Pan Pearl River Delta Physics Olympiad 2012
2012 年泛珠三角及中华名校物理奥林匹克邀请赛
Part-1 (Total 6 Problems) 卷-1（共 6 题）
(9:00 am – 12:00 pm, 02-02-2012)

Q1 (5 points) 题 1 （5 分）

It is found that the speed v of the neutrinos with 10 MeV of total energy is $(1-v/c)<2\times10^{-9}$. Estimate the mass of the neutrinos in terms of eV, and determine whether the value you find is an upper or lower limit.
总能量为 10MeV 的微中子的速度 v 为 $(1-v/c)<2\times10^{-9}$。估算以电子伏特为单位的微中子质量，并决定你计算得到的值是上限还是下限。

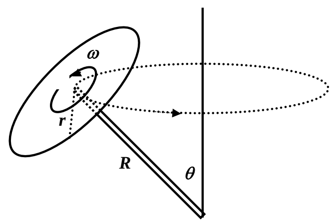
Q2 (10 points) 题 2 （10 分）

Usually we only consider the motion of a simple pendulum of length L in one dimension, while in fact the point mass can move in the horizontal plane, *i. e.*, with two degrees of freedom. Find an initial condition for the point mass such that its simple harmonic motion trajectory in the horizontal plane is (a) a straight line of length D; (b) a circle of radius R; and (c) an ellipse with long axis a and short axis b. All the length scales of the motion are much smaller than L.
通常我们只考虑长度为 L 的单摆一维的运动，但是实际上质点是可以在水平面上运动的，也就是说，有两个自由度。现在水平面上，所有运动的长度量级都远小于 L 的条件下，给出质点的初始速度和位置，使它的简谐运动轨迹是(a)长度为 D 的直线；(b)半径为 R 的圆形；(c)长轴为 a、短轴为 b 的椭圆。

Q3 (9 points) 题 3 （9 分）

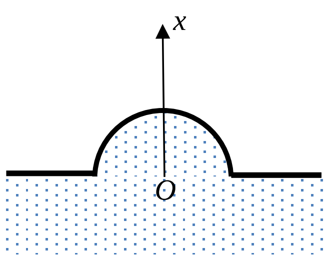
A satellite of mass m is revolving around Earth on a circular orbit of radius R with angular frequency ω_0. It is then hit by a meteoroid with a small impulse I in the inward radial direction. Determine the motion of the satellite afterwards in terms of the position as function of time.
质量为 m 的卫星原来围绕着地球运动，轨道半径为 R，角频率为 ω_0。一颗具有很小冲量 I 的流星沿向内的径向方向撞击了卫星。求此后卫星的位置与时间的关系。

Q4 (6 points) 题 4 （6 分）



As shown in the figure, a gyroscope consists of a uniform disk of radius r and an axle of length R through its center and along its axis. The other end of the axle is hinged on a table but is otherwise free to rotate in any direction. The mass of the disk is much larger than that of the axle. The gyroscope is spinning with angular velocity ω with the axle inclined to the vertical direction. Let g be the gravitational acceleration. Find its angular velocity of precession.
如图所示，一陀螺仪由半径为 r 的均匀盘和长度为 R 的转轴组成，转轴的质量远小于均匀盘。转轴的另外一端铰合在桌上，但是转轴可以在任何方向上自由转动。陀螺仪以角速度 ω 转动，转轴倾斜于垂直方向。令 g 为重力加速度。求陀螺仪进动的角速度。

Q5 (10 points) 题 5 （10 分）

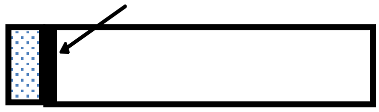


A point charge q is at $x_0 = 3R/2$ on the X-axis in front of a grounded conductor hemisphere of radius R on a large conductor plate perpendicular to the X-axis and in the Y-Z plane. The center of the hemisphere is at (0, 0, 0). Find the potential energy of the point charge. (Note: You must verify that the boundary conditions are preserved if you use image charge(s).)
垂直于 X 轴并在 Y-Z 平面的大导体平板上有一半径为 R 的接地导体半球。

半球的中心位于(0, 0, 0)。在半球前 X 轴上$x_0 = 3R/2$处有电荷量为 q 的点电荷。求点电荷的势能。（注意：如果运用镜像电荷，则必须证明满足边界条件。）

Q6 (10 points) 题 6 （10 分）

As shown, a thermally insulated smooth container with a locked heavy piston contains n moles of single atom ideal gas at temperature T_0 in the left chamber and vacuum in the right chamber. The piston is then released and eventually sticks to the right wall of the container. A η portion of the kinetic energy of the piston is eventually absorbed as heat by the gas. The volume of the whole container is κ times the original volume of the gas.



(a) Find the kinetic energy of the piston right before it hits the wall, and verify your answer for $\kappa \to \infty$. (3 points)

(b) Find the change of entropy of the gas, and proof that the change is positive. (6 points)

(c) Verify your answer in (b) for the case $\eta = 1$. (1 point)

如图所示，一个绝热的光滑容器带有一个被锁定的重活塞。该容器左边的腔室内有 n 摩尔的单原子理想气体，气体温度为T_0，右边腔室则为真空。然后，活塞被释放，撞击容器右壁后与之相连。最终，活塞动能中的某部分能量（假设该值为η）被气体以热能的方式吸收。整个容器的体积是原来气体体积的κ倍。

(a) 求在活塞撞击右壁之前瞬间活塞的动能，并用$\kappa \to \infty$验证你的答案。（3 分）

(b) 求气体熵的变化，并证明该变化为正值。（6 分）

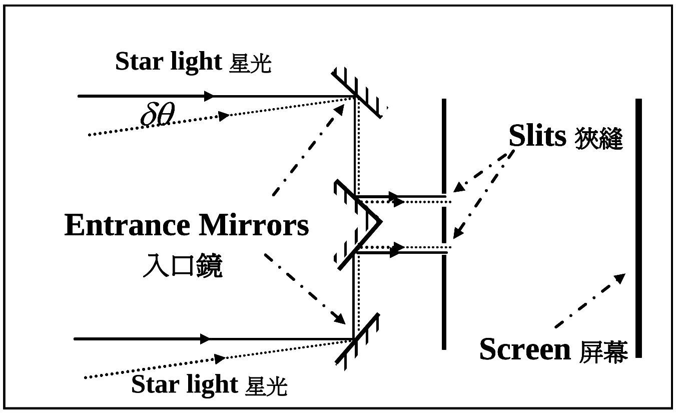
(c) 用$\eta = 1$验证你(b)中的答案。（1 分）

《THE END 完》

Pan Pearl River Delta Physics Olympiad 2012
2012 年泛珠三角及中华名校物理奥林匹克邀请赛
Part-2 (Total 3 Problems) 卷-2（共 3 题）
(2:30 pm – 5:30 pm, 02-02-2011)

Q1 Stella Interferometer (8 points) 题 1 恒星干涉仪（8 分）

A Stella Interferometer is used to accurately measure the angular separation between two close-by stars. As shown, the light from the stars can be treated as two broad parallel light waves of 5.0×10^{-7} meters in wavelength; one (wave-1) is at normal incidence and the other (wave-2) is off by a small angle $\delta\theta = 3.0\times10^{-6}$ degrees. Each wave is then split into two by the two entrance mirrors. The distance between the mirrors is *D*. There is no phase difference between the two waves split from wave-1 at the entrances. The waves at the entrances are then brought to the two narrow slits of a Young's interference experiment without introducing additional path difference. The entrance mirrors are moved slowly to increase the distance *D* until the fringes on the screen disappear.



Stella Interferometer 星光干涉儀

(a) Find the value of *D*. (6 points)
(b) If an optical telescope is used to observe the two stars, what should be the minimum diameter of the primary lens or mirror? (2 points)

恒星干涉仪可用来精确地测量双星之间微小的角距离。如图所示，从两颗恒星发射出来的光可当做两束宽大的平行光，其波长为 5.0×10^{-7} 米；光束-1 正入射，而光束-2 偏移一个微小的角度 $\delta\theta = 3.0\times10^{-6}$ 度入射。随后，每束光被两个入口镜分裂成两束光。镜子间的距离为 *D*。在入口处，光束-1 分裂出的两束光之间没有相位差。光束随后被引入到杨氏干涉实验的狭缝上，并且在此过程中没有引入任何另外的光程差。慢慢移动入口镜，增加彼此之间的距离 *D* 直到屏幕上的条纹消失。

(a) 求出 *D* 的值。（6 分）
(b) 如果用光学望远镜观测这两颗恒星，初级镜的最小直径应该是多少？（2 分）

Q2 Y-Meson (12 points) 题 2 Y 介子（12 分）

Y-mesons with rest mass $m_Y = 1.058\times10^{10}$ eV are produced by colliding electrons with positrons head-on in the reaction $e^+ + e^- \to Y$. Each Y-meson will decay immediately into a pair of B-mesons: $Y \to B^+ + B^-$. The rest mass of the B-mesons is $m_B = 5.28\times10^9$ eV and their lifetime is $\tau_0 = 1.5\times10^{-12}$ seconds. The rest mass of electrons and positrons is 5.11×10^5 eV. When the momentum of the electrons is the same as the positrons in the laboratory frame, the Y-mesons are at rest. ($c = 3.0\times10^8 m/s$)

(a) How large is the decay length (the distance it travels) of the B-mesons in the laboratory? (2 points)
(b) Assume that the B-mesons are moving along the electron-positron trajectory. To increase the decay length of half of the B-mesons, the Y-mesons need to be given momentum in the laboratory frame. This is done by colliding electrons with positrons with different energies. What momentum (in the unit of eV/*c*) should the B-mesons have if they will have decay length of 0.20 mm? (3 points)
(c) What is the total energy of the Y-mesons before they decay? (4 points)

(d) What should be the energies of the electrons and the positrons in order to produce the Y-mesons in (c)? (3 points)

通过电子和正电子对撞，即反应$e^+ + e^- \to Y$，可以产生静质量为1.058×10^{10} eV 的 Y 介子，而 Y 介子会立刻衰变成一对 B 介子，即反应$Y \to B^+ + B^-$。B 介子的静质量为5.28×10^9 eV，他们的寿命为$\tau_0 = 1.5\times10^{-12}$秒。电子和正电子的静质量为$5.11\times10^5$ eV。在实验室中，当电子的动量与正电子相一致时，Y 介子处于静止状态。（$c = 3.0\times10^8 m/s$）

(a) 在实验室中，B 介子的衰变长度（即走过的距离）是多少？（2 分）

(b) 设 B 介子只沿着电子-正电子的轨迹运动，为了增加一半 B 介子的衰变长度，需要给 Y 介子动量。这可以通过用不同能量的正电子与电子碰撞来实现。如果希望 B 介子的衰变长度为 0.20 mm，那么 B 介子的动量应该为多少（以 eV/*c* 为单位）？（3 分）

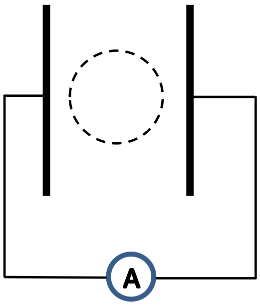
(c) 在衰变前，Y 介子的总能量是多少？（4 分）

(d) 为了得到(c)中的 Y 介子，电子和正电子的能量分别应为多少？（3 分）

Q3 Penning Trap (30 points) 潘宁势井 （30 分）

(a) Consider an ion (mass m and charge q) with initial velocity v in the X-Y plane in a uniform magnetic field B along the Z-axis. Find its angular frequency ω_c (cyclotron frequency), and the kinetic energy in terms of ω_c and the orbital radius r_0. (1 point)

(b) Consider a point charge q between two grounded conductor plates perpendicular to the X-axis separated by a distance D. It can be shown that the induced charge on plate-1 and plate-2 are $Q_1 = q\left(\frac{x}{D} - 1\right)$ and $Q_2 = -q\frac{x}{D}$, respectively, where x is the distance between the charge and plate-1. Referring to the figure, find the electric current in the circuit if the center of the ion orbit in (a) is at $x = D/2$. To obtain larger current, should the orbital radius be bigger or smaller? (2 points)



(c) An AC electric field $E(t) = E_0 \cos(\omega_c t)$ is applied across plate-1 and plate-2. Suppose the orbit of the ion remains nearly circular for each revolution and the energy gained from the AC electric field in each revolution is much smaller than the kinetic energy of the ion. The charges induced by the ion on the plates can be ignored. After time T which is much longer than the orbital period, find the radius of the orbit R. (3 points)

(d) The AC electric field in (c) is then turned off when $2R$ is still smaller than D. So far the ion is free to move along the z-direction, which means that it can escape from the uniform magnetic field region if it has initial velocity along the z-axis. To prevent that, another electric potential field in the form of $V(\vec{r}) = V_0(z^2 + \beta x^2 + \beta y^2)/z_0^2$, where $V_0 > 0$, is applied so that in the Z-direction the ion can only oscillate around $z = 0$. Find the oscillation frequency ω_z. (2 points)

(e) For the potential field in (d) to be valid in the region of empty space, find the constant β in (d). (1 point)

Such a combination of electric and magnetic field is called a Penning Trap. It is a device to trap an ion for a long time so its cyclotron frequency ω_c, and therefore its q/m ratio, can be measured with very high precision. Use ω_c and ω_z as known quantities for the remaining part of the question.

(f) The ion is now in a Penning Trap as above. Derive the differential equations for the position of the ion $x(t)$ and $y(t)$ in the X-Y plane. (4 points)

(g) Let $u(t) \equiv x(t) + iy(t)$, where $i \equiv \sqrt{-1}$. Find the differential equation for $u(t)$. (2 points)

(h) Try solution $u(t) = Ae^{-i\omega t}$, determine the two possible frequencies ω_+ and ω_-, with $\omega_+ > \omega_-$. (2 points)

(i) The general solution is then $u(t) = A_+ e^{-i\omega_+ t} + A_- e^{-i\omega_- t}$. Suppose the electric trap potential is turned on after the magnetic field is on for a while. When the trap is turned on at $t = 0$ the ion is at $x = R$ on the X-axis, and the center of its orbit is at the origin of the X-Y plane. Determine A_+ and A_-, and take the approximation that $\omega_c >> \omega_z$. (5 points)

(j) To see what the trajectory of the ion looks like in (i), let us go to a rotating reference frame with angular frequency Ω. Using the definition of $u(t)$ in (g), find its expression $\tilde{u}(t)$ in the rotating frame. (3 points)

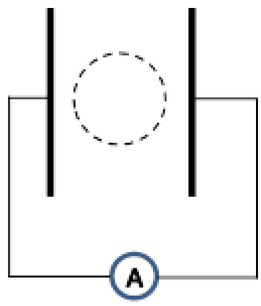
(k) Apply your answer in (j) to the answer in (i), and let $\Omega = \omega_-$. Draw a schematic diagram of the ion trajectory in the rotating frame on the X-Y plane. (1 point)

(l) Draw a schematic diagram of the ion trajectory on the X-Y plane in the laboratory frame. (1 point)

(m) Give three possible shapes of the ion trajectory on the X-Y plane in a rotating frame with angular frequency $\omega_c/2$ if the initial conditions are appropriate. (3 points)

(a) 考虑一个在 X-Y 平面上、初速度为 v 的离子（质量为 m，电量为 q），沿着 Z 方向有均匀的外磁场 B。求它的角频率 ω_c（回旋频率），并且用 ω_c 和轨道半径 r_0 表达它的动能。（1 分）

(b) 假设现有两个垂直于 X 轴的接地导体板，板之间的距离为 D，之间放置了一电量为 q 的点电荷。可以证明，板-1 和板-2 上的感应电荷分别为 $Q_1 = q(\frac{x}{D} - 1)$ 和 $Q_2 = -q\frac{x}{D}$，其中 x 是点电荷到板-1 的距离。如图所示，如果(a)中离子的轨道中心在 $x = D/2$ 处，求电路中的电流。为了获得更大的电流，轨道半径应该变小还是变大？（2 分）



(c) 现在板-1 和板-2 之间加上交流电场 $E(t) = E_0 \cos(\omega_c t)$。假设在每次旋转中，离子的轨道还是保持接近于圆形，并且在每周期从交流电场获得的能量远小于离子的动能，同时忽略导体板上的感应电荷，求经过时间 T（远长于轨道周期）后轨道半径 R。（3 分）

(d) 在 $2R$ 大于 D 之前将(c)中的交流电场关掉。到目前为止，离子可以沿着 Z 轴方向自由运动，这就意味着，如果离子在 Z 方向上有初速度，那么它可以逃离均匀磁场区。为了防止此发生，需要再加一个电势场 $V(\vec{r}) = V_0(z^2 + \beta x^2 + \beta y^2)/z_0^2$，其中 $V_0 > 0$，使离子在 Z 方向上只能在 $z = 0$ 附近振动。求振动频率 ω_z。（2 分）

(e) 要使(d)中的电势场在真空中仍然有效，常数 β 应为何值？（1 分）

上述的电磁场组合称为潘宁势井。它可以长时间地捕获住离子，所以能够非常精确地测量出离子的回旋频率以及离子的 q/m 比值。
在解下面的问题时可把 ω_c 和 ω_z 当作是已知量。

(f) 现在，离子处于(e)中所描述的潘宁势井中。推导出离子在 X-Y 平面上位置 $x(t)$ 、$y(t)$ 所满足的微分方程。（4 分）

(g) 令 $u(t) \equiv x(t) + iy(t)$，其中 $i \equiv \sqrt{-1}$ 。导出 $u(t)$ 所满足的微分方程。（2 分）

(h) 尝试解 $u(t) = Ae^{-i\omega t}$，求出两个可能的频率 ω_+ 和 ω_-，其中 $\omega_+ > \omega_-$ 。（2 分）

(i) 令通解为 $u(t) = A_+ e^{-i\omega_+ t} + A_- e^{-i\omega_- t}$ 。假设在开启磁场后经过一段时间再加上电势场。在 $t = 0$ 时刻加上电势场时，离子处在 $x = R$ 处的 X 轴上，其轨道中心位置在 X-Y 平面的原点。计算 A_+ 和 A_-，并取近似 $\omega_c >> \omega_z$ 。（5 分）

(j) 为了方便确定(i)中离子运行轨迹的简图，假设我们处在一个角频率为 Ω 的旋转参考系中。利用(g)中 $u(t)$ 的定义，求在旋转参考系中它的表达式 $\tilde{u}(t)$ 。（3 分）

(k) 将你在(j)中得到的答案应用于(i)中，并令 $\Omega = \omega$。画出离子运动轨迹在旋转参考系中 X-Y 平面上的简图。（1 分）

(l) 画出离子在实验室参考系中 X-Y 平面上的运动轨迹简图。（1 分）

(m) 当角频率为 $\omega_c / 2$ 时，若初始条件合适，给出离子在旋转参考系中三种可能的 X-Y 平面上的运动轨迹。（3 分）

《THE END 完》