is equal to what is generated by a point charge of $(q_1 + q/\varepsilon_1)$ at position (0, 0, d). The field in medium-1 is equal to the total field by a point charge q/ε_1 at position (0, 0, d) *and* a point charge q_2 at position (0, 0, $-d$).

已知介质-1、介质-2 为介电常数分别为 ε_1、ε_2 的普通绝缘体。一点电荷 q 在介质-1 内，离两介质之间的界面的距离为 d。坐标 Z 轴与界面垂直，点电荷 q 在 Z 轴上，界面位置为 z = 0。求解本题需要用到两个虚拟电荷 q_1、q_2。在介质-2 内的电场由在(0, 0, d)的点电荷 $q_1 + q/\varepsilon_1$ 产生；在介质-1 内的电场由在(0, 0, d)的点电荷 q/ε_1 和在(0, 0, $-d$)的点电荷 q_2 共同产生。

(i)　Apply the above boundary conditions to determine q_1 and q_2. (3 points)
利用上述边界条件，求 q_1、q_2。（3 分）

(ii)　Determine the ***total*** surface charge density at the interface. (1 point)
求界面上的**总**电荷面密度。（1 分）

(iii)　Verify your answer with the special condition $\varepsilon_1 = \varepsilon_2$. (1 point)
用特殊情况 $\varepsilon_1 = \varepsilon_2$ 来验证你的答案。（1 分）

(iv)　Verify your answer with the special condition $\varepsilon_1 << \varepsilon_2$. (1 point)
用特殊情况 $\varepsilon_1 << \varepsilon_2$ 来验证你的答案。（1 分）

(b)　A magnetic dipole is made of a south pole and a north pole. Interestingly, so far no magnetic monopoles, namely the objects carrying only a south pole or a north pole, have ever been discovered. If a magnetic monopole of 'magnetic charge' g ever exists, it will generate a magnetic field in the same way as an electric point charge generates an electric field. Recently, pseudo particles behaving like magnetic monopoles generated by the collective motions of electrons in a particular type of materials, called the 'topological insulators', was theoretically predicted. Such particle can be induced by an external electric point charge. Similar to part-a, a point electric charge q is placed at a distance d from the interface between an ordinary dielectric medium (medium-1) with dielectric constant ε_1 and magnetic permeability μ_1, and a topological insulator (medium-2) with dielectric constant ε_2, magnetic permeability μ_2, and a magneto-electric coupling constant β. The relations between various fields in medium-1 are $\vec{D}_1 = \varepsilon_1 \vec{E}_1$, and $\vec{H}_1 = \frac{\vec{B}_1}{\mu_1}$.Those in medium-2 are $\vec{D}_2 = \varepsilon_2 \vec{E}_2 - \beta \vec{B}_2$, and $\vec{H}_2 = \frac{\vec{B}_2}{\mu_2} + \beta \vec{E}_2$. The electric field and the magnetic field in both media can again be found by using image charges as in part-a. For simplicity, let us use a set of specially chosen units such that the electric field at position $\vec{r}$ generated by a point electric charge q at position $\vec{r}_0$ is $\vec{E}(\vec{r}) = q\frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|^3}$, and the magnetic field produced by a magnetic monopole of magnetic charge g in a similar spatial setting is $\vec{B}(\vec{r}) = g\frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|^3}$. In medium-2, the electric field is generated by a point electric charge of $(q_1 + q/\varepsilon_1)$ at position (0, 0, d), and the magnetic field is generated by a magnetic monopole with charge g_1 at position (0, 0, d). In medium-1, the electric field is generated by a point electric charge of q/ε_1 at position (0, 0, d) *and* a point charge q_2 at position (0, 0, $-d$), and the magnetic field is generated by a magnetic monopole with charge g_2 at position (0, 0, $-d$).

磁偶极子由一对南、北极组成。有趣的是，到现在为止我们还没能找到只带南极或北极的所谓磁单极子。一个带‘磁荷’ g 的磁单极子产生的磁场和一个点电荷产生的电场的表达式是一样的。最近，有理论预言，在一种叫拓扑绝缘体（topological insulator）的物质里，电子的运动可以产生象磁单极子这样的准粒子。这种粒子可由点电荷感应产生。与(a)部分类似，一点电荷 q 在普通的介质-1 内，离介质-1 与拓扑绝缘体（介质-2）的界面的距离为 d。介质-1 的介电常数为 ε_1，磁化率为 μ_1。介质-2 的介电常数为 ε_2，磁化率为 μ_2，电磁偶合系数为 β。介质-1 内各个场之间的关系为 $\vec{D}_1 = \varepsilon_1 \vec{E}_1$，$\vec{H}_1 = \frac{\vec{B}_1}{\mu_1}$。介质-2 内各个场之间的关系为 $\vec{D}_2 = \varepsilon_2 \vec{E}_2 - \beta \vec{B}_2$，$\vec{H}_2 = \frac{\vec{B}_2}{\mu_2} + \beta \vec{E}_2$。与(a)部分类似，介质里的电、磁场也能用虚拟电、磁荷来求解。为方便起见，我们选用一套特殊的单位，使一位于 $\vec{r}_0$ 的点电荷 q 在 $\vec{r}$ 处的电场为 $\vec{E}(\vec{r}) = q\frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|^3}$，而磁荷 g 产生的磁场为 $\vec{B}(\vec{r}) = g\frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|^3}$。在介质-2 内的电场由位于(0, 0, d)的电荷 $q_1 + q/\varepsilon_1$ 产生，磁场由位于(0, 0, d)的磁荷 g_1 产生。在介质-1 内的电场由位于(0, 0, d)的电荷 q/ε_1 和位于(0, 0, $-d$)的电荷 q_2 共同产生，磁场由位于(0, 0, $-d$)的磁荷 g_2 产生。

(i) Use the boundary conditions to determine q_1, q_2, g_1, and g_2. (10 points)
利用边界条件，求 q_1、q_2、g_1、g_2。（10 分）

(ii) Find the **total** surface electric charge density at the interface. To shorten the answers, you may treat q_1 and g_1 as known. (1 point)
求界面上的**总**电荷面密度。为简化起见，可把 q_1、g_1 当作已知。（1 分）

(iii) Find the **total** electric current density at the interface. As the electric current density is a vector, you should find both the components parallel and perpendicular to the interface. To shorten the answers, you may treat q_1 and g_1 as known. (3 points)
求界面上的**总**电流密度。由于电流密度是矢量，你必须给出电流密度垂直和平行于界面的分量。为简化起见，可把 q_1、g_1 当作已知。（3 分）

《THE END 完》