Pan Pearl River Delta Physics Olympiad 2013
2013 年泛珠三角及中华名校物理奥林匹克邀请赛
Sponsored by Institute for Advanced Study, HKUST
香港科技大学高等研究院赞助
Part-1 (Total 5 Problems) 卷-1（共 5 题）
(9:00 am – 12:00 pm, Feb. 15, 2013)

Q1 (9 points)

A bead of mass m and initial speed v_0 hits a uniform thin rod of mass m and length L perpendicularly at one end, which initially rests on a horizontal plane.

a) If the other end of the rod is fixed on a hinge which allows the rod to rotate freely in the horizontal plane, and the bead stays on the rod after collision, find the mechanical energy loss due to the collision.
b) If the rod is free to move on the plane and the bead stays on the rod after collision, find the mechanical energy loss due to the collision.
c) The rod is free to move on the plane. The collision is elastic. The velocity of the bead is perpendicular to the rod right after the collision. Find the angular speed of the rod, and the speeds of the bead and the center of mass of the rod.

第一题 (9 分)

一质量为 m 的小球以初速度 v_0 垂直撞击一个质量同为 m 长度为 L 的均匀细杆端点。细杆初始静止于水平面上。

a) 如果杆的另一端固定在一个光滑铰链上使它可以在水平面自由转动，碰撞后小球黏在杆上，计算碰撞所带来的动能损耗。
b) 如果杆可在平面自由运动，碰撞后小球黏在杆上，计算碰撞所带来的动能损耗。
c) 如果杆可在平面自由运动，而碰撞是弹性的。在碰撞后的瞬间，小球的速度垂直于杆。计算碰撞后杆的角速度、质心速度和小球的速度。

Q2 (10 points)

Consider a pair of stars, with masses m_1 and m_2, respectively, bound by their mutual gravity (binary system). Denote their positions as $\vec{r}_1(t)$ and $\vec{r}_2(t)$, respectively.

a) Write down the differential equations for $\vec{r}_1(t)$ and $\vec{r}_2(t)$.
b) Let $\vec{r}_c(t) = \dfrac{m_1\vec{r}_1(t) + m_2\vec{r}_2(t)}{m_1 + m_2}$, $\vec{r}(t) = \vec{r}_1(t) - \vec{r}_2(t)$, find the differential equations for $\vec{r}_c$ and $\vec{r}$.
c) Suppose initially $\vec{r}_c(t=0) = 0$, and $\dfrac{d\vec{r}_c}{dt}(t=0) = 0$, solve the equation in b) for $\vec{r}_c(t)$.
d) Assume that $|\vec{r}(t)| = a$ and a is a time-independent constant, find the solution in b) for $\vec{r}(t)$ with initial condition $\vec{r}(t=0) = a\vec{x}_0$.
e) The period of the orbital motion of a binary system, which is made of two neutron stars each having 1.0 solar-mass, is $T = 3.0\times10^4\,s$. Find the distance between the stars. ($G = 6.7 \times 10^{-11}$ N m^2/kg^2, Mass of the sun = 2.0×10^{30} kg,)

第二题 (10 分)

一对质量分别为 m_1 和 m_2 的星体由万有引力束缚在一起 (双星系统)。用 $\vec{r}_1(t)$ 和 $\vec{r}_2(t)$ 分别表示它们的位置。

a) 写出 $\vec{r}_1(t)$ 和 $\vec{r}_2(t)$ 所遵循的微分方程。

b) 定义 $\vec{r}_c(t)=\dfrac{m_1\vec{r}_1(t)+m_2\vec{r}_2(t)}{m_1+m_2}$， $\vec{r}(t)=\vec{r}_1(t)-\vec{r}_2(t)$，写出 $\vec{r}_c(t)$ 和 $\vec{r}(t)$ 遵循的微分方程。

c) 如果 $\vec{r}_c(t)$ 的初始状态为 $\vec{r}_c(t=0)=0$ 、 $\dfrac{d\vec{r}_c}{dt}(t=0)=0$，求 b)中微分方程的解 $\vec{r}_c(t)$ 。

d) 如果 $|\vec{r}(t)|=a$ 而且 a 是与时间无关的常数，求 b)中微分方程的解 $\vec{r}(t)$ 。其初始状态为 $\vec{r}(t=0)=a\vec{x}_0$ 。

e) 一个由一对中子星(质量各为 1.0 太阳质量)构成的双星系统的轨道周期是 $T=3.0\times10^4 s$ 。求两中子星之间的距离。(G = 6.7 × 10^{-11} N m^2/kg^2, 太阳质量 = 2.0 × 10^{30} kg)

Q3 (12 points)

A perfect conductor shell of radius R is charge neutral. Its center is at the origin of the X-Y coordinate. The answers for the following should be expressed in terms of L, R, Q, and a fundamental constant, where $L < R$.

a) Find the work done by electrostatics to move a point charge $q_1 = Q$ from the shell center to (L, 0).

b) Another point charge $q_2=\dfrac{L}{R}Q$ is fixed at ($\dfrac{R^2}{L}$, 0), find the work done by electrostatics to move the point charge q_1 from the shell center to (L, 0).

c) If charge q_1 is fixed at (L, 0), find the work done by electrostatics to move charge q_2 from (∞, 0) to ($\dfrac{R^2}{L}$, 0).

d) If charge q_1 is fixed at (L, 0), find the work done by electrostatics to move charge q_2 from $(0,\infty)$ to $(0,\dfrac{R^2}{L})$.

第三题(12 分)

一个中性的理想导体球壳半径为 R，中心位于坐标原点。在下述题目中结果须用 L 、 R 、 Q 和相关的基本常数表示。这里 $L<R$ 。

a) 计算把一个点电荷 $q_1=Q$ 从球壳中心移到 $(L,0)$ 的过程中静电力所做的功。

b) 如果另一个点电荷 $q_2=\dfrac{L}{R}Q$ 固定在 $(\dfrac{R^2}{L},0)$，计算把 q_1 从球壳中心移到 $(L,0)$ 的过程中静电力所做的功。

c) 如果固定 q_1 在 $(L,0)$，计算将点电荷 q_2 从 $(\infty,0)$ 移到 $(\dfrac{R^2}{L},0)$ 静电力所做的功。

d) 如果固定 q_1 在 $(L,0)$，计算将点电荷 q_2 从 $(0,\infty)$ 移到 $(0,\dfrac{R^2}{L})$ 静电力所做的功。

Q4 (9 points)

Consider a faraway object with speed v and distance L from us. At one time, it is seen by us at certain position in the sky. After some time Δt, it is seen to have moved by a small angular displacement $\Delta\alpha$. Visually, the lateral distance covered by the object is $L\cdot\Delta\alpha$, and its lateral visual speed is $v_{visual} = \dfrac{L\cdot\Delta\alpha}{\Delta t}$. The actual velocity of the object $\vec{v}$ is at an angle θ to the line-of-sight.

a) If we observe the object by detecting the light it emits, find v_{visual}. Verify if v_{visual} exceeds the speed of light c for $v = 0.9c$ and $\theta = 45^\circ$.

b) If we observe the object by the sound (sound speed is v_s) it emits, like a bat tracking a moth, find v_{visual}, and the range of the object speed such that the object seems to move in the opposite direction as in a).

c) If we observe the object by detecting the solid beads emitted from it, and the relative speed between the beads and the object is v_b. All speeds are much smaller than that of light. Find v_{visual}. Can the object seems to move in the opposite direction as in a)?

Answers should be expressed only in terms of θ and the relevant speeds v, c, v_s, or v_b.

第四题(9 分)

一个遥远的物体速度 v，离我们的距离 L。在某一时刻，我们观察到该物体位于天空中的某一点。过了时间 Δt 之后，我们观察到它移动了一个小小的角位移 $\Delta\alpha$。我们测到的物体横向移动距离为 $L\cdot\Delta\alpha$，而其视觉横向速度为 $v_{視覺} = \dfrac{L\cdot\Delta\alpha}{\Delta t}$。物体运动的真实速度与观察方向的夹角为 θ。

a) 如果以物体发出的光作为观察信号，求物体的 $v_{視覺}$，并判断当 $v = 0.9c$ 、$\theta = 45^\circ$ 时 $v_{視覺}$ 是否超过光速 c。

b) 如果以物体发出的声音(其速度为 v_s)作为观察信号，如同蝙蝠追踪飞蛾那样，求物体的视觉横向速度 $v_{視覺}$， 和视觉横向速度与 a) 的视觉运动方向相反时物体的真实速度范围。

c) 如果以物体发射出来的小球作为观察信号，小球与物体间的相对速度为 v_b，所有的速度都远小于光速，求物体的视觉横向速度 $v_{視覺}$。这时可以观测到物体与 a)中视觉运动方向相反的运动么？

答案须以 θ 和相应的速度 v 、c 、v_s 或者 v_b 表示。

Q5 (10 points)

Consider the process of pumping a flat tire. The atmospheric temperature is T_a. The atmospheric pressure is P_a. The initial pressure inside the tire is P_i, and the temperature inside is also T_a. After pumping, it reaches a maximum pressure $P_{\max}$ and temperature $T_{\max}$. The tire then cools slowly back to temperature T_a, and the pressure drops to the desired pressure P_f.

Note:

1. Air is ideal gas with heat capacity ratio γ.
2. The volume of the tire is a constant V_0.
3. The process of pumping is adiabatic.
4. Express your answer in terms of γ, V_0, P_i, T_a, P_a, P_f, R, and V_p for part-b.

(a) Pumping by a large compressor (4 points)

The pressure of the air inside the compressor is P_c. The temperature is T_a. Both remain the same during pumping. Find (i) the work done by the compressor, (ii) the internal energy gained by the air inside the tire (including the air pumped into the tire), and (iii) the minimum P_c required to obtain the final desired pressure P_f.

(b) Pumping by a small hand pump (6 points)

The volume of the pump is V_p. The air inside is initially at atmospheric pressure P_a. Because V_p is very small, a large number of strokes of the hand pump is required. Assume that in the j-th stroke, the pressure inside the tire is increased from P_{j-1} to P_j, where $P_0 = P_i$. Our model is that the air inside the pump is first compressed adiabatically so that its pressure increases from P_a to P_j. Then it is transferred into the tire under constant pressure P_j. (i) Find the number of strokes needed to fill up the tire. (ii) Find the internal energy gained by the gas after the j-th stroke in terms of P_j and the given parameters. (iii) By using integration as approximation of discrete sum, show that the pressure inside the tire just after pumping is

$$P_{max} = P_i\left[1+\left(\frac{P_a}{P_i}\right)^{\frac{1}{\gamma}}\left(\frac{P_f-P_i}{P_a}\right)\right]^{\gamma}$$

第五题(10分)

已知环境温度为T_a，大气压力为P_a，一个轮胎中的初始压力为P_i，温度还是T_a。刚打完气时轮胎最大压强为$P_{\max}$，这时的温度为$T_{\max}$，之后慢慢冷却到T_a，压强则下降到最终的目标压强P_f。

条件：

1. 空气是理想气体，其热容比为γ。
2. 轮胎的体积为常数V_0。
3. 打气过程是绝热过程。
4. 答案用γ、V_0、P_i、T_a、P_a、P_c、P_f、R 和 V_p (b-部分) 表示。

   a) 用一个大型空气压缩机打气。(4分)

   设压缩机中的气压P_c在打气的过程中维持不变，其温度为T_a。求(i)完成打气压缩机须做的功，(ii)轮胎中空气(包括被打入轮胎的空气)内能的增加，(iii)为了得到最终压强P_f所需要的最小P_c。

   b) 用小型手动打气筒打气。(6分)

   打气筒的体积为V_p，其中气体的初始压强为P_a。因为V_p很小，因此需要很多次打气冲程。假设在第j次冲程，轮胎中的压强从P_{j-1}增加到P_j， $P_0 = P_i$。这里我们假设打气过程如下：空气在打气筒中先绝热压缩使得压强由P_a增加到P_j，而后在恒定压强P_j下输进轮胎。 (i)求冲程总数。(ii)求第j次冲程后胎内空气内能的增加(用P_j和已知量表达)。(iii)用积分代替离散求和，证明刚打完气时轮胎的压强$P_{\max}$为

$$P_{max} = P_i\left[1+\left(\frac{P_a}{P_i}\right)^{\frac{1}{\gamma}}\left(\frac{P_f-P_i}{P_a}\right)\right]^{\gamma}$$

《THE END 完》

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香港科技大学高等研究院赞助
Part-2 (Total 2 Problems) 卷-2（共 2 题）
(2:00 pm – 5:00 pm, Feb. 15, 2013)

Q1 Inertia Inductance and Negative Refraction Medium 惯性电感与负折射介质

A conductor cylinder has certain electromagnetic inductance due to Faraday Effect. Its exact calculation is quite challenging in mathematics. Its order of magnitude, however, is the same as the model in a).
由于法拉第电磁感应效应，一个圆柱形导体是有一定电感的。该电感的严格求解在数学上相当困难。但电感的数量级可以用以下模型来计算。

a) Consider an infinitely long cylinder of radius *r*. The electric current is uniformly distributed in the cross section perpendicular to the cylinder axis. On the surface of the cylinder there is a thin shell of conduct insulated from the cylinder that carries the current in the opposite direction. Find the inductance per length. (1 point)
一个无限长的圆柱导体，其半径为 *r*。电流在导体内部均匀分布，并沿垂直于导体横截面的方向流动。在导体的表面有一层很薄的与圆柱导体绝缘的导体壳，里面的电流沿反方向流动。求导体单位长度的电感。（1 分）

For the rest of the problem there is no shell on the cylinder as in a).
In a real conductor electrons (mass *m*) driven by external electric fields are constantly colliding with defects and impurities. The average effect of such collisions is like a viscosity force $\vec{f}_c = -m\vec{v}/\tau$, where τ is a constant parameter called collision time.

<u>解以下题目时不用考虑薄导体壳。</u>

实际导体内的电子(质量 *m*)在外电场作用下运动，并不断与导体中的缺陷、杂质碰撞。碰撞的平均效果相当于粘滞阻力 $\vec{f}_c = -m\vec{v}/\tau$，其中 τ 为常数，称作碰撞时间。

b) Write down the differential equation for the velocity $\vec{v}(t)$ of an electron in the conductor in an external electric field $\vec{E}(t)$. Ignore electron-electron interactions. (1 point)
给出导体中电子在外电场 $\vec{E}(t)$ 作用下运动的微分方程。电子间的相互作用可忽略。（1 分）

c) Suppose $\vec{E}(t) = \vec{E}_0 e^{i\omega t}$, find the velocity of the electron. (2 points)
若外加电场为 $\vec{E}(t) = \vec{E}_0 e^{i\omega t}$，求电子运动的速度。（2 分）

d) Suppose the electron number density in the conductor is *n*, find the electric current density *J*. (2 points)
导体中电子的数量密度为 *n*，求电流密度 *J*。（2 分）

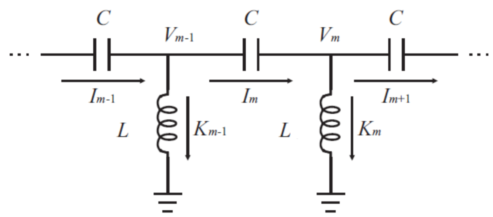
e) For a conductor cylinder (length *D* and radius *r*), find the relation between the voltage across the two ends and the electric current. The inductance in a) can be ignored for the time being. (2 points)
求一长为 *D*，半径为 *r* 的圆柱导体两端的电压与电流的关系。忽略 a)中所求的电感。（2 分）

f) From the answer in e), identify the term which is like a resistor and the one which is like an inductor L_I. L_I is called inertia inductance because it is not due to Faraday's law. (2 points)
在 e)中所得的答案中，找出相当于电阻、电感 L_I 的项。该电感 L_I 称为惯性电感，与法拉第定律无关。（2 分）

g) The resistance of a 1.0 μm long metal wire is 1.0 ohm and the collision time is $\tau = 2.0\times10^{-9}$ s at low temperature, find the value of L_I, and compare it with the inductance due to Faraday Effect. Which one is much bigger? ($\mu_0 = 4\pi\times10^{-7} N/A^2$) (2 points)

在低温下，长为 1.0 μm 的金属导线的电阻为 1.0 ohm，其电子碰撞时间为 $\tau = 2.0\times10^{-9}$ 秒。求惯性电感 L_I，并与电磁感应所形成的电感比较。哪个更大？($\mu_0 = 4\pi\times10^{-7} N/A^2$)。（2 分）

For a wave $A = A_0\cos(kx-\omega t)$, where $\omega = ck$, c is the wave speed. For conventional materials, c is almost a constant, so k increases with increasing ω. However, for some extraordinary artificial materials, c is a strong function of ω, so much so that k decreases with increasing ω. Such phenomenon is called Negative Refraction. It is like having a refraction index that is negative. Consider an infinitely long chain of inductor-capacitor (LC) as shown in the figure. The length of each LC segment is a.



考虑一波动 $A = A_0\cos(kx-\omega t)$，其中 $\omega = ck$，c 为波速。对于传统材料，c 为常数，故 k 随 ω 的增加而增加。但是，对于某些超常人工材料，c 可以是 ω 的函数，以至于在 ω 增加时，k 反而减小。该现象称作负折射。如图所示，考虑一个无限长的电感电容(LC)电路。每个 LC 单元的长度为 a。

h) Find the differential equation that relates $V_{m-1}(t)$, $V_m(t)$, and $V_{m+1}(t)$. (5 points)

给出 $V_{n-1}(t)$、$V_n(t)$、$V_{n+1}(t)$ 所满足的微分方程。（5 分）

i) Assume a wave-like solution $V_m(t) = V_0 e^{i(\omega t - mka)}$, find the relation between ω and k. (2 points)

设方程有波动形式的解 $V_m(t) = V_0 e^{i(\omega t - mka)}$，找出 ω 和 k 的关系。（2 分）

j) Does the answer in i) imply Negative Refraction? (1 point)

请问 i)中答案是否意味着负折射的存在？（1 分）

Q2 Gravitational Waves 引力波 (30 points)（30 分）

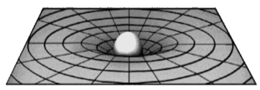
Gravitational waves (GW) are distortion, such as extension/compression, of space. The amplitude of a GW is given by $\varepsilon = \Delta L / L$, where L is the original length of space and ΔL is the change of length. So GW is like elastic waves in solids, except that here space is the medium for the waves.

The necessary constants for answering the following questions are given below.

引力波描述的是空间的扭曲变形(拉伸/压缩)。引力波的振幅为 $\varepsilon = \Delta L / L$，其中 L 是空间的原有长度，ΔL 是引力波所引起的长度变化。所以引力波类似于固体中的弹性波，只不过引力波是以空间为介质。求解以下题目时会用到以下物理常数：

Speed of light in vacuum 真空光速 $c = 3.0\times10^8 m/s$	Mass of the sun 太阳质量 $= 2.0\times10^{30}$ kg
Gravitational constant 引力常数, $G = 6.7\times10^{-11}$ N m^2/kg^2	Sun-Earth distance 太阳−地球距离 $= 1.5\times10^{11}$ m
Boltzmann constant 波茨曼常数 $= 1.4\times10^{-23}$ J/K	Mass of Earth 地球质量 $= 6.0\times10^{24}$ kg

- Generation of gravitational waves by a binary star system 双星系统产生的引力波



a) Shown in the figure is the space distortion by a heavy mass. Consider two stars bound by their mutual gravity on a circular orbit. Their masses are M_1 and M_2, respectively. If viewed sideway (in the orbital plane) from Earth, when the (visual) separation between two stars is largest/smallest, is the space between the stars stretched or compressed? (1 point)

图中所示为一重物对空间产生的扭曲。考虑两个由自身引力束缚并作圆周运动的星体 。它们的质量分别为 M_1 和 M_2。从侧面观察（沿轨道所在平面），可见双星的距离随时间变化，并有最大和最小值。问空间何时被拉伸，何时被压缩？（1 分）

b) Such space disturbance in a) is propagating through space as gravitational waves. Suppose the angular frequency of the orbital motion in a) is ω, and the radius of the orbit is R, then the GW emission power L_G of the binary is given by the expression $L_G = A^a B^b M^2 R^4 \omega^6$, where M is called the reduced mass, $M \equiv M_1 M_2 / (M_1 + M_2)$. The constant A is a fundamental physics constant related to Relativity, and constant B is a fundamental physics constant related to Gravity. Determine the two constants and their exponent powers a and b by using dimension analysis. (2 points)
双星对空间的扰动会以引力波的形式向外传播。已知轨道运动的角频率为 ω，轨道半径为 R，则引力波的辐射功率为 $L_G = A^a B^b M^2 R^4 \omega^6$，其中 $M \equiv M_1 M_2 / (M_1 + M_2)$ 是约化质量。常数 A 是与相对论相关的物理常数，B 为与引力相关的物理常数。用量纲分析找出 A、B 和指数 a、b。（2 分）

c) Suppose the motion and the mechanical energy of the binary can be described by classical mechanics, find the total mechanical energy of the orbital motion of the binary system in terms of their orbital period. (2 points)
用经典力学求双星运动的总机械能。答案用公转周期表示。（2 分）

d) Estimate the numerical value of the orbital period decrease rate $\frac{dT}{dt}$ due to gravitational wave emission of a pair of neutron stars, each having 1.0 solar-mass, with orbital period $T = 3.0 \times 10^4 s$. (4 points)
考虑一个由两个相同中子星组成的双星系统，一个中子星的质量等于太阳质量。双星公转周期为 $T = 3.0 \times 10^4 s$。因引力波辐射，其周期会以 $\frac{dT}{dt}$ 逐渐减小。估算 $\frac{dT}{dt}$ 的数值。（4 分）

e) The Sun-Earth can also be considered as a binary system. Find the time needed, in terms of years, for Earth orbital period to reduce by 1 second. (2 points)
太阳-地球同样可被视为双星系统。估算地球公转周期减小 1 秒所需的时间，以年作单位。（2 分）

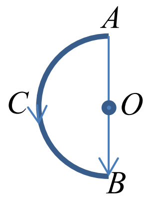
- Estimation of the emission power of a pair of black holes right before coalescence occurs
估算两个黑洞融合前的引力波辐射功率。

f) A black hole can be regarded as a point object of mass M. Find the distance R_s from the black hole where the escape velocity is equal to the speed of light c. (1 point)
黑洞可看作是质量为 M 的质点。距离该质点 R_s 的逃逸速度为光速 c。求 R_s。（1 分）

g) Estimate the numerical value of the emission power of a pair of identical black holes, each having 2 times the solar mass, right before coalescence occurs by using the following procedure: i) Start from the result in b), replace $R\omega$ by orbital speed v; ii) Take the distance between the two black holes as $2R_s$; iii) Replace M by R_s; iv) Take $v \approx c$. (3 points)
通过以下步骤，估算黑洞双星系统即将融合时的引力波辐射功率，一个黑洞的质量等于 2 倍太阳质量：i) 用 b)的结果，用轨道线速度 v 代替 $R\omega$；ii) 设两黑洞间距离为 $2R_s$； iii) 用 R_s 代替 M；iv)取 $v \approx c$。（3 分）

- Detection of gravitational waves 引力波的探测

h) Estimate the space elongation around a black hole by using the following procedure. As shown in the figure, consider a light beam from A to B. Without the black hole at



O the light will travel in a straight line AOB. With the black hole, the space is severely distorted and the light beam will travel along the semi-circle ACB. The distortion of the space ε is close to 0.1, 1, or 10? (2 points)
通过以下步骤，估算黑洞附近空间的拉伸。如图所示，考虑一束光从 A 传播至 B。若黑洞不存在，则光线沿直线 AOB 传播。若 O 点有一黑洞，则空间的扭曲使光线沿半圆 ACB 传播。空间的扭曲ε接近于 0.1, 1, 还是 10？（2 分）

i) Suppose the GW amplitude at the distance R_s from a black hole is the value you choose in h), given that the energy flux of GW is proportional to the square of amplitude, estimate the amplitude of G-wave at 10^4 light years away. (3 points)
假设引力波在距离黑洞 R_s 处的振幅为你在 h)中所选的值。已知引力波的能量通量与振幅的平方成正比，估算该引力波传播至 10^4 光年外的振幅。（3 分）

j) One type of GW detector is a straight uniform aluminum bar. As the bar is elongated by a GW by ΔL, such distortion remains in the bar for several thousand seconds as a standing sound wave, which can be detected by sensitive electronics. In the GW detection project AURIGA in Italy, the length of the bar is 3.0 meters and the speed of sound in aluminum is 6.4×10^3 m/s, determine the lowest resonant frequency of the bar. (2 points)
一根均匀铝棒可作为一种引力波探测器。铝棒被引力波拉伸变长ΔL，该扰动会以声波的驻波形式在铝棒内振荡数千秒钟。在意大利的 AURIGA 引力波探测项目中，铝棒的长度为 3.0 米，铝中声速为 6.4×10^3 m/s。求铝棒的最低共振频率。（2 分）

k) The length of the bar does not remain constant even without the GW because of the thermal fluctuation. The amplitude of the length fluctuation can be determined by treating the vibration of the bar as two half masses (1.1×10^3 kg) joint by a spring with the natural frequency equal to the resonant frequency of the bar in j). Find the average vibration amplitude at 4.2 K due to thermal fluctuation. (4 points)
由于热涨落，铝棒的长度即使在没有引力波的情况下仍然会改变。此时铝棒的运动可被看作两个质量相等的重物(1.1×10^3 kg)在一个弹簧连接下的振动，其频率等于 j)中所求得的共振频率。求温度在 4.2K 时，热涨落导致的铝棒长度涨落。（4 分）

l) Find the maximum distance within which AURIGA can detect a black hole coalescence event in g). (2 points)
用 AURIGA 探测 g)中两个黑洞的融合。求 AURIGA 所能探测的最大距离。（2 分）

m) At such distance in l), suppose one millionth (10^{-6}) of the energy released in the coalescence process in g) is in the form of electromagnetic (EM) waves instead of GW, compare the EM wave energy intensity Earth would be exposed to with that from the sun, which is 1.4×10^3 W/m^2. Are we in danger of global annihilation if this happens? (2 points)
根据 l)中所求得的距离，如果黑洞融合的能量中有百万分之一(10^{-6})是以电磁波的形式释放出来的，相比于地球上接受到太阳的电磁辐射（1.4×10^3 W/m^2），该电磁辐射是否会造成地球生物大灭绝？（2 分）

《THE END 完》