Note: The marking scheme is intended for grading when the final answer is wrong. If the final answer is correct, full mark should be given regardless of the procedure.

PART I

Q1 (8 points)

In a general variable q, if the potential energy $U_p = \frac{1}{2}aq^2$, and the kinetic energy $U_k = \frac{1}{2}b\dot{q}^2$,

then we have frequency $\omega = \sqrt{\frac{a}{b}}$.

(a)

$U_p = mgL(1-\cos\theta) + mg\frac{L}{2}(1-\cos\theta) = \frac{3}{2}mgL(1-\cos\theta) = \frac{1}{2}(\frac{3}{2}mgL)\theta^2$, (1 point)

$U_k = \frac{1}{2}mL^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2 = \frac{1}{2}(\frac{4}{3}mL^2)\dot{\theta}^2$, here $I = \frac{1}{3}mL^2$. (1 point)

Then $a = \frac{3}{2}mgL$, and $b = \frac{4}{3}mL^2$, then $\omega = \sqrt{\frac{a}{b}} = \frac{3}{2\sqrt{2}}\sqrt{\frac{g}{L}}$. (2 points)

Alternative solution:

$I = mL^2 + \frac{1}{3}mL^2 = \frac{4}{3}mL^2$. (1 point)

Torque $\tau = mgL\theta + \frac{1}{2}mgL\theta = \frac{3}{2}mgL\theta = K\theta$. (1 point)

$\omega = \sqrt{\frac{K}{I}} = \sqrt{\frac{3\cdot 3mgL}{2\cdot 4mL^2}} = \frac{3}{2\sqrt{2}}\sqrt{\frac{g}{L}}$. (2 points)

(b)

$$U_p = \frac{q^2}{4\pi\epsilon_0}\left(\frac{1}{x+\delta x} + \frac{2}{L-x-\delta x}\right)$$

$$\simeq \frac{q^2}{4\pi\epsilon_0}\left[\left(\frac{1}{x} + \frac{2}{L-x}\right) - \left(\frac{1}{x^2} - \frac{2}{(L-x)^2}\right)\delta x + \left(\frac{1}{x^3} + \frac{2}{(L-x)^3}\right)\delta x^2\right].\ \text{(1 point)}$$

By properly choosing zero-energy as $\frac{q^2}{4\pi\epsilon_0}\left(\frac{1}{x_0} + \frac{2}{L-x_0}\right)$, and making coefficient of δx zero

to find the balance point x_0, i. e., $\frac{1}{x_0^2} - \frac{2}{(L-x_0)^2} = 0 \Rightarrow x_0 = (\sqrt{2}-1)L$, we have

$$U_p = \frac{q^2}{4\pi\epsilon_0}\left(\frac{1}{\left(\sqrt{2}-1\right)^3 L^3} + \frac{2}{\left(L-\left(\sqrt{2}-1\right)L\right)^3}\right)\delta x^2 = \frac{(24+17\sqrt{2})q^2}{8L^3\pi\epsilon_0}\delta x^2.\ \text{(1 point)}$$

$U_k = \frac{1}{2}m\delta\dot{x}^2$. (1 point)

Then, $a = \frac{(24+17\sqrt{2})q^2}{4\pi L^3\epsilon_0}$ and $b = m$. $\omega = \sqrt{\frac{a}{b}} = \left[\frac{(24+17\sqrt{2})q^2}{2L^3\pi\epsilon_0 m}\right]^{\frac{1}{2}} = 1.96\sqrt{\frac{q^2}{L^3 m\epsilon_0}}$. (1 point)

Alternative solution

Force $F = \frac{q^2}{4\pi\epsilon_0}\left(\frac{1}{x^2} - \frac{2}{(L-x)^2}\right)$. (1 point)

Balance point at $F = 0$, $x_0 = (\sqrt{2}-1)L$. (1 point)

The elastic constant $K=-\frac{dF}{dx}|_{x=x_0}=\frac{q^2}{2\pi\epsilon_0 L^3}\left(\frac{1}{(\sqrt{2}-1)^3}+\frac{2}{(2-\sqrt{2})^2}\right)\approx 3.82\frac{q^2}{\epsilon_0 L^3}$. (1 point)

$\omega=\sqrt{\frac{K}{m}}=1.95\sqrt{\frac{q^2}{\epsilon_0 m L^3}}$. (1 point)

Q2 (9 points)

Relativistic total energy is $E=\sqrt{p^2c^2+m^2c^4}$ (1 point)

Here mc^2 is the rest energy. The kinetic energy is given by $K=\sqrt{p^2c^2+m^2c^4}-mc^2$. (1 point)

So the momentum is $p=\frac{1}{c}\sqrt{(K+mc^2)^2-m^2c^4}=\frac{1}{c}\sqrt{K^2+2Kmc^2}$. (1 point)

(i) $K<<mc^2$, so

$p\approx\frac{1}{c}\sqrt{2mc^2K}=\frac{1}{c}\sqrt{2\times0.511\times10^{-6}}=1.0\times10^{-3}MeV/c$. (2 points)

(ii) $p=\frac{1}{c}\sqrt{K^2+2mcK}=\frac{1}{c}\sqrt{1+2\times0.511}=1.4MeV/c$. (2 points)

(iii) $K>>mc^2$. $p\approx\frac{K}{c}=1.0\times10^6 MeV/c$. (2 points)

Q3 (8 points)

(a) Let one cable carry line charge density λ, and the other carry $-\lambda$. Choose the line joining the cable centers as the X-axis and choose $x=0$ at the middle point between the cables, the total electric field on the X-axis is $E=\frac{\lambda}{2\pi\epsilon_0}\left(\frac{1}{d/2+x}+\frac{1}{d/2-x}\right)$. (1 point)

The voltage difference between the two cables is

$V=\int_{-(d/2-R)}^{d/2-R}E\cdot dx=\frac{\lambda}{2\pi\epsilon_0}\left[\ln\left(\frac{d}{2}+x\right)-\ln\left(\frac{d}{2}-x\right)\right]_{-(d/2-R)}^{d/2-R}=\frac{\lambda}{\pi\epsilon_0}\ln\left(\frac{d-R}{R}\right)\approx\frac{\lambda}{\pi\epsilon_0}\ln\left(\frac{d}{R}\right)$. (2 points)

$C=\frac{Q}{V}=\pi\epsilon_0/\ln\left(\frac{d}{R}\right)$. (1 point)

(b) Employ the method of image charge, the electric field is $E=\frac{\lambda}{2\pi\epsilon_0}\left(\frac{1}{d+x}+\frac{1}{d-x}\right)$. The voltage difference is (1 point)

$V=\int_{-(d-R)}^{0}E\cdot dx=\frac{\lambda}{2\pi\epsilon_0}\left[\ln(d+x)-\ln(d-x)\right]_{-(d-R)}^{0}=\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{2d-R}{R}\right)\approx\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{2d}{R}\right)$ (2 points)

$C=2\pi\epsilon_0/\ln\left(\frac{2d}{R}\right)$. (1 point)

Q4 (10 points)

(a)

(i) The force of plate-1 on plate-2 is determined by the field generated by plate-1 only. Apply Gauss law to the conducting plate, with a pillbox boundary for $\oint_S \vec{E}\cdot d\vec{S}=\frac{Q}{\varepsilon_0}$.

Since we are only considering the field of one plate, which is the same on both side of the plate, the field is then $\vec{E}=\frac{\sigma}{2\varepsilon_0}\vec{z}_0$. (1 point)

One can also use integration to calculate the field directly.

$$E = \frac{\sigma}{4\pi\varepsilon_0}\int_0^\infty \frac{2\pi rL dr}{(L^2+r^2)^{3/2}} = \frac{-\sigma}{2\varepsilon_0}\frac{L}{(L^2+r^2)^{1/2}}\Big|_0^\infty = \frac{\sigma}{2\varepsilon_0}.$$

The force per unit area is $\vec{F} = \frac{\sigma^2}{2\varepsilon_0}\vec{z}_0$. (1 point)

(ii) The total electric field is the superposition of the field generated by the two plates. The field between the plates is $\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{z}_0$. Outside the plates the field is zero. (1 point)

(iii) The work done on an area A of the plate is

$\delta W = AF\cdot\delta D = \frac{\sigma^2 A}{2\varepsilon_0}\delta D$. (1 point)

(iv) The volume of the space where the field is non-zero is decreased by $A\delta D$. Let the energy density of the field by H. Then the energy stored in the field is decreased by $HA\delta D$. By energy conservation, $HA\delta D = \frac{\sigma^2 A}{2\varepsilon_0}\delta D$. So the energy density is given by $H = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2$, which is positive. (2 points)

(b)

(i) Gauss law for gravitational field $\oint\!\!\oint_S \vec{g}\cdot d\vec{S} = 4\pi GM$

In the same way as part (a), we have $\vec{g} = 2\pi G\sigma\vec{z}_0$ (1 point)

The force per unit area is $\vec{F} = 2\pi G\sigma^2\vec{z}_0$. (1 point)

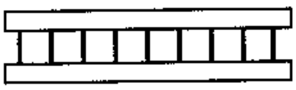
(ii) The field in the space between the plates is zero. Outside the field is $g = 4\pi G\sigma$

(iii) $\delta W = AF\cdot\delta D = 2\pi G\sigma^2 A\delta D$. (1 point)

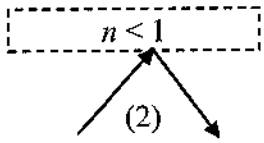
(iv) The volume of space where the field is non-zero is ***increased*** by $A\delta D$. The energy stored in the field is decreased by $-HA\delta D$. So $H = -2\pi G\sigma^2 = -2\pi G\frac{g^2}{(4\pi G)^2} = -\frac{1}{2}\frac{g^2}{4\pi G}$. (2 points)

Q5 (5 points)

A homogenous sheet with $n > 1$ is incorrect. Possible solutions are given below. Any other ones that are correct in principle should also be given full mark.



(1)



(2)

Q6 (10 points)

(a) $PV^\gamma = C \Rightarrow NKTV^{\gamma-1} = C$, (1 point)

Then $T_2/T_3 = T_1/T_4 = (V_1/V_2)^{\gamma-1}$. (1 point)

(b) Internal energy of ideal gas is $U = N\epsilon_0 + Nc_V T$, (1 point)

Then $Q = \Delta U = Nc_V(T_2 - T_1)$ and $c_V = \frac{k}{\gamma-1}$. (1 point)

Work is

$W = \int P dV = \int CV^{-\gamma} dV = \frac{C}{1-\gamma}V^{1-\gamma}$. (1 point)

For path A: $C = C_A = P_1V_2^\gamma = NkT_1V_2^{\gamma-1}$, and for path C: $C = C_C = P_2V_2^\gamma = NkT_2V_2^{\gamma-1}$. Then we have

$$W_A = \frac{C_A}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right) = \frac{Nk}{1-\gamma}\left(1-\left(\frac{V_1}{V_2}\right)^{1-\gamma}\right)T_1 = \frac{Nk}{1-\gamma}\left(T_1 - T_4\right) \text{ (1 point)}$$

$$W_C = -\frac{C_C}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right) = -\frac{Nk}{1-\gamma}\left(1-\left(\frac{V_1}{V_2}\right)^{1-\gamma}\right)T_2 = -\frac{Nk}{1-\gamma}\left(T_2 - T_3\right). \text{ (1 point)}$$

Then

$W_A / Q = -(T_1 - T_4)/(T_2 - T_1)$, and $W_C / Q = (T_2 - T_3)/(T_2 - T_1)$. (1 point)

(c)

$$\eta = \frac{W_A + W_C}{Q} = \frac{(T_2 + T_4) - (T_1 + T_3)}{T_2 - T_1}, \text{ (1 point)}$$

or $\eta = \dfrac{k}{c_V(1-\gamma)}\left(1-\left(\dfrac{V_1}{V_2}\right)^{1-\gamma}\right) = \left(1-\left(\dfrac{V_1}{V_2}\right)^{1-\gamma}\right)$. (1 point)

PART II

Question 1 (10 points)

Solution:

(a) The total energy is much larger than the rest energy. So the answer is '1', i. e., the neutrino is moving at speed very close to that of light. (3 points)

(b)

$E_1 - E_2 = \sqrt{p^2c^2 + m_1^2c^4} - \sqrt{p^2c^2 + m_2^2c^4}$ (1 point)

$\approx pc(1 - \frac{1}{2}\frac{m_1^2c^4}{pc}) - pc(1 - \frac{1}{2}\frac{m_2^2c^4}{pc}) = \frac{1}{2}\frac{(m_1^2 - m_2^2)c^4}{pc} = \frac{1}{2}\frac{\Delta m^2c^4}{pc}$ (4 points)

$T = \dfrac{h}{|E_1 - E_2|} = \dfrac{2Pch}{\Delta m^2c^4}$. (2 points)

If the expansion is done in (c) instead in (b), full mark should still be given to (b).

(c) From expression of T in (b), we have $\Delta m^2 = \dfrac{2Pch}{Tc^4}$, (1 point)

Substitute $P = E/c$ (1 point) into above: $\Delta m^2c^4 = \dfrac{2hcE}{Tc}$.

On the other hand, $cT/2 = 2R_{\text{earth}}$, (1 point) and $R_{\text{earth}} = 6\times10^6 m$. (1 point)

Then $\Delta m^2c^4 = \dfrac{2hcE}{4R_{\text{earth}}} = \dfrac{100\times10^6\times1.24\times10^{-6}}{2\times6\times10^6} = (1.0\pm0.5)\times10^{-5}\ \text{eV}^2$. (1 point)

Note: Any answer within the given range should get full mark.

Question 2 (15 points)

Solution:

(a)

(i) The impulse perpendicular to the path is $I_\perp = \int F_\perp(t)dt = \int F(t)\cos\theta(t)dt$. (1 point)

$\tan\theta = x/b$ and $x = vt$, we then have $dt = \dfrac{1}{v}dx = \dfrac{b}{v}d(\tan\theta) = \dfrac{b}{v}\dfrac{1}{\cos^2\theta}d\theta$. (1 point)

Then $I = \int_{-\pi/2}^{\pi/2}\dfrac{GMmb}{b^2v}\cos\theta d\theta = \dfrac{2GMm}{bv}$. (1 point)

The impulse along the path is

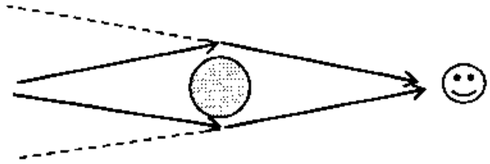
$I_{//} = \int F_{//}(t)dt = \int F(t)\sin\theta(t)dt = \dfrac{GMmb}{b^2v}\int_{-\pi/2}^{\pi/2}\sin\theta d\theta = 0$ (1 point)

(ii) $p = mv$, then $\theta \simeq \frac{I_\perp}{p} = \frac{2GM}{bv^2}$. (1 point)

(b)

(i) The angular diameter of the sun is $\theta_{sun} = R_{sun} / d = \frac{1.4\times10^6}{1.5\times10^8} = 9.3\times10^{-3}$. (1 point)

(ii) Correct drawing (2 points)



(iii)

$$\delta\theta = \frac{2GM}{Rc^2} = \frac{2\times6.67\times10^{-11}\times2\times10^{30}}{1.4\times10^6\times(3\times10^8)^2} = 2.1\times10^{-3},$$

(4 points)

The angular distance is then $\theta = 2\theta_{sum} + 2\delta\theta = 2.280\times10^{-2}$. (1 point)

(iv) No, because the sun is too bright. (1 point)

(v) No, because the angular diameter of the moon would only be half of the sun. (1 point)

Q3 (20 points)

(a)

(i) $\vec{E}_1 = \frac{x\vec{x}_0 + y\vec{y}_0 + (z-d)\vec{z}_0}{\left[x^2+y^2+(z-d)^2\right]^{3/2}}\frac{q}{\varepsilon_1} + \frac{x\vec{x}_0 + y\vec{y}_0 + (z+d)\vec{z}_0}{\left[x^2+y^2+(z+d)^2\right]^{3/2}} q_2, z > 0.$ (0.5 points)

$\vec{E}_2 = \frac{x\vec{x}_0 + y\vec{y}_0 + (z-d)\vec{z}_0}{\left[x^2+y^2+(z-d)^2\right]^{3/2}}\left(\frac{q}{\varepsilon_1} + q_1\right), z < 0.$ (0.5 points)

Full marks should be given if $\frac{1}{4\pi\varepsilon_0}$ appears in the above answers.

$\vec{D}_2 = \varepsilon_2\vec{E}_2$, $\vec{D}_1 = \varepsilon_1\vec{E}_1$

$\vec{D}_1^\perp = \vec{D}_2^\perp \Rightarrow \varepsilon_1 q_2 - q = -\varepsilon_2\left(\frac{q}{\varepsilon_1} + q_1\right)$ (1) (0.5 points)

$\vec{E}_1^{//} = \vec{E}_2^{//} \Rightarrow q_2 = q_1$ (2) (0.5 points)

Solving (1) and (2), $q_1 = q_2 = \left(\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}\right)\left(\frac{q}{\varepsilon_1}\right)$. (1 point)

(ii) $\sigma = 4\pi(E_1^\perp - E_2^\perp) = 4\pi\left(\frac{d}{\left[x^2+y^2+d^2\right]^{3/2}}\left(q_2 - \frac{q}{\varepsilon_1} + \frac{q}{\varepsilon_1} + q_1\right)\right) = \frac{8\pi d q_1}{\left[x^2+y^2+d^2\right]^{3/2}}$. (1 point)

Full mark should be given if ε_0 instead of 4π is given.

(iii) $\varepsilon_1 = \varepsilon_2$, $q_2 = q_1 = 0$. No interface and no image charge. (1 point)

(iv) $q_1 = q_2 = -\frac{q}{\varepsilon_1}$. $\vec{E}_2 = 0$, no electric field in perfect metal. $\vec{E}_1$ is determined by an image charge $q_2 = -\frac{q}{\varepsilon_1}$ and a real charge $\frac{q}{\varepsilon_1}$. (1 point)

(b)

(i) $\vec{D}_2 = \varepsilon_2\vec{E}_2 - \beta\vec{B}_2$, $\vec{D}_1 = \varepsilon_1\vec{E}_1$, $\vec{H}_2 = \frac{\vec{B}_2}{\mu_2} + \beta\vec{E}_2$, $\vec{H}_1 = \frac{\vec{B}_1}{\mu_1}$.

$\vec{E}_1 = \frac{x\vec{x}_0 + y\vec{y}_0 + (z-d)\vec{z}_0}{\left[x^2+y^2+(z-d)^2\right]^{3/2}}\frac{q}{\varepsilon_1} + \frac{x\vec{x}_0 + y\vec{y}_0 + (z+d)\vec{z}_0}{\left[x^2+y^2+(z+d)^2\right]^{3/2}} q_2, z > 0.$ (1 point)

$$\vec{E}_2 = \frac{x\vec{x}_0 + y\vec{y}_0 + (z-d)\vec{z}_0}{\left[x^2+y^2+(z-d)^2\right]^{3/2}}\left(\frac{q}{\varepsilon_1}+q_1\right),\ z<0.\ \text{(1 point)}$$

$$\vec{B}_1 = \frac{x\vec{x}_0 + y\vec{y}_0 + (z+d)\vec{z}_0}{\left[x^2+y^2+(z+d)^2\right]^{3/2}} g_2,\ z>0.\ \text{(1 point)}$$

$$\vec{B}_2 = \frac{x\vec{x}_0 + y\vec{y}_0 + (z-d)\vec{z}_0}{\left[x^2+y^2+(z-d)^2\right]^{3/2}} g_1.\ z<0.\ \text{(1 point)}$$

At z = 0, apply the boundary conditions.

(i) $\vec{B}_1^{\perp} = \vec{B}_2^{\perp}$

$$\vec{B}_1^{\perp} = \frac{g_2 d\vec{z}_0}{\left[x^2+y^2+d^2\right]^{3/2}},\ \vec{B}_2^{\perp} = \frac{-g_1 d\vec{z}_0}{\left[x^2+y^2+d^2\right]^{3/2}}.\ \Rightarrow\ g_1 = -g_2 \qquad (1).\ \text{(1 point)}$$

(ii) $\vec{D}_1^{\perp} = \vec{D}_2^{\perp}$

$$\vec{D}_1^{\perp} = \frac{-q d\vec{z}_0}{\left[x^2+y^2+d^2\right]^{3/2}} + \frac{\varepsilon_1 q_2 d\vec{z}_0}{\left[x^2+y^2+d^2\right]^{3/2}},$$

$$\vec{D}_2^{\perp} = \frac{-\left(\dfrac{q}{\varepsilon_1}+q_1\right)\varepsilon_2 d\vec{z}_0}{\left[x^2+y^2+d^2\right]^{3/2}} + \frac{\beta g_1 d\vec{z}_0}{\left[x^2+y^2+d^2\right]^{3/2}}.$$

So $\varepsilon_1 q_2 - q = \beta g_1 - \varepsilon_2\left(\dfrac{q}{\varepsilon_1}+q_1\right)$ (2). (1 point)

(iii) $\vec{E}_1^{//} = \vec{E}_2^{//}$

$$\vec{E}_1^{//} = \frac{x\vec{x}_0 + y\vec{y}_0}{\left[x^2+y^2+d^2\right]^{3/2}}\frac{q}{\varepsilon_1} + \frac{x\vec{x}_0 + y\vec{y}_0}{\left[x^2+y^2+d^2\right]^{3/2}} q_2,$$

$$\vec{E}_2^{//} = \frac{x\vec{x}_0 + y\vec{y}_0}{\left[x^2+y^2+d^2\right]^{3/2}}\left(\frac{q}{\varepsilon_1}+q_1\right).$$

So $q_2 = q_1$ (3) (1 point)

(iv) $\vec{H}_1^{//} = \vec{H}_2^{//}$

$$\vec{H}_1^{//} = \frac{\vec{B}_1^{//}}{\mu_1} = \frac{x\vec{x}_0 + y\vec{y}_0}{\left[x^2+y^2+d^2\right]^{3/2}}\frac{g_2}{\mu_1},$$

$$\vec{H}_2^{//} = \frac{\vec{B}_2^{//}}{\mu_2} + \beta\vec{E}_2^{//} = \frac{x\vec{x}_0 + y\vec{y}_0}{\left[x^2+y^2+d^2\right]^{3/2}}\frac{g_1}{\mu_2} + \frac{x\vec{x}_0 + y\vec{y}_0}{\left[x^2+y^2+d^2\right]^{3/2}}\beta\left(\frac{q}{\varepsilon_1}+q_1\right).$$

So $\dfrac{g_2}{\mu_1} = \dfrac{g_1}{\mu_2} + \beta\left(\dfrac{q}{\varepsilon_1}+q_1\right)$ (4). (1 point)

Putting (1) – (3) into (4), we finally get

$$q_1 = q_2 = \left(\frac{(\varepsilon_1-\varepsilon_2)(1/\mu_1+1/\mu_2)-\beta^2}{(\varepsilon_1+\varepsilon_2)(1/\mu_1+1/\mu_2)+\beta^2}\right)\left(\frac{q}{\varepsilon_1}\right),\ \text{(1 point)}$$

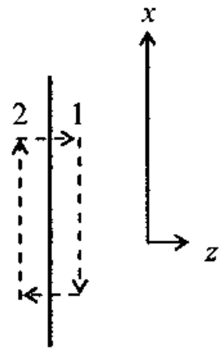
$$g_2 = -g_1 = \left(\frac{\beta}{(\varepsilon_1+\varepsilon_2)(1/\mu_1+1/\mu_2)+\beta^2}\right)\left(\frac{q}{\varepsilon_1}\right).\ \text{(1 point)}$$

In the unit system used here, the Gauss's Law become $\nabla \cdot \vec{E} = 4\pi\rho$, where ρ is the total charge density.

The electric charge sheet density is

$$\sigma = 4\pi(E_1^{\perp} - E_2^{\perp}) = 4\pi\left(\frac{d}{\left[x^2+y^2+d^2\right]^{3/2}}(q_2 - \frac{q}{\varepsilon_1} + \frac{q}{\varepsilon_1} + q_1)\right) = \frac{8\pi d q_1}{\left[x^2+y^2+d^2\right]^{3/2}}. \text{ (1 point)}$$

Likewise, the Ampere's Law now is $\nabla \times \vec{B} = 4\pi\vec{J}$, where $\vec{J}$ is the electric current density. Do not deduct any points if μ_0 is used instead of 4π. Choose a point on the X-axis, and take a loop with length L and nearly zero height.



$\vec{B}_1^{//} = \frac{g_2}{x^2}\vec{x}_0 = -\frac{g_1}{x^2}\vec{x}_0$, $\vec{B}_2^{//} = \frac{g_1}{x^2}\vec{x}_0$. For a loop perpendicular to the X-axis and along the Y-axis (pointing out of the paper plane), the path integral

$\oint_l \vec{B} \cdot d\vec{l} = \vec{B}_1^{//} \cdot L\vec{y}_0 - \vec{B}_2^{//} \cdot L\vec{y}_0 = 0 - 0 = 0$. (1 point)

For a loop parallel to the X-axis as shown in the figure,

$\oint_l \vec{B} \cdot d\vec{l} = -\vec{B}_1^{//} \cdot L\vec{x}_0 + \vec{B}_2^{//} \cdot L\vec{x}_0 = \frac{Lg_1}{x^2} + \frac{Lg_1}{x^2} = \frac{2Lg_1}{x^2}$.

Le the surface current density be K, then $\frac{2Lg_1}{x^2} = \oint_l \vec{B} \cdot d\vec{l} = 4\pi KL$. (1 point)

Using the right hand rule, we have $\vec{K} = -\frac{g_1}{2\pi x^2}\vec{y}_0$, i. e., pointing into the paper plane. In general, we have $\vec{K} = \frac{g_1}{2\pi}\frac{(y\vec{x}_0 - x\vec{y}_0)}{(x^2+y^2)^{3/2}}$. (1 point)

Any other ways to reach the above expression for the current density are fine.